How does the pressure of the water in a cylinder container connected to an open-ended tube affect the volumetric flow rate of the laminar flow of the water.

Background

This investigation discusses the relationship between water pressure and the volumetric flow rate of laminar flow of water. Laminar flow is a pattern for the movement of water flow. For laminar flow, the fluid particles move in layers, sliding over each other and thus causing a smooth and streamlined flow.

Volumetric Flow Rate

For a fluid flowing in a cylinder, the flow velocity can be calculated using the flow rate formula. Flow rate is the volume of fluid passing through a given cross sectional area per unit time. Equation 1 gives the formula for the flow rate:

$$Q = \frac{dV}{dt}$$

Figure 1 below shows the water flow in a cylinder tube where the red shaded area is the volume of water passing point P in an unit time t. The volume of the red shaded area is the volume of fluid V.

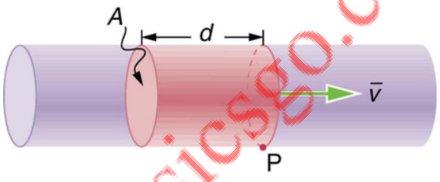


Figure 1: Figure for the flow rate in a tube

Table 1 below lists the variables in the flow rate formula.

Variables	Meaning	Unit
Q	flow rate of the fluid	$m^3 \cdot s^{-1}$
V	volume of the fluid	m^3
t	elapsed time	s

Table 1: Volume Flow Rate Variables

Poiseuille's Law

The relationship between the pressure of water and the volumetric flow rate can be described using Poiseuille's Law .There is a directly proportional relationship between the pressure applied and the fourth power of the diameter of the tube. Water flow is caused due to the pressure difference. The relationship between horizontal flow and the pressure can be stated as:

$$Q = \frac{P_2 - P_1}{R}$$

where R is conducted from everything other than pressure which affects the flow rate. It consists of three properties of the flow: the dynamic viscosity of the fluid, the hydraulic diameter of the fluid and the length of the tube which the fluid flows. Therefore, the resistance R to a laminar flow having dynamic viscosity of μ through a horizontal tube with a uniform radius of r and length l is given by :

$$R = \frac{8 \cdot \mu \cdot l}{\pi \cdot r^4}$$

Combing this equation with the flow rate equation, we obtain:

$$Q = \frac{(P_2 - P_1) \cdot \pi \cdot r^4}{8 \cdot \mu \cdot l}$$

We can abbreviate the pressure differences as ΔP. Thus we obtain:

$$Q = \frac{\Delta P \cdot \pi \cdot r^4}{8 \cdot \mu \cdot l}$$

Table 2 below lists the variables in Poiseuille's Law formula:

Variables	Meaning	Unit
Q	volumetric flow rate	$m^3 \cdot s^{-1}$
r	radius of the tube	m
μ	dynamic viscosity of the fluid	$kg \cdot m^{-1} s^{-1}$
l	length of the tube	m
ΔP	pressure difference on the two end of the tube	$kg \cdot m^{-1} \cdot s^{-2}$

Table 2: Poiseuille's Law Variables

Water pressure equation

In this investigation, the higher-pressure end of the tube is connected to a water tank filled with water at different heights. The pressure is calculated by the pressure equation:

$$P = \rho \cdot g \cdot h$$

Table 3 below lists the variables used in the hydrostatic pressure equation:

Variables	Meaning	Unit
P	fluid pressure	$kg \cdot m^{-1} \cdot s^{-2}$
h	height of the fluid	m
ρ	density of the fluid	$kg \cdot m^{-3}$

Table 3: Water Pressure Equation Variables

Volume for a cylinder

For a cylinder, the volume is calculated as :

$$V = \pi \cdot r^2 \cdot h$$

Table 4 below lists the variables used in the equation above:

Variables	Meaning	Unit
V	volume of the cylinder	m^3
r	radius of of the cylinder	m
h	height of the cylinder	m

Table 4: Volume Variables

Hypothesis

According to Poiseuille' s law, the volumetric flow rate is calculated as:

$$Q = \frac{\Delta P \cdot \pi \cdot r^4}{8 \cdot \mu \cdot l}$$

We can rearrange the equation as:

$$Q = \frac{\pi \cdot r^4}{8 \cdot \mu \cdot l} \cdot \Delta P$$

In this investigation, the difference of the pressure is the water pressure according to the water height, thus we obtain:

$$Q = \frac{\pi \cdot r^4}{8 \cdot \mu \cdot l} \cdot P$$

where Q is the volumetric flow rate and $h(t)$ is the height of the water in the container.

