IB Physics Internal Assessment

Research question: How does the density of $100m^3$ salt water ($1.05g/m^3$,$1.10g/m^3$, $1.15g/m^3$,$1.20g/cm^3$,$1.25g/m^3$) affect the specific heat capacity?

Introduction

Saltwater is a common material we may frequently use in the kitchen. We use it to cook a variety of food. On the other hand, during cooking, we may find that we may use different times to heat up salt water with varying densities to boil, even with the same size of flame. In this experiment, I will investigate the relationship between the density of salt water and its specific heat capacity.

Background

1. In the physics lesson, we learned that the formula of heat energy is given by

$$Q = mc\Delta T$$

By rearranging the formula, we can obtain the relationship between the specific heat capacity and mass of salt water which is given in equation 1:

$$\Delta T = \frac{Q}{mc} \ (1)$$

2. Also, the formula of the density of a substance is given in equation 2:

$$\rho = \frac{m}{V} \ (2)$$

By rearranging this formula, we can obtain the formula of mass of salt water which is

$$m = \rho v$$

Table 1 below lists the symbols of the quantity and their units

Variable	Meaning	Unit
Q	The amount of thermal energy	J
m	The mass of the salt water	kg
c	The specific heat capacity	$J/kg\Delta T$
ΔT	Amount of temperature change	K
ρ	The density of salt water	g/cm^3
V	The volume of salt water	m^3

<u>Deriving the relationship between the density and specific heat capacity</u>

By inserting equation 2 into equation 1, we can obtain the theoretical relationship between the specific heat capacity of salt water and its density which is

$$\rho = \frac{Q}{vc\Delta T}$$

The specific heat capacity of salt water increases as its density decreases.

Methodology

Table 2 below lists the variables and how and why they are considered

Variables	**Ceuter**
Independent	The density of salt water in the beaker (2.5%,5%,10%,15%,20%). This will be produced by adding different maps of salt into 100g of water.
Dependent	The time that the solution of salt water needs to increase 10°C. This will be measured by measuring the amount of time that the different salt water solution required to reach a fixed temperature.
Control variables	**Why and how to control the variable**
The volume of water	According to the background, the density of salt water solution is determined by both the mass and amount of salt. The water with different masses may affect the time required for the salt water to reach the fixed temperature and hence reduce the precision of the result. It will be kept constant by using the same volume of 100m^3 of water. The salt in solution is so little and hence it can be seen as negligible.
The thickness of the beaker	The beaker will affect the time that it takes for the saltwater to reach the final temperature as the beaker with varying thickness may have a different rate to conduct heat energy and may therefore affect the precision of the result. It will be kept constant by using the same beaker throughout the experiment.
Surface area of beaker	The surface which is heated may not be covered totally which can increase the time required for the salt water to reach the final temperature. This may affect the accuracy of the result. It will be kept constant by using the beaker so that its surface area can all be covered.
Rate of heating	The rate of heating may directly affect the time which the salt water reach the final temperature, may affect the precision of the result. It will be kept constant by using the same boiler and heated the initial water at a same rate.
The starting and final temperature of water	The difference of final temperature may affect the time that the time that the salt water reaches the final

	temperature. This may affect the accuracy of the experiment. It will be kept constant by preheating the salt water at the same temperature and measured by a temperature probe.

In addition, some methodological strategies will be applied to the incoming procedure. First, before the experiment, the water will be preheated at the same temperature which is similar to the room temperature, as it can reduce the temperature loss and reduce systematic error, which makes the result more accurate. Second, a dry cap may be put on the water to prevent some water droplets from dropping inside and increasing the mass of water which causes a systematic error. It increases the accuracy of the experiment.

