RQ: What effect does a tension force (10.0N, 20.0N, 30.0N, 40.0N, 50.0N) on a guitar string have on its fundamental frequency by using an oscilloscope?

Background Information

As a guitar string is tied on two fixed end points, it performs *simple harmonic motion*[1] *(Simple Harmonic Motion – Concepts)* when it is plucked and produces standing waves where some particular points remain the same position while others between them vibrate with the maximum amplitude. The science behind SHM is the vibration of the string, which is also the source of musical sounds from a guitar. Frequency, the number of oscillations completed by the string in one second, of which the primary frequency (the lowest frequency of a periodic waveform) is a crucial determinant to the pitch of the sound. Either for musicians or amateurs of guitars, it is vital to modify the tension of each string on the guitar to ensure the pitch is in a standard range. Hence, this experiment aims to investigate the relationship between the tension force acting on a guitar string and the fundamental frequency during the vibration.

<u>Frequency of a Vibrating Spring Formula:</u>

Eq. (1) is given to be able to showcase the variables that affect the frequency of the guitar string, the tension force on the string is one of which.

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \text{(Eq. (1))}$$

Variable	Meaning	Unit
f	Frequency of the string	Hz
L	Length of the string	m
T	Tension force on the string	N
μ	The mass per unit distance on the string	kg/m

The above variables are visualized in Figure 1 below.

Figure 1. Guitar string with variables for understanding formula

The method of deriving the relationship between the primary frequency and the tension force on the string:

Since Frequency of a Vibrating Spring Formula (Eq. (1)) is not a composite formula, the relationship of the frequency and the tension force is derived from other three formulas, which will be explained as below:

As the motion of the guitar string acts as a wave when plucked, for any wave, the following relationship, Eq. (2), which is also the *Wave Equation*[2] *(Speed of Sound, Frequency, and Wavelength, Physics)* could be applied:

$$v = f\lambda \quad \text{(Eq. (2))}$$

Where:

v – speed of the wave traveling through the medium [m/s]
f – frequency of one oscillation [Hz]
λ – wavelength [m]

To derive the relationship between the length of the string and the wavelength for the standing waves, the following mode, Figure 2, offers an explanation:

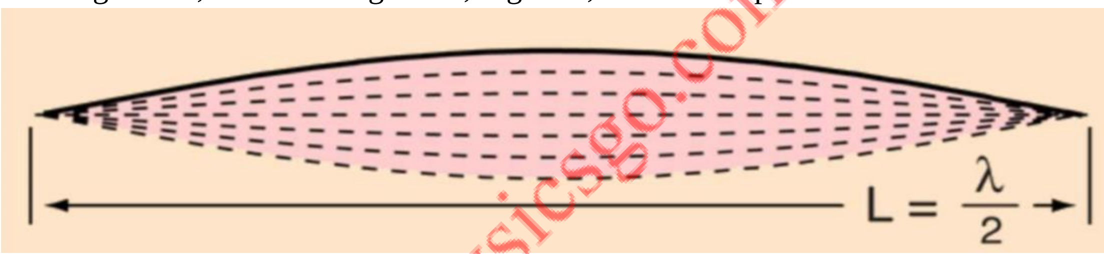

Figure 2. Length of the guitar string and the wavelength

Hence, *Fundamental Frequency of a String Equation*[3] *("Fundamental Frequency of a String")* combining the mode above is extracted as Eq. (3):

$$f = \frac{v}{2L} \quad \text{(Eq. (3))}$$

According to the speed of the wave traveling through the medium, we determine *Wave Speed on a String Formula*[4] *(The Wave Equation and Wave Speed - Physclips Waves and Sound)* to showcase its relation to the tension force on the string:

$$v = \sqrt{\frac{T}{\mu}} \quad \text{(Eq. (4))}$$

Where:

μ – mass per unit length of the string [kg/m]

From Eq. (2), Eq. (3) and Eq. (4), *Frequency of a Vibrating Spring Formula* (Eq. (1)) is ultimately derived.