In this investigation, the volumetric flow rate is calculated by $Q = \frac{dV}{dt}$. Thus, the equation can be rewritten:

$$\frac{dV}{dt} = \frac{\pi \cdot r^4}{8 \cdot \mu \cdot l} \cdot P$$

Table 5 below lists the values of the constants in the equation:

Variables	Value
ρ	$998.2\ kg \cdot m^{-3}$
g	$9.81\ m \cdot s^{-}2$
μ	$0.0010016\ kg \cdot m^{-1} \cdot s^{-1}$
l	$0.17\ m$
r	$0.0025\ m$

Table 5: Constants List

Substituting these constants into the equation, we can obtain:

$$\frac{dV}{dt} = \frac{3.1415 \cdot 0.0025m^4}{8 \cdot 0.0010016kg \cdot m^{-1} \cdot s^{-1} \cdot 0.17m} \cdot P$$

$$\frac{dV}{dt} = 9.01 \cdot 10^{-8} \cdot P$$

According to this calculation, it could be hypothesized that as the height of the water increases then the pressure on the higher end of the tube will increase and the volumetric flow rate of water will increase. When plotting the pressure of the water against the volumetric flow rate of water, a linear line would be expected.

Methodology

Table 6 below lists the independent variable and dependent variable in this experiment.

Variables	Details
Independent Variable	The pressure of the water on the higher-pressure end of the tube. This will be determined by the video analysis on Logger Pro and deduction using hydrostatic pressure equation.
Dependent Variable	The volumetric flow rate of the laminar flow of the water. This will be determined by video analysis on Logger Pro.

Table 6: Independent and Dependent Variables

Table 7 below lists the controlled variables in this experiment and the further explanations on these variables.

Variables	Details
The density of fluid	The volumetric flow rate would be affected by the density of the fluid. It must be controlled so that the change of the volumetric flow rate is solely due to the change of the water pressure.Density is the measurement of how tightly matter is packed together and is constant for the same matter under the same temperature and pressure. The density of the fluid will be kept constant by always using water throughout the experiment.
Dynamic viscosity of fluid	The volumetric flow rate would be affected by the dynamic viscosity of the fluid. It must be controlled so that the change of the volumetric flow rate is solely due to the change of the water pressure. Dynamic viscosity is measure of a fluid's resistance when an external force is applied and is constant for the same fluid under the same temperature. The dynamic viscosity of the fluid will be kept constant by always using water throughout the experiment.
Diameter of the tube	The volumetric flow rate would be affected by the diameter of the tube. It must be controlled so that the change of the volumetric flow rate is solely due to the change of the water pressure. The same cylinder tube will be used throughout the experiment to keep the diameter of the tube constant.
Length of the tube	The volumetric flow rate would be affected by the length of the tube. It must be controlled so that the change of the volumetric flow rate is solely due to the change of the water pressure. The same cylinder tube will be used throughout the experiment to keep the length of the tube constant.

Table 7: Controlled Variables

Note that there are some methodological strategies to be applied to the upcoming procedure. Considering the limitations of the measurement instruments, I will determine the volumetric of the fluid flow in my experiment using Logger Pro to track the decrease of the height of water in the water tank and deduce the decrease of the volume of water from this height.

Apparatus

Table 8 below lists the apparatus for the experiment.

Apparatus	Quantity	Uncertainty
Glass tube with a fixed length (10.0cm)	1	±0.05cm
Water Tank with a valve (diameter = 0.25cm, length = 7cm)	1	± 0.05cm
Water	/	/
Camera Recorder	1	/
Thermometer	1	± 0.5 °C
Logger Pro	/	/

Table 8: Apparatus List

Figure 2 below shows the setup of the experiment for the apparatus:

Figure 2: Experimental Setup

Procedure

1. Measure the room temperature using the thermometer and record the corresponding dynamic viscosity and density of water.
2. Plug the water tube with the valve at the bottom of the water tank.
3. Fill the water tank with 8 liters of water.
4. Turn on the camera recorder and start recording.

