RQ: How does the temperature (20,22,24,26,28,30℃/±1℃) of an iron wire of a specific length and diameter affect its resistivity?

Background:

This exploration will investigate the effect temperature has on the resistivity of a wire. Resistivity ρ is defined as the electrical resistance of a conductor of unit cross-sectional area and unit length. When the temperature of a conductor increases, the average kinetic energy of atoms of the conductor will also increase. This means the electrons in these atoms will have more energy which means more capability to move from one atom to another to form a current. In this way, the resistivity ρ of the conductor will decrease. Figure 1 below shows the process:

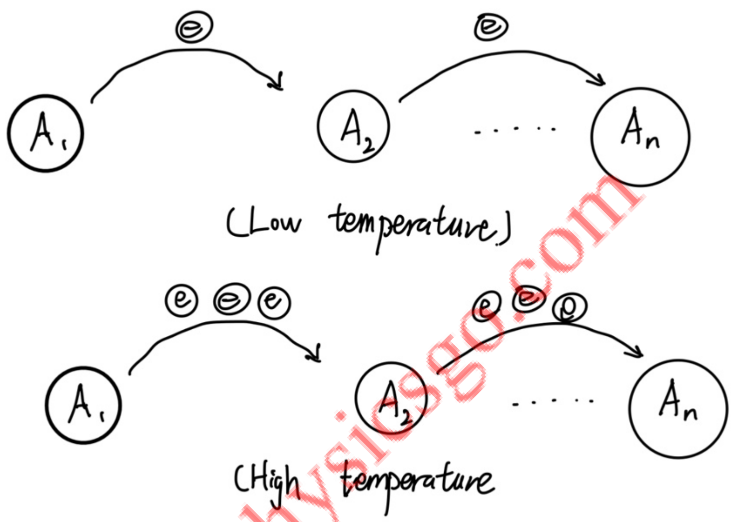


Figure 1.

This brings the iron some changes in its resistivity ρ, which is marked as 'α'. The relationship between ρ_T and ρ_0 is often described as $\rho_T = \rho_0[1 + \alpha(T - T_0)]$. However, for iron wire, it cannot be determined whether ρ_T will decrease or increase after a positive change in temperature unless an experiment is conducted.

What's more, there is an important concept in the calculation of resistance, which is the law of resistance:

$$R = \rho\frac{L}{A}$$

R is the resistance of the wire measured in Ohms Ω, U is the voltage over the two terminals of the wire measured in volts V, and I is the current flowing through the wire measured in ampere A. This equation indicates the proportional relationship between resistance and resistivity and length of the conductor. It also indicates the negative proportional relationship between the resistance and the cross area of the conductor

Table 1 below shows the meanings of physical quantities I will use and their units:

Variable	Meaning	Unit
ρ_T	Resistivity of the material at temperature.	°C
ρ_0	Resistivity of the material at a reference temperature.	°C
α	Temperature coefficient of resistivity for the material.	$°C^{-1}$
T	Temperature after being changed.	°C
T_0	Reference temperature.	°C
R	Resistance of the conductor	Ω
ρ	Resistivity of the conductor	Ω m
L	Length of conductor	m
A	Cross area of the conductor	m^2

Table 1.

Hypothesis:
According to the equation, as the temperature of the iron wire increases, the resistivity of the iron wire of a fixed length and diameter decreases.

Methodology:
Independent variable: The temperatures of the iron wire.
(20,22,24,26,28,30°C/$\pm$1°C).   The temperatures are determined by measuring them before each experiment. The temperatures can be changed by heating the conductor in a heated liquid and then dry it quickly.

Dependent variable: The resistance of the conductor. This could be measured by calculating the ratio of the potential difference between the two ends to the current to the current which travels through it.

Table 2 below shows the variables which should be controlled in this experiment:

Controlled variables:

Controlled variable	Reason to control	How to control
Conductor material	To make sure this is the conductor's temperature is the only variable that changes the resistance of the conductor.	Use conductors with the same materials.

Conductor length	Conductor's length affects its resistance. Thus, it is necessary to keep it the same.	Measure the length of wire before every trial of the experiment.
Conductor diameter	Conductor's diameter affects its resistance.	Thus, it is necessary to keep it the same. Measure the diameter of wire before every trial of the experiment.
Wire resistance	It would be difficult to obtain the resistance of a conductor according to Ohm's law if the resistance of the wire changes.	Use the same wire throughout the investigation.