Apparatus

Table 3 lists the materials I will use in this investigation and their uncertainties

Apparatus	Quantity	Uncertainty
Electrical balance	1	$0.01g$
Measuring cylinder	1	$0.000001m^3$
Beaker	1	$0.05m^3$
Heater	2	0.1
Temperature probe	1	0.1
Water	/	/
Salt	/	/
Paper towel	/	/
Glass rod	1	/
Stopwatch	1	/

Procedure

1. Rinse the beaker with water. Dry and clean it thoroughly.
2. Measure $0.0001m^3$ water by the measuring cylinder and place it into the beaker.
3. Measure 5g salt by the electrical balance and pour it into the water.
4. Stir the water with a glass rod to ensure that the salt is completely dissolved.
5. Place the temperature probe into the water and turn it on.
6. Preheat the beaker to 30℃ (303K).
7. Heat the water again and start the stopwatch.
8. Stop the heater when the water reaches 40℃ (323K), stop the stop watch, and record the time.
9. Wait until the temperature falls to 30℃, and repeat steps 6-8 for three times.
10. Repeat steps 1-9 for the same amount of water with 10g, 15g, 20g, and 25g salt.

Safety, environmental, and ethnic issues

The heater in the experiment may cause burns or cause water to flow out. Therefore safety glasses and a pair of gloves must be worn during the experiment.

Raw data

Qualitative data-what is observed when the concentration of salt increased.

When salt is added to the water, there isn't much change in the color of the water. As the concentration of salt increases, the time which the solution needs to increase 10℃ increases.

Quantitative data-Raw data table

Table 4 below lists the recorded data of the time that the solution required to increase 10*K*.

Table 4: mass of salt and measured time to increase 10*K*

Trial #	Mass of salt to be dissolved into the water (g±0.1g)	Time taken to increase 10℃ (±0.01℃)
1	0g	60.46
2		62.41
3		61.79
1	5.0g	61.42
2		57.35
3		60.27
1	10.0g	58.09
2		57.03
3		56.63
1	15.0g	54.76
2		54.97
3		54.81
1	20.0g	51.15
2		52.90
3		52.14
1	25.0g	50.70
2		49.15
3		48.67

Sample calculation

In my investigation, density is defined as

$$\frac{mass\ of\ salt}{volume\ of\ water}$$

A sample calculation based on a mass of 5g salt is shown below:

$$Density\ (\rho) = \frac{mass\ of\ solution\ (kg)}{volume\ of\ \ solution\ (m^3)} = \frac{0.105}{0.0001} = 1050kg/m^3$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = \frac{0.00001}{0.105} + \frac{0.0000001}{0.0001} = 0.001kg/m^3$$

$$\frac{\Delta\rho}{\rho}\rho = 1050 * 0.001009 = 1.06kg/m^3$$

$$\Delta\rho = 1.06kg/m^3$$

As the uncertainty of density of the solution is small compared to the concentration of volume, the uncertainty of density will be ignored in the following parts of the investigation.

The average value of the time is calculated:

$$\Delta T = \frac{61.42 + 57.35 + 60.27}{3} = 59.62s$$

The uncertainty of the time is estimated using half range measurement:

$$\Delta(\Delta T) = \frac{Max(\Delta T) - Min(\Delta T)}{2} = \frac{61.42 - 57.35}{2} = 2s$$

By repeating the procedure upward, I get the data for other values. They are all summarized in table 5..

Density of solution, kg/m^3	Average time used (s)	Uncertainty of time used (s)
1000	62	2
1050	60	2
1100	57	1
1150	55	1
1200	53	1
1250	50	1

Table 5-Density of solution vs. Time required to increase 10°C

*Uncertainty of time used is kept as one significant figure. The average time used is adjusted according to the significant figures in its uncertainty.

In order to determine the rate of heat transfer, I use the time which the pure water requires to increase $10K$. The specific heat capacity of pure water is approximately $4200J/kg * K$.

I first calculate the total thermal energy the the pure water was transferred.

$$Q = mc\Delta T = 0.1 * 4200 * 10 = 4200J$$

Then, I calculate the rate of transferring

$$\frac{Q}{t} = \frac{4200J}{62s} = 67.742J/s$$

The uncertainty of rate of transferring is calculated as:

$$\frac{\frac{\Delta Q}{t}}{\frac{Q}{t}} = \frac{\Delta Q}{Q} + \frac{\Delta t}{t}$$

$$\frac{\Delta Q}{t} = \frac{Q}{t}\left(\frac{\Delta Q}{Q} + \frac{\Delta t}{t}\right) = 67.742 * \left(\frac{2}{62}\right) = 2.19J/s$$