5. Open the valve.
6. Turn off the camera recorder and store the video when all the water flows out of the water tank.
7. Determine the height of the water surface in the cylinder container every three seconds using the video and Logger Pro. Record this number and its corresponding time.

Safety, environmental and ethical issues

Tubes are made of glass in this experiment and the glass may shatter during the process if it is hit too hard or drops to the grounds and then shatters, leading to related injuries.

Raw Data

Qualitative Data - what is observed when the water is flowing

As the water level drops, the flow rate of the water flowing out starts to decrease and the rate of change of the height of the water decreases gradually.

Quantitative Data - Raw Data Table

Table 9 below lists the raw data of trial 1 of the experiment:

Time (s,±0.1s)	Height (m,±0.0005m)	Time (s,±0.1s)	Height (m,±0.0005m)
0.	0.2005	45.	0.0901
3.	0.1898	48.	0.0856
6.	0.1799	51.	0.0805
9.	0.1701	54.	0.0771
12.	0.1616	57.	0.0723
15.	0.1530	60.	0.0691
18.	0.1456	63.	0.0652
21.	0.1374	66.	0.0616
24.	0.1302	69.	0.0591
27.	0.1236	72.	0.0560
30.	0.1178	75.	0.0524
33.	0.1111	78.	0.0495
36.	0.1058	81.	0.0479
39.	0.0996	84.	0.04484
42.	0.0951	87.	0.0429
		90.	0.0406

Table 9: Raw Data Table

Processed Data

Sample Calculation

The volumetric flow rate is calculated via the cylinder volume equation and the height of the water:

$$Q = \frac{dV}{dt}$$

In my experiment, the water is held in a cylindrical container. Thus the volume of the water in the volumetric flow rate equation can be calculated as:

$$Q = \frac{dV}{dt} = \pi \cdot R^2 \cdot \frac{dh}{dt}$$

where R is the radius of the cylinder container which is 0.1275 m ,and h is the height of the water.

A sample calculation based on the first two rows of the raw data ($t = 0s, h = 0.2005m; t = 3s, h = 0.1898m$)is given below:

$$Q = \pi \cdot R^2 \cdot \frac{dh}{dt} = \pi \cdot 0.1275m^2 \cdot \frac{0.2005m - 0.1898m}{3s - 0s}$$

$$Q \approx 1.822 \cdot 10^{-4} m^3 \cdot s^{-1}$$

$$\frac{\Delta Q}{Q} = \frac{\Delta R}{R} \cdot 2 + \frac{\Delta h}{dh} + \frac{\Delta t}{dt} = \frac{0.0005m}{0.1275m} \cdot 2 + \frac{0.0005m}{0.0107m} + \frac{0.1s}{3s} \approx 0.0879 m^3 \cdot s^{-1}$$

$$\Delta Q = 0.0879 \cdot 1.822 \cdot 10^{-4} \approx 1.601 \cdot 10^{-5} m^3 \cdot s^{-1}$$

$$\Delta Q \approx 8\%$$

The value of the pressure is calculated by the hydrostatic pressure equation:

$$P = \rho \cdot g \cdot h$$

where ρ is approximately 998.2 $kg \cdot m^{-3}$, and g is 9.81 $m \cdot s^{-2}$ A sample calculation with the height of 0.2 m is given below:

$$P = \rho \cdot g \cdot h = 998.2 \cdot 9.81 \cdot 0.2 \approx 1985.46 kg \cdot m^{-1} \cdot s^{-2}$$

$$\frac{\Delta P}{P} = \frac{\Delta h}{h} = \frac{0.0005m}{0.2m} = 0.0025 kg \cdot m^{-1} \cdot s^{-2}$$

$$\Delta P = 0.0025 \cdot 1985.46 \approx 4.96 kg \cdot m^{-1} \cdot s^{-2}$$

$$\Delta P = 0.24\%$$

As the uncertainty of the pressure of the water is exceedingly small, in the later parts of the report, the uncertainty of the pressure is ignored.