Table 2.

Note that there are some methodological strategies to be applied to the upcoming procedure. First, this experiment will be conducted quicky in each trial to prevent the iron wire to cool down and cause a systematic error. Secondly, the iron wire will be placed on a piece of wood to insulate it from the external environment in order to prevent safety problems and systematic errors made by interactions with the surrounding objects. Thirdly, paper will be used to dry the iron wire after the conductor is heated it in a hot water bath in order to prevent safety problems caused by short circuits and systematic errors caused by currents flowing through water.

Table 3 below lists the materials used in this investigation:

Number	Apparatus	Quantity	Uncertainty
1	Iron wire	length:10.00m cross area:$3\times 10^{-6}m^2$	$\pm 0.01m$ $\pm 0.5\times 10^{-6}m^2$
2	6.0V battery	1	$\pm$0.1V
3	Voltmeter (0~6V)	1	$\pm$0.1V
4	Ammeter (0~3.0A)	1	$\pm$0.01A
5	Heater	1	$\pm$1℃
6	Thermometer (0~100℃)	1	$\pm$1.0℃
7	Water bath（1L）	1	/
8	Wire	1	/
9	Wood	1 piece	/

10	Tissues	/	/
11	Watch	/	/
12	Ruler	1	±0.001m

Table 3.

Figure 2 below shows the materials used in this investigation:

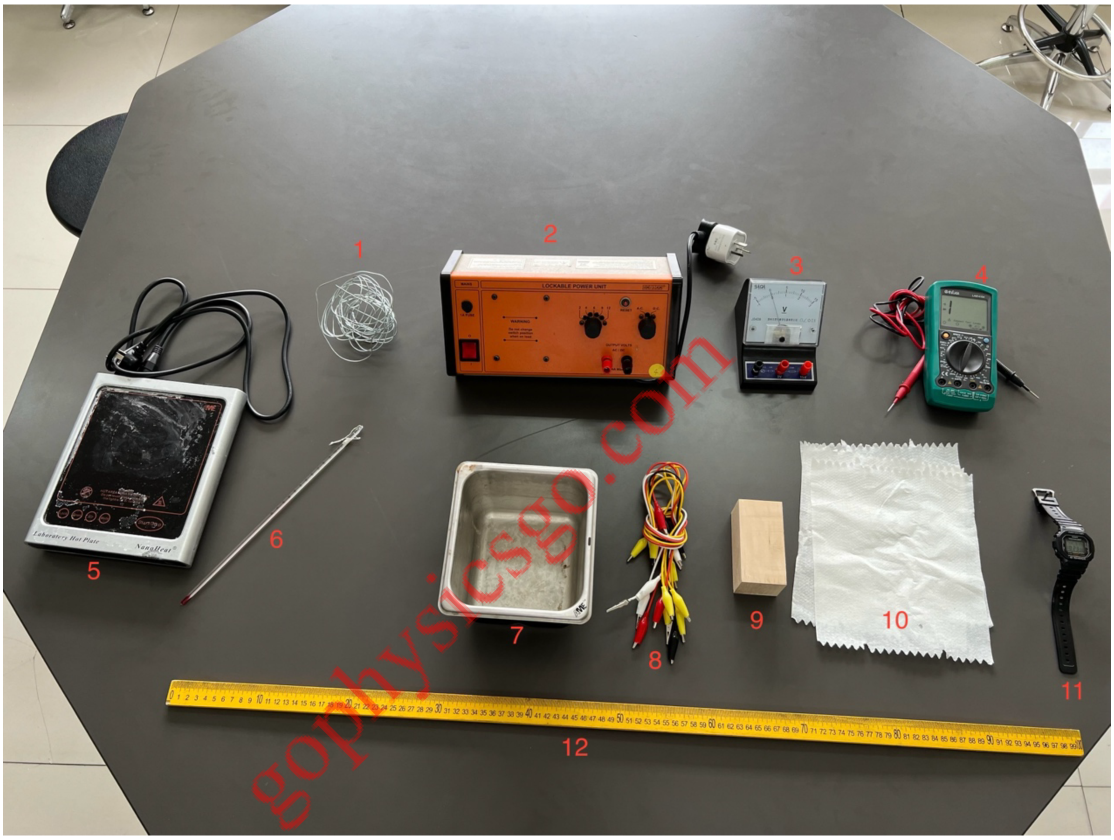


Figure 2.