Hypothesis

From the derived formula (Eq. (1)), it is hypothesized that when the tension force on the guitar string increases, the fundamental frequency of the vibrating guitar string will increase. When plotting f against the tension force on the string in N to the power of 0.5, a linear line, where the coordinate points are determined by μ and L, would be expected.

Methodology

Table 1 below lists the independent variable and the dependent variable:

Variables	Details
Independent	The tension force, starting with one end point of the guitar string towards the direction away from the string and parallel to it (10.0N, 20.0N, 30.0N, 40.0.N, 50.0N ± 0.5N). This could be manipulated by using the force gauge to measure the value of the tension force
Dependent	The primary frequency of the vibrating guitar string measured by the phone application Tunable (± 0.1 Hz)

Table 1: The independent variable and the dependent variable

Figure 2 shows the interface of the phone application Tunable:

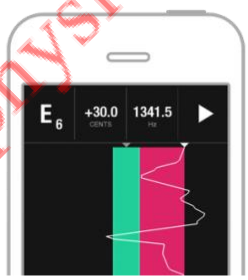

Figure 2. Photo of the application Tunable

Table 2 below lists the control variables and reasons, methods to control:

Control Variables	Why and how to control the variables
The length of the guitar string	According to Eq. (1), $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ indicates that L, the length of the guitar string affects the frequency. It must be controlled so as to ensure the frequency solely being influenced by the tension force on the string. The method to control is to select the guitar string with the length of 0.50 m ± 0.01 m and use the meter ruler to measure it.

The linear density of the guitar string	According to Eq. (1), $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ indicates that μ, the linear density of the guitar string affects the frequency. It must be controlled in order to ensure the frequency solely being influenced by the tension force on the string. The method to control is to measure the guitar string with same thickness and the same mass of 0.20 g ± 0.01g by using the electrical balance to guarantee the mass per unit length on the string is the same.
The temperature of the guitar string	Since metals take up a high proportion of all the materials in the guitar string, it expands with an increase in temperature and shrinks with a decrease, which significantly affects the fundamental frequency of the string. The method to control is to turn on and set the air conditioner at 22.0 °C ± 0.5 °C.

Table 2: Control variables

Note that there are plenty of errors occurring during the measurements, particularly when exerting the tension force on the guitar string by using the force gauge. Theoretically, although the mass of the gauge itself is able to eliminate the effect its mass has on the measurements, the gauge is still poised to be positioned vertically to ensure its able of elimination can function well.

<u>Apparatus</u>

Table 3 below lists the apparatus required with quantities and uncertainties:

Apparatus	Quantities	Uncertainties
Guitar string	1	/
Force gauge	1	± 0.25 N
Electrical balance	1	± 0.01 g
Meter ruler	1	± 0.005 m
Phone being able to detect the frequency	1	/
Phone app for frequency measurement	1	± 0.1 Hz
Stand with clamps for string tensioning	1 set	/
Weights (10N, 20N, 30N, 40N, 50N)	5	/
Protractor or angle measurer	1	Manufacturer spec

Table 3: Apparatus required to conduct the experiment

Procedure

1. Tie one end point of the guitar string to the stand and attach the force gauge to another end.
2. Use the meter ruler to measure the unstressed length of the guitar string.
3. Set up the frequency measurement app and prepare for the data collection.
4. Exert a 10 N tension force on the guitar string by adding a 10 N weight under the force gauge. Ensure the force is parallel and away from the orientation of the string and make sure the string is a straight line.
5. Gently pluck the center (the mid-point) of the guitar string and then use the phone app Tunable to accurately measure the primary frequency of the vibrating string. Record the value of the frequency.
6. Solely attach the weight of 20 N, 30 N, 40 N, 50 N to replace the 10 N weight in the step four and repeat step 4-5.
7. Repeat step 5 two extra times.
8. After collecting all the measurements to the tension force and the frequency, compile the data into a chart for analysis and identify the relationship between the frequency and the tension force.

Figure 3 shows the image of materials and lab setup:

Figure 3. Image of materials and lab set up

Safety, environmental, and ethical issues

1. Safety is the priority, especially avoiding the case in which the tension force on the string surpasses the maximum range it can carry to trigger out a snap and injuries. Plus, use the force gauge carefully to prevent damaging.
2. For potential ethical issues, take more considerations to others around when processing the experiment. For instance, it is better to minimize the noise which probably has a negative impact on others.