Quantitative Data - Raw Data Table

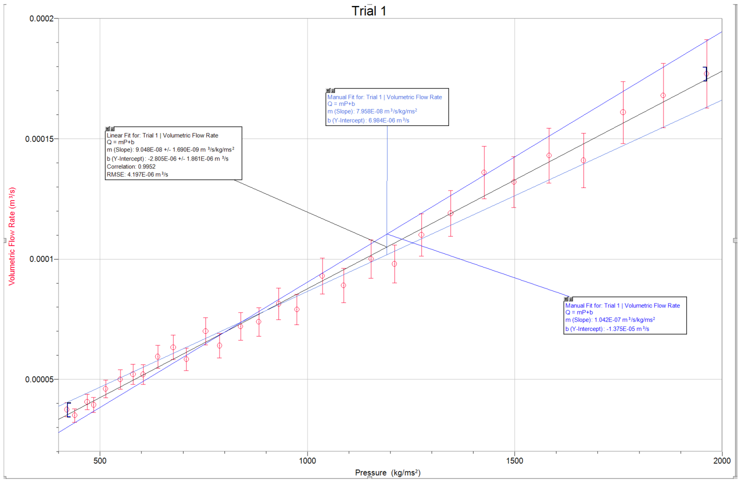
Table 10 below lists the processed data of trial 1 of the experiment calculated according to the sample calculation.

Pressure $(kg \cdot m^{-1} \cdot s^{-2})$	Volumetric Flow Rate $(m^3 \cdot s, \pm 8\%)$	Pressure $(kg \cdot m^{-1} \cdot s^{-2})$	Volumetric Flow Rate $(m^3 \cdot s, \pm 8\%)$
1962.87	$1.77 \cdot 10^{-4}$	882.69	$7.39 \cdot 10^{-5}$
1858.26	$1.68 \cdot 10^{-4}$	838.95	$7.20 \cdot 10^{-5}$
1761.37	$1.61 \cdot 10^{-4}$	788.28	$6.40 \cdot 10^{-5}$
1666.05	$1.41 \cdot 10^{-4}$	754.54	$7.00 \cdot 10^{-5}$
1582.83	$1.43 \cdot 10^{-4}$	708.25	$5.83 \cdot 10^{-5}$
1498.23	$1.32 \cdot 10^{-4}$	676.72	$6.33 \cdot 10^{-5}$
1426.22	$1.36 \cdot 10^{-4}$	639.26	$5.94 \cdot 10^{-5}$
1345.52	$1.19 \cdot 10^{-4}$	604.11	$5.20 \cdot 10^{-5}$
1275.02	$1.10 \cdot 10^{-4}$	579.5	$5.21 \cdot 10^{-5}$
1210.11	$9.80 \cdot 10^{-5}$	548.69	$5.00 \cdot 10^{-5}$
1153.55	$1.00 \cdot 10^{-4}$	513.55	$4.60 \cdot 10^{-5}$
1087.52	$8.90 \cdot 10^{-5}$	484.78	$3.94 \cdot 10^{-5}$
1035.86	$9.30 \cdot 10^{-5}$	469.15	$4.06 \cdot 10^{-5}$
974.88	$7.90 \cdot 10^{-5}$	439.17	$3.50 \cdot 10^{-5}$
930.9	$8.14 \cdot 10^{-5}$	420.23	$3.74 \cdot 10^{-5}$

Table 10: Processed Data Table

Analysis

The processed data including the uncertainty are plotted on Logger Pro. Graph 1 below shows the relationship of volumetric flow rate and pressure:



Graph 1: Volumetric Flow Rate vs. Pressure with uncertainty

The best-fit line reveals a relationship of:

$$Q = (9.046 \cdot 10^{-8}) \cdot P - (2.895 \cdot 10^{-6})$$

The worst-fit lines reveals a relationship of:

$$Q = (7.985 \cdot 10^{-8}) \cdot P - (6.984 \cdot 10^{-6})$$

$$Q = (1.402 \cdot 10^{-7}) \cdot P - (1.375 \cdot 10^{-5})$$

The uncertainty of the gradient is:

$$\frac{1.402 \cdot 10^{-7} - 7.985 \cdot 10^{-8}}{2} = 3.018 \cdot 10^{-8}$$

The uncertainty of the Y-intercept is:

$$\frac{1.375 \cdot 10^{-5} - 6.984 \cdot 10^{-6}}{2} = 3.383 \cdot 10^{-6}$$

Therefore:

$$Q = (9.046 \cdot 10^{-8} \pm 3.018 \cdot 10^{-8}) \cdot P - (2.895 \cdot 10^{-6} \pm 3.383 \cdot 10^{-6})$$

The relationship reveals that the water pressure has a proportional linear relationship with the volumetric flow rate of water.