Procedure:

1. Turn on the water bath, heat it to 30°C. Make sure the water temperature is stable and place the iron wire in and heat it for five minutes. Remove and dry it.
2. Mount the iron wire on the piece of wood. Attach the wire ends to the power supply through the rheostat. Connect it to the wire, battery, ammeter, and switch in series within a single circuit. Connect the voltmeter with the iron wire in parallel. Keep the switch open.
3. Close the switch. Measure the initial voltage across the wire and the current flowing through it using the ammeter. Take five trials and record these values (Heat it for a while before every trial).
4. Using the hot water bath, gently heat the iron wire evenly from 30°C to 40°C. Remove it and repeat procedures one to three .

5. Repeat this experiment at 50°C, 60°C, 70°C, 80°C and 90°C.
Compare the results with the theoretical background on the temperature coefficient of resistance for different materials.

Figure 3 below shows the setup of the investigation:

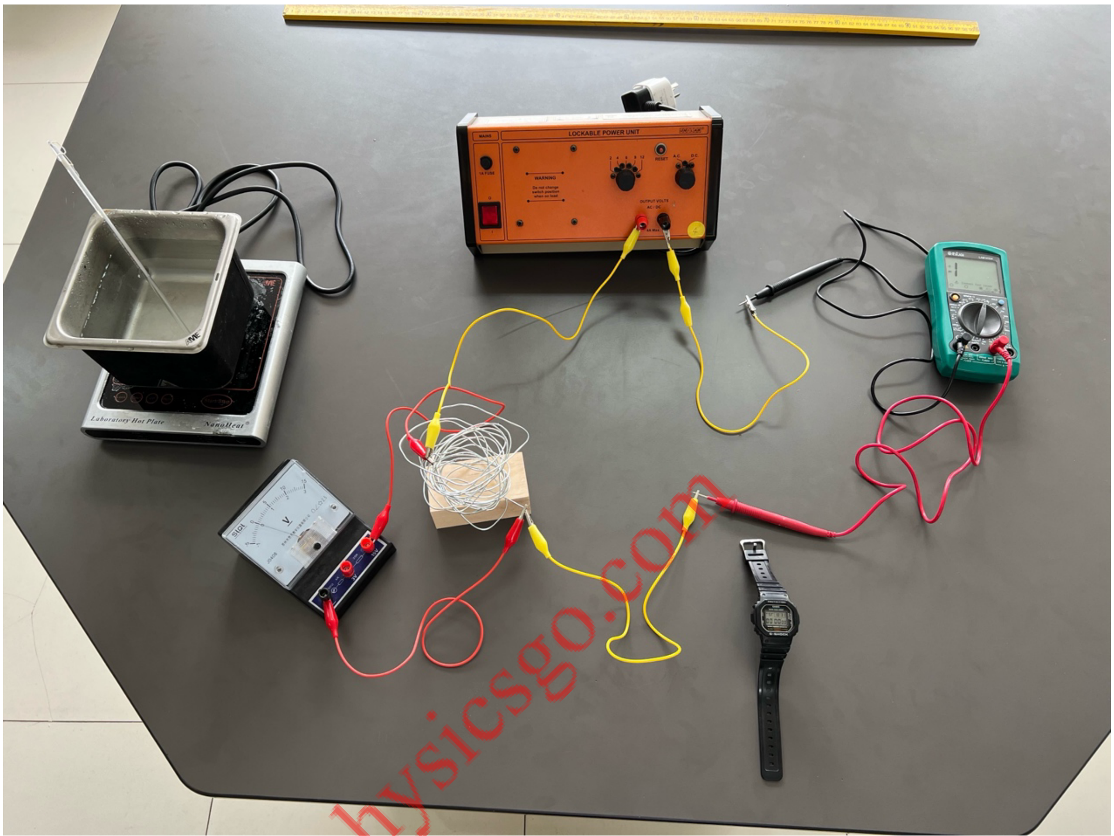


Figure 3.

Safety, Environmental and Ethical Issues:
As hot water bath and electricity is used in this experiment, it might lead to scalding and electric shock as safety problems. However, it can be avoided by wearing gloves. Besides this, there are no more environmental or ethical issues.