Raw Data Collection

Qualitative Data

When the tension force is exerting on the guitar string, it starts to extend a bit. When plucked, the string is seen to be vibrating in terms of standing waves. The increase in the tension force causes a higher primary frequency while vibrating.

Quantitative Data

Table 4 below lists the data for the guitar string:

	Mass (g ± 0.01g)	**Length (cm ± 0.05cm)**
Guitar string	0.34	38.60

Table 4: Basic information about the guitar string

Hence, μ, the linear mass can be derived:

$$\mu = \frac{m}{L} = \frac{(0.34 \times 10^{-3})kg}{(38.60 \times 10^{-2})m} = 8.81 \times 10^{-4}\ kg \cdot m^{-1}$$

Table 5 below lists the raw data:

Tension force (N ± 0.5N)	**Pitch trial 1 (Hz ± 0.1Hz)**	**Pitch trial 2 (Hz ± 0.1Hz)**	**Pitch trial 3 (Hz ± 0.1Hz)**	**Mean pitch (Hz)**
10.0	138.3	141.2	134.5	138 ± 3
20.0	190.8	188.4	194.3	191 ± 3
30.0	237.9	233.8	241.1	238 ± 4
40.0	279.2	275.0	283.9	279 ± 4
50.0	313.7	312.1	315.5	314 ± 2

Table 5: Raw data showing fundamental frequency with respect to the tension force

Data Processing

<u>Sample Calculation</u>

In my investigation, the relationship between the tension force on the guitar string and the fundamental frequency is expressed as Eq. (1):

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \text{(Eq. (1))}$$

A sample calculation when the tension force is 40.0 N is shown below.

$$\sqrt{T} = 6.32\sqrt{\text{N}} \qquad \Delta T = 0.5\text{N}$$

$$\frac{\Delta T}{T} = \frac{0.5\text{N}}{40\text{N}} \times 100\% = 1.25\%$$

$$\sqrt{\frac{\Delta T}{T}} = \frac{1}{2} \cdot \frac{\Delta T}{T} = \frac{1}{2} \times 1.25\% = 0.625\%$$

$$\sqrt{\Delta T} = \sqrt{T} \cdot \sqrt{\frac{\Delta T}{T}} = 6.32\text{N} \times 0.625\% = 0.04\text{N}$$

As the uncertainty of the tension force is not small enough compared to the sample value of the tension force, and its percentage uncertainty varies when the value of tension force changes. Hence, the uncertainty of the tension force cannot be ignored yet.

The mean value of the measured primary frequency is calculated:

$$f = \text{Fundamental Frequency} = \frac{279.2 + 275.0 + 283.9}{3} = 279.4 \text{ Hz}$$

The uncertainty value of the primary frequency using half range method is calculated:

$$\Delta f = \frac{\text{MAX}(f) - \text{MIN}(f)}{2} = \frac{283.9 - 275.0}{2} = 4 \text{ Hz}$$

With the same procedure, the data processing for other values are calculated in the same method as above, which are resulted in Table 6.

Tension (N ± 0.5N)	**Square root of Tension ($\sqrt{N}$)**	**Mean pitch (Hz)**
10.0	3.16 ± 0.08	138 ± 3
20.0	4.47 ± 0.06	191 ± 3
30.0	5.48 ± 0.05	238 ± 4
40.0	6.32 ± 0.04	279 ± 4
50.0	7.07 ± 0.04	314 ± 2

Table 6: Processed data - square root of Tension vs. Mean pitch

Data Analysis

The mean value of primary frequency is graphed with respect to the square root of the tension force on the guitar string, with error bars shown. The best fit line (black) and two worst fit lines (pink, green) are drawn according to the measured data. All above are shown on the Figure 4.