$$\frac{Q}{t} = 68J/s \pm 2J/s$$

Then, by multiplying the average time of each trail required to increase 10K and the rate of transferring, I get the average total energy transferred in the solution of density 1.05g/cm^3:

$$Q = \frac{Q}{t}t = 67.742 * 60 = 4064.52J$$

The uncertainty of thermal energy transferred is calculate by:

$$\frac{\Delta Q}{Q} = \frac{\frac{\Delta Q}{t}}{\frac{Q}{t}} + \frac{\Delta t}{t}$$

$$\Delta Q = Q\left(\frac{\frac{\Delta Q}{t}}{\frac{Q}{t}} + \frac{\Delta t}{t}\right) = 4164.52 * \left(\frac{2}{68} + \frac{1}{60}\right) = 261J$$

Hence, we can obtain the average thermal energy transferred in the solution of

$1.05\mathrm{g/cm^3}$: $\mathrm{Q} = 4065\mathrm{J} \pm 261\mathrm{J}$

After we obtained the total energy transferred, we can determine the specific heat capacity of solution when density is $1.05\mathrm{g/cm^3}$:

$$c = \frac{Q}{m\Delta T} = \frac{4062}{0.105 * 10} = 3869 J/kg\ K$$

The uncertainty of the specific heat capacity is given by:

$$\frac{\Delta c}{c} = \frac{\Delta Q}{Q} + \frac{\Delta m}{m} + \frac{\Delta T}{T}$$

$$\Delta c = c * \left(\frac{\Delta Q}{Q} + \frac{\Delta m}{m} + \frac{\Delta T}{T}\right) = 3869 * \left(\frac{261}{3869} + \frac{0.00001}{0.105} + \frac{0.1}{10}\right) = 300 J/kg\ K$$

Consequently, I obtain the specific heat capacity of solution $1.05\mathrm{g/cm^3}$ with uncertainty: $\mathrm{c} = 4100\mathrm{J/kg\ K} \pm 300\ \mathrm{J/kg\ K}$

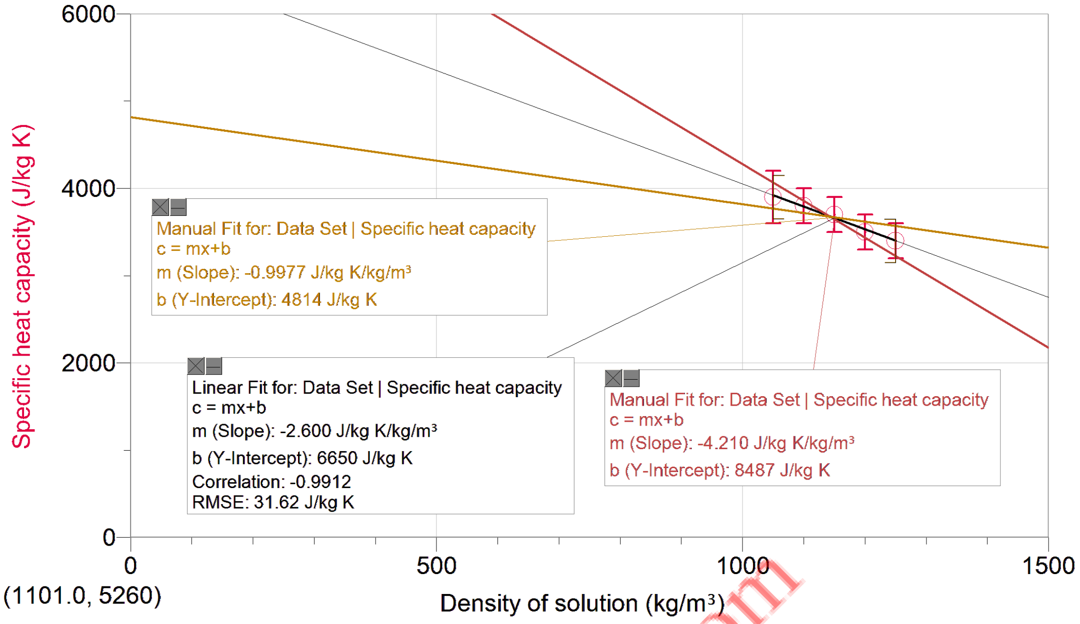
By repeating the procedure upward, I get the data for other values. They are all summarized in table 3.