Conclusion

The investigation aims to calculate the relationship between the pressure of the water and the volumetric flow rate of the water pouring out from an open-ended tube. It was hypothesized that there is a linear relationship between the value of the water pressure in the cylinder container and the volumetric flow rate of the water.

The processed experimental data was plotted and revealed a relationship of $Q = (9.046 \cdot 10^{-8} \pm 3.018 \cdot 10^{-8}) \cdot P - (2.895 \cdot 10^{-6} \pm 3.383 \cdot 10^{-6})$ and it indicates that as the pressure increase $100kg \cdot m^{-1} \cdot s^{-2}$, the volumetric flow rate would increase by $6.151 \cdot 10^{-6} \pm 3.65 \cdot 10^{-7} m^3 \cdot s^{-1}$. This calculation supports the hypothesis of a positive linear relationship between the pressure of the water and the volumetric flow rate of the water.

The line of best fit does not pass the origin According to the background and hypothesis, there isn't a y-intercept for their relationship. There is a constant systemic error presented in the experiment. This system error is probably due to the limitation of measurements so the data calculated on Logger Pro may show a constant deviation from the actual calculation.

The background and hypothesis predict a positive gradient and its value to be $\frac{8 \cdot \mu \cdot l}{\pi \cdot r^4}$ This is a combination of several physical factors of everything other than pressure that affects the flow rate. The precise value is $9.01 \cdot 10^{-8}$. This is consistent with the produced gradient from the best-fit line. This shows that the coefficient is precise and generally believed to be reliable.

However, the uncertainty of the gradient is about 30% which is too much so the values are inaccurate to a certain degree. This uncertainty and accuracy is probably due to the same reason which causes the y-intercept in my best-fit line. The values in this experiment are really small and any small error would be enlarged in the calculation. Since the heights are marked by naked eyes through Logger Pro, a random error will be created and thus causes the inaccuracy in my data.

Evaluation

The experiment shows inaccuracy with the hypothesis. The reason for it is the weaknesses throughout the experiment. At the same time, there are some strengths in my experiment. Table 11 below shows the strength in this investigation:

Strength	Significance
The calculation records data every three seconds	According to Poiseuille' s law, the volumetric flow rate calculates the rate of change of the volume. Therefore during calculation and measurement, the more changes are calculated, the more precise the value of the flow rate is. Three seconds is a time period which is long enough for a substantial change of the height of the water surface and short enough for frequent measurements.
The same set of the apparatus is used throughout the experiment	The thermometer, water tank and tube, along with other apparatus were kept the same throughout the experiment, This ensures the data to be closer together and more precise.
Videos are used for a more accurate measurement	Since the time frame for each recorded value is three seconds, I will not be able to record and measure at the same time when the water is flowing down from the tube. So I use video recording to measure the values afterwards.
Thermometer are used for the value of water density and dynamic viscosity	Although the effect of temperature is small, the water density and dynamic viscocity at different temperatures are slightly different. Therefore, I use a thermometer to measure the precise temperature during the experiment for a precise value.

Table 11: Strength Table

Table 12 below shows the weakness and limitation in this investigation:

Weaknesses and limitations and their consequences	Improvements
The diameter of the tube is too large. Some air is left in the tube which will affect the pressure differences of two sides of the tube. Although this effect is small and neglect able in my experiment, it will affect the calculation in some degree.	use tubes with smaller diameters.
Logger Pro is not accurate enough for small measurements. All the values are small in the experiment and the use of Logger Pro with the naked eye to record the height would cause substantial amount of random errors. This is one reason why the result is precise but not accurate	use measurement instruments for water pressure and flow rate that can retrieve accurate data.

Table 12: Weakness Table

References

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