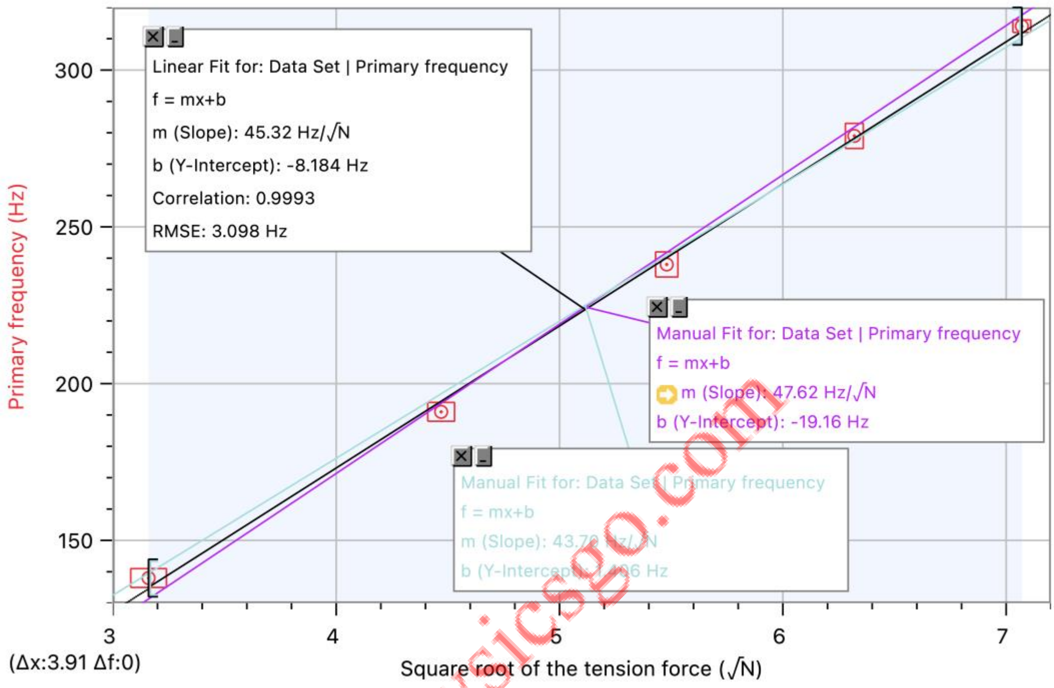

Figure 4. Primary frequency (Hz) VS Square root of the tension force $(\sqrt{\mathrm{N}})$ with uncertainties

The best fit line indicates the relationship:

$$f = 45.32 \cdot \sqrt{T} - 8.184$$

The worst fit lines indicate the relationship:

$$f = 43.70 \cdot \sqrt{T} + 1.406, f = 47.62 \cdot \sqrt{T} - 19.16$$

The uncertainty of the gradient is:

$$\frac{47.62 - 43.70}{2} = 1.96 \approx 2 \, [\mathrm{Hz} \cdot \sqrt{\mathrm{N}}^{-1}]$$

The uncertainty of the Y-Intercept is:

$$\frac{1.406 - (-19.16)}{2} = 10.283 \approx 10 \text{ Hz}$$

Therefore,

$$f = (45 \pm 2)\sqrt{T} + (-8 \pm 10) \, [\mathrm{Hz}]$$

Such a relationship demonstrates that the primary frequency of the vibrating guitar string is directly proportional to the square root of the tension force exerted on the string.

Conclusion

The experiment aims to investigate the effect of the tension (N) acting on the guitar string on its fundamental frequency (Hz) when pluck the string by conducting the experiments and as well calculating the coefficient of the primary frequency per square root of the tension via the linear regression line so that the frequency at different tensions can be determined. It is hypothesized that when the tension force increases, the fundamental frequency also increases, and there is a linear relationship between the fundamental frequency and the square root of the tension.

The data processing part gives a plotted and revealed relationship as $f = (45 \pm 2)\sqrt{T} + (-8 \pm 10)$ [Hz] and it indicates that when the square root of tension increases by $1\ \sqrt{\mathrm{N}}$, the primary frequency will increase by $45\ \mathrm{Hz} \pm 2\ \mathrm{Hz}$. The calculation result supports the hypothesis of a positive linear correlation between the tension force and the fundamental frequency. The reason behind this is an increase in the tension triggers out a higher wave velocity of the vibrating guitar string, and due to its fixed length, the frequency increases to adapt to that higher wave velocity with no change in wavelength (Griffith and Brosing)[5].