Raw Data

Table 4 below shows the raw data collected from the experiment about the voltage and the currents:

U=6V(±0.1V)					
Temperature (°C) (±1°C)	Current A (±0.01A)				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
20	0.98	1.00	0.97	1.00	0.98
22	1.03	1.07	1.05	1.07	1.05
24	1.33	1.28	1.27	1.28	1.30
26	1.43	1.40	1.36	1.43	1.46
28	1.58	1.62	1.66	1.71	1.67
30	1.94	2.06	1.88	1.87	1.94

Table 4.

Sample Calculation:

According to 'Physics for the IB Diploma (London) (John Allum, Paul Morris)', to determine the uncertainty using voltage and current, Ohm's law should be utilized:

$$R = \frac{U}{I}$$

where *R* is the resistance of the wire, *U* is the voltage drop between the two terminals of the wire, and *I* is the current flowing through the wire.

Then, according to the law of resistance:

$$R = \rho \frac{L}{A}$$

Where *R* is the resistance of the wire, ρ is the resistivity of the wire, L is the length of the wire and *A* is the cross area of the wire. Thus, as R is equal, the following relationship can be established:

$$\frac{U}{I} = \rho \frac{L}{A}$$

$$\rho = \frac{UA}{IL}$$

According to uncertainty's law, the percentage uncertainty of each resistance from the raw data table can be calculated by [1]:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta U}{U} + \frac{\Delta I}{I} + \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

which indicates that the absolute uncertainty of each of the resistance is:

$$\Delta\rho = (\frac{\Delta U}{U} + \frac{\Delta I}{I} + \frac{\Delta A}{A} + \frac{\Delta L}{L})\rho$$

In this way, by substituting the values for voltage (*U*),current (*I*),length (*L*) and cross sectional area (*A*) along with their uncertainties (ΔU, ΔI, ΔL, ΔA), we can compute the resistivity and its absolute uncertainty.

For instance, the calculation to obtain the resistance and its absolute uncertainty when *T*=20°C, *I*=0.98A:

$$\rho = \frac{UA}{IL}$$

$$\rho = \frac{6.0\text{V} \times 3 \times 10^{-6}\text{m}^2}{0.98\text{A} \times 10.00\text{m}}$$

$$\rho = 1.84 \times 10^{-6}\Omega\text{m}$$

$$\Delta\rho = (\frac{\Delta U}{U} + \frac{\Delta I}{I} + \frac{\Delta A}{A} + \frac{\Delta L}{L})\rho$$

$$\Delta\rho = (\frac{0.1\text{V}}{6.0\text{V}} + \frac{0.01\text{A}}{0.98\text{A}} + \frac{0.5 \times 10^{-6}\text{m}^2}{3 \times 10^{-6}\text{m}^2} + \frac{0.01\text{m}}{10.00\text{m}})2 \times 10^{-6}\Omega\text{m}$$

$$\Delta\rho = 0.36 \times 10^{-6}\Omega\text{m}$$

$$\therefore \rho = 1.84 \times 10^{-6} \pm 0.36 \times 10^{-6}\Omega\text{m}$$

Table 5 below shows the results of the resistivities and their absolute uncertainties:

Temperature (°C)	Current (A)	Resistivities(Ωm)	Absolute resistance uncertainty (±Ωm)
20	0.98	1.84	0.36
20	1.00	1.80	0.35
20	0.97	1.86	0.36
20	1.00	1.80	0.35
20	0.98	1.84	0.36
22	1.03	1.75	0.34
22	1.07	1.68	0.33
22	1.05	1.71	0.33
22	1.07	1.68	0.33
22	1.05	1.71	0.33
24	1.33	1.52	0.28
24	1.28	1.47	0.27
24	1.27	1.49	0.27
24	1.28	1.48	0.27
24	1.30	1.51	0.28
26	1.43	1.29	0.24
26	1.40	1.32	0.25
26	1.36	1.32	0.25
26	1.43	1.28	0.24
26	1.46	1.29	0.24
28	1.58	1.14	0.22
28	1.62	1.11	0.21
28	1.66	1.08	0.21
28	1.71	1.05	0.20
28	1.67	1.08	0.21
30	1.94	0.93	0.18
30	2.06	0.87	0.17
30	1.88	0.96	0.18
30	1.87	0.96	0.18
30	1.94	0.93	0.18

Table 5.