Density of solution/ρ, $\mathrm{kg/m^3}$	Specific heat capacity/C, J/kg K	Uncertainty of specific heat capacity/ΔC, J/kg K
1000	4200	/
1050	3900	300
1100	3800	200
1150	3700	200
1200	3500	200
1250	3400	200

Table 6-Density of solution v.s. Specific heat capacity

*Uncertainty of specific heat capacity is kept as one significant figure. The specific heat capacity is adjusted according to the significant figures in its uncertainty.

The specific heat capacity is graphed against the density of solution. The best fit line is obtained by the software used. The 2 worst fit lines were obtained by determining the steepest and gentlest straight line which interests with all error bars.



Graph 1– Density of solution VS specific heat capacity of solution

The best fit line reveals the relationship:

$$c = -2.600kg/m^3 + 6650kg/m^3$$

The worst fit line reveals the relationship:

$$c = -4.210kg/m^3 + 8487kg/m^3$$

$$c = -0.998kg/m^3 + 4814kg/m^3$$

The uncertainty of the gradient is:

$$\frac{4.210 - 0.998}{2} = 1.606kg/m^3$$

The uncertainty of the y-intercept is:

$$\frac{8487 - 4814}{2} = 1837J/kg\ K$$

Therefore,

$$c = -(2.6 \pm 1.6)kg/m^3 + (6650J/kg\ K \pm 1837\ J/kg\ K)$$

The relationship reveals that the time that the salt water required to increase 10K has a inverse proportional linear relationship with the density of the solution.

Conclusion:

This investigation aims to calculate the effect of density $(\mathrm{k}g/m^3)$ of salt water on its specific heat capacity $(J/kg\ K)$ by performing experiments and also calculate the coefficient of total thermal energy transferred during the process of heating. Through the formula of density of specific heat capacity, it was hypothesized that there is a negative liner correlation between the density of salt water and specific heat capacity of it.

The proceeded experiment data was plotted and received a relation as $c = -(2.6 \pm 1.6s)kg/m^3 + (6650J/kg\ K \pm 1837\ J/kg\ K)$ And it indicated that when the density increase by $1\mathrm{k}g/m^3$, the specific heat capacity will increase by $2.6 \pm 1.6J/kg\ K$. The calculation supports the hypothesis of a negative linear correlation between the density of salt water and the specific heat capacity of it. This is because the diluted salt becomes ions, which hold a rigid cage of water molecules around them which are not allowed to move freely. These cages cause the reduction in the vibration degrees of freedom and result in the decrease in specific heat capacity.[1]
The slope the of correlation has a unit of $J/cm^3\ K$, which suggest that the $3600 \pm 2.06J$ is required to increase 1K per cm^3 volume of salt water. Using the slope with uncertainty, I can calculate the percentage uncertainty of the slope as:

$$\frac{1.6}{2.6} * 100\% = 62\%$$

The uncertainty of she slope is 62%, which is greater than 10% and indicates a big error bar of the data. This means that the experiment has a big random error, and suggests that my data is not precise.
Also, the negative linear relationship has an constant of $6650J/kg * K$, which suggests that it does not pass through the origin, indicates that there's a positive systematic error in my investigation.
The theoretical coefficient of thermal energy required to increase 1K per $m^3$of volume is $\frac{4.2J}{m^3}K$ [2], while the experimental thermal energy is$-\frac{(2.6\pm1.6)J}{cm^3}K$. The percentage error can hence be calculated by using the formula

$$Percentage\ error = \left|\frac{(literature\ value - \ experimental\ value)}{literature\ value}\right| * 100\%$$

Therefore, substituting the theoretical value and experimental value of the thermal energy required to increase 1K per cm^3 value, the percentage error is

$$Percentage\ error = \left|\frac{3.7 - 2.6}{3.7}\right| * 100\%$$

$$Percentage\ error = 30\%\ to\ 2\ significant\ figures$$

The percentage error was calculated based the an assumption that there's very little energy lost. The percentage error indicates a generally satisfactory accuracy.