The coefficient m of the linear expansion refers to the directly proportional relationship between the primary frequency of the vibrating guitar string and the square root of the tension force exerted on the string, which is $45\ \mathrm{Hz} \cdot \sqrt{\mathrm{N}}^{-1} \pm 2\ \mathrm{Hz} \cdot \sqrt{\mathrm{N}}^{-1}$.

Two worst fit lines fit within all the error bars, where the uncertainty of the slope, the coefficient in the graph, can be calculated. The percentage uncertainty is $\frac{2}{45} \times 100\% = 4.44\%$, less than 10%, which indicates the high precision and liability of the experimental data, as well as a small random error existing on the data.

The best fit line does not pass through the origin, but this is expected. The processed data illustrates a negative y-intercept value, and it is estimated according to the linear regression line (Figure 3) to be -8.184 Hz. However, the range between y-intercepts value cut by the two worst fit lines includes the origin. Hence, there is no apparent systematic error in the data.

The literature value for the coefficient of the linear regression line is calculated:

$$k = \text{literature value} = \frac{1}{2L\sqrt{\mu}} = \frac{1}{2 \times 0.3860 \times \sqrt{8.81 \times 10^{-4}}} = 43.64\ [\frac{1}{\sqrt{\mathrm{kg} \cdot \mathrm{m}}}]$$

The experimental coefficient of the linear regression line is estimated through the processed data as 45.32. The percentage error can be calculated by using the formula:

$$\text{Percentage error} = \left|\frac{\text{literature value} - \text{calculated value}}{\text{literature value}}\right| \times 100\% \quad (Eq.(5))$$

Therefore, all the values can be applied to Eq. (5) to determine the answer of the percentage discrepancy, which is:

$$\text{Percentage discrepancy} = \left|\frac{43.64 - 45.32}{43.64}\right| \times 100\% = 3.8\%$$

The percentage discrepancy is calculated based on an assumption that the mass and the linear mass of the guitar string which infers the literature value have no uncertainty. According to the result, the percentage error indicates a satisfactory accuracy.

Evaluation

Although the hypothesis, which states that the increase in the tension force exerted on the guitar string will increase the fundamental frequency when plucking the string, corresponds to the processed data and the result by calculating and estimating, the calculated coefficient of the linear regression line differs from the theoretical value, with a value of 3.8% percentage error combining the experimental uncertainties. The possible reasons for such a discrepancy are the weaknesses in the process of the experiment. On the contrary, there are some strengths leading to the success of this investigation. The strengths as well as weaknesses in the investigation are given in Table 7 below:

Strengths	Significance
The meter ruler and the electrical balance only have $0.05\ \text{cm}$ and $0.01\ \text{g}$ uncertainties respectively.	Such extremely small uncertainties of measuring apparatus prevent the possible fluctuation of the actual mass and linear mass of the guitar string. The values are then more settled. Hence, it decreases the random error of the value of the mass and the linear mass of the string and increases the precision of the collected data.
The same method of setting the apparatus and of processing the trials is used.	The guitar string and the set of the force gauge are kept the same throughout the whole process of all trials, which guarantees the collected data is closer together with a less random error and a higher precision.
The force gauge is hanged vertically on the bottom point of the guitar string.	The force gauge has the capability of eliminating the effect of the mass itself on the measured force value, but the premise is to position it vertically. The additional value of the mass will shift all the value of the mass of the guitar string measured. Hence, positioning it vertically will somewhat decrease the random error and increase the accuracy of the data.

Table 7. Strengths of the investigation

Weaknesses or limitations and their impacts	Improvements
When the guitar string is vibrating, there are various frequencies. Hence, when a too small force is exerted, the primary frequency is not obvious to be detected by the phone. It decreases the precision and causes the random error of the collected primary frequency value.	Turn the range of the independent variable, the tension higher from 60N to 100N to make the amplitude of the primary frequency louder. Then the receiver will be