It is better to determine the average value of the resistances at each temperature and their absolute uncertainties to verify the accuracy of the raw data. This can be determined using the following two equations:

$$\rho_{ave} = \frac{\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5}{5}$$

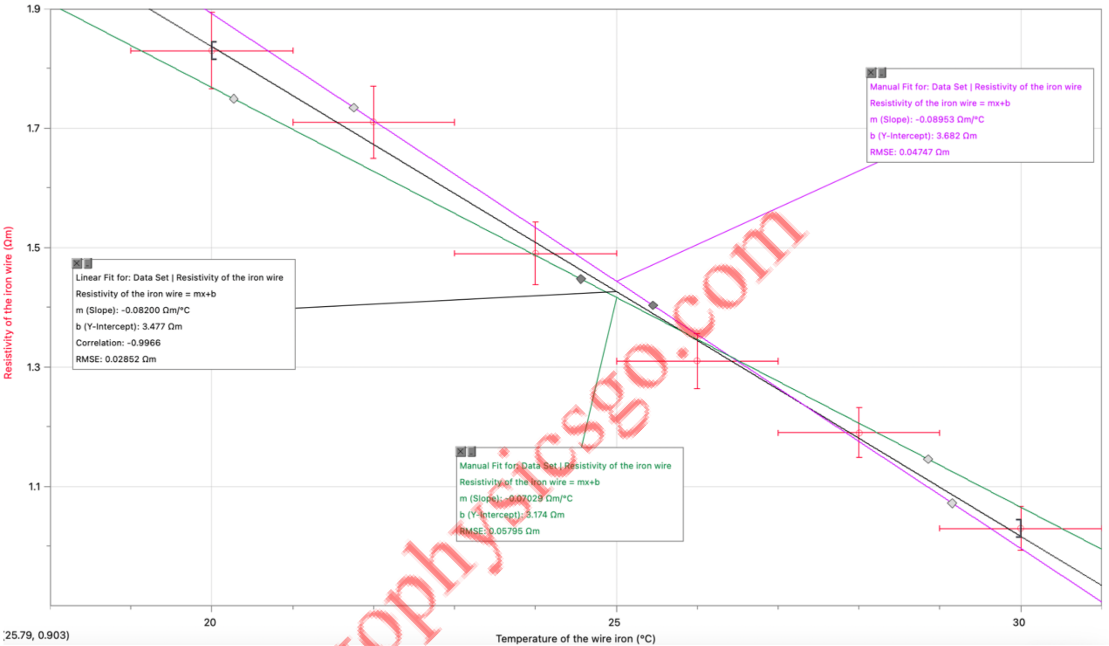
$$\Delta\rho_{ave} = \frac{\Delta\rho_1 + \Delta\rho_2 + \Delta\rho_3 + \Delta\rho_4 + \Delta\rho_5}{5} [1]$$

Table 6 shows the average value of resistivity and its absolute uncertainty:

Temperature (°C)	Average resistance (Ω)	Resistance Uncertainty (Ω)
20	1.83	0.36
22	1.71	0.33
24	1.49	0.27
26	1.31	0.24
28	1.09	0.21
30	0.93	0.18

table 6.

Graph 1 shows the linear function and its graph of the function:



Graph 1.

The best fit line reveals the following relationship:

$$\rho = -0.08200T + 3.477$$

The worst fit line reveals the following relationship:

$$\rho = -0.07029T + 3.174, \ \rho = -0.08953T + 3.682$$

The uncertainty of the gradient is

$$\frac{-0.07029 - (-0.08953)}{2} = 0.00962\Omega\text{m}/\text{°C}$$

The uncertainty of the Y-intercept is :

$$\frac{3.682 - 3.174}{2} = 0.254\Omega\text{m}$$

Therefore,

$$\rho = -(0.08200 \pm 0.00962)T + (3.477 \pm 0.254)$$

This relationship reveals that the resistivity of iron wire has a negative linear relation with the temperature.

Conclusion:

This investigation aims to calculate the effect of the change in temperature(℃) of an iron wire on its resistivity by performing experiments and also calculate the coefficient of linear decrease of the resistivity of the iron wire. It was hypothesized that as the temperature of the iron wire increases, the resistivity of the iron wire of a fixed length and diameter decreases.