Evaluation

Although the hypothesis is corroborated by the results of the theory of ion, which states that an increase in density of salt water will increase its specific heat capacity, the calculated value of thermal energy required to increase the temperature 1K per m^3 volume of salt water differs from the theoretical value, with a value of 15% percentage error together with experimental uncertainties. The possible reasons for this discrepancy may be the weakness throughout the experiment. At the same time, there are some strengths in the processes of experiments.

The experiment contains some systematic error. At the room temperature of the day I collected the data is around 291K, which is cooler than the beaker during heating. There will always be some thermal energy transferred to the surroundings and more thermal energy will be needed to increase the temperature by 10K, which may decrease the rate of transferring heat and therefore increase the specific heat capacity of solution measured, and increase the accuracy of experiment. This systematic error may be very difficult to avoid in the lab conditions as there's still a systematic error even if I've used a Styrofoam cup to reduce the error. However, I think the heat loss may be reduced further if I start the experiment at a temperature near the room temperature. For example, starting at 291K at the time when the room temperature is also 291K. This can reduce the energy transfer to the surroundings as there's no temperature difference at the beginning of the experiment. As a result, the convection at the start of the experiment can be avoided and reduce the heat lost to the surroundings. Consequently, the time required to increase the temperature can be reduced which is closer to the ideal value, and make the data more accurate.

Second, the controlled variable of the rate of heating may not be controlled perfectly. To be specific, when I heated the solution for the first time, the heater was kept at room temperature. However, when I heated the solutions on the following times, the surface temperature of the heater had increased above the surface temperature, which may result in thermal conduction between the beaker and the surface of the heater and two heating process occurred at the same time. Therefore, the rate of heating in the following times is faster than in the first time and the time require to increase 10K in the following times greater than the actual time. This increases the amount of thermal energy and the calculated specific heat capacity become greater. This error may be reduced if I use the heater after its surface cools off to room temperature. However, this may take too much time to do so. In order to deal with this problem, a beaker with ice water can be placed on the heater's surface after turning the heater off in order to cool if off in less time. Due to the temperature difference, there will be a thermal conduction between the surface of the heater and beaker, which can cool down the heater to room temperature quickly, therefore, reducing the systematic error of the heating process and increasing the accuracy of experiment.

Third, there may be some random errors in my experiment as I only repeat each procedure three times. Some random errors may occur and affect and precision of the result. In order to reduce these random errors, I should repeat each procedure for five times and calculate and the average value in order to make the result more precise.

Fourth, the range of the density may be too small and only be applied to a specific density between $1000kg/m^3$ and $1250kg/m^3$. In order to make my result applicable to a wider range of density, I should use liquid with varying densities such as $1300kg/m^3$ and $1400kg/m^3$, as it can make my exploration's result be applicable to a wider range of density.

In addition, I used a stop watch to measure the time in my experiment. I pressed the start button when the temperature is at 303K, and the pressed stop when the temperature is at 313K. This means that, due to my time of reaction, I may not press the button at the exact time when the temperature change. If I press the start button late, I may underestimate the time. Conversely, if I press the stop button late, I may overestimate the time. Therefore, the time of reaction is a random error and decreases the precision of the result. This problem can be solved by using the temperature probe and connected to the computer. By connecting the temperature probe to the computer , I can see the diagram which shows the time when the the temperature reaches 303K and 313K, determine the time this requires this to increase 10K, and increase the precision of the result.

Citations:

1. Zwicky, F. “Theory of the Specific Heat of Electrolytes.” *Proceedings of the National Academy of Sciences*, vol. 12, no. 2, 1 Feb. 1926, pp. 86–92, https://doi.org/10.1073/pnas.12.2.86. Accessed 26 June 2020.
2. Nayar, Kishor, et al. “Desalination.” Water Treatment, vol. 390, 2016, pp. 354–380, web.mit.edu/seawater/2017_MIT_Seawater_Property_Tables_r2b.pdf, https://doi.org/10.1016/j.desal.2016.02.024.