This processed experimental data was plotted and revealed a relationship of $\rho = -(0.08200 \pm 0.00962)T + (3.477 \pm 0.254)$ and it indicates that when the temperature increases by 1℃, the resistivity of the iron wire decreases by

$-0.08200\Omega\text{m}$. This calculation proves the hypothesis by proving the negative linear

relationship between the temperature and resistivity of the iron wire. This is because the increased temperature represents an increase in the kinetic energy of atoms in the iron wire, causing the electrons to have more energy to move from one molecule to another, resulting in a lower resistivity.

The coefficient of the linear fit is $-(0.08200 \pm 0.00962)\Omega\text{m}/℃$. The percentage

uncertainty of this coefficient is $\frac{0.00962\Omega\text{m}/℃}{0.08200\Omega\text{m}/℃} \times 100 = 11.732\%$. As when the

percentage uncertainty of the coefficient is under or around 10%, the coefficient is precise and reliable [1]. My coefficient of linear decrease of the resistivity of the iron wire can be considered as precise and accurate. Plus, considering the tightly spread of points around the best fit line on the graph, it also indicates the reliability of the results. There could be some potential existence of systematic error in this experiment, but the repeated experiments could eliminate this as much as possible. The factors and consequences will be discussed further in the evaluation section.

What's more, other investigations also indicates a similar relationship between the resistivity of the iron wire and its temperature. By using magnetic fields to help examine the resistance in the iron wire, this essay successfully indicates that not only in normal temperature, but also in low temperatures like 1 to 4.2K, the resistivity of iron wires with a diameter of 1×10^{-6}m to 3×10^{-6}m all show a linear relationship between the temperature and its resistivity (Taylor, Isin, & Coleman, 1968). This experiment used a wider range of temperatures and more precise methodologies to determine the relationship between the resistivity of the iron wire and its temperature.

Evaluation:

The hypothesis is corroborated by the results from the experiment, which states that an increase in temperature will decrease the resistivity of the iron wire. However, there are still some strengths and their significances, and also weaknesses and limitations and their consequences.

First, several successes of procedures and their significances should be discussed. In the first place, the experiment's procedure fully considered the safety issues. For instance, a piece of wood was used to insulate it from the external environment in order to prevent safety problems caused by electricity, and insulating gloves were used to prevent potential safety caused by heat. These fully stressed the importance of safety issues so that the experiment could be carried out smoothly without much problems. Second, it applied direct measurement methods for data collection. For example, the raw data was collected by an ammeter directedly. Avoid using complex steps can significantly decrease the random or systematic errors which could be created. Third, the procedure fully considered the importance of eliminating systematic errors. For instance, it stressed the significance of measuring quickly. A quick measurement can decrease the change in temperature of the iron wire after moved away from the water bath, which can eliminate the potential systematic errors as much as possible.

However, several weaknesses and their consequences can also be identified, prompting suggestions for improvements. First, the repeated bending of the metal wire to stretch and place the wire back to the water bath in the experiment may cause fractures, causing parts of the wire's cross-sectional area to change. This could introduce random errors into the resistivity measurements as an intractable problem of metal fatigue. Second, with the presence of water in this experiment. Water could remain in the cracks of the wire caused by metal fatigue. This will cause random errors to the experiment as water's resistivity is different than wire. A suggestion for this is to use a fan heater to evaporate the water. Another source of systematic error is the recording time. As the iron wire is out of the water bath to the outside where the temperature is much lower, the temperature would continue to decrease. The longer time it takes to record a value, the lower the actual temperature would be compared to the ideal temperature. This would cause a systematic error of lower-measuring. To solve this, an improvement is to use a fan heater to consistently heat the iron wire in the process to keep the temperature as well as recoding raw data. Finally, the issue of thermal expansion of the iron wire affecting the measurements may cause systematic error. As the cross-sectional area expands, the current flowing through it will increase, according to the law of resistance. This will ultimately cause the resistivity to increase with reference to the equation $\rho = \frac{UA}{IL}$, causing a systematic error of over-measuring. A possible improvement to this is to measure the change in the wire's cross-sectional area and substitute it into the equation to eliminate the systematic error theoretically.

Citations:

1. Physics for the IB Diploma (London) (John Allum, Paul Morris, published by Hodder education in 2023)
2. Taylor, G.R., Isin, A. and Coleman, R.V., 1968. Resistivity of iron as a function of temperature and magnetization. *Physical review*, *165*(2), p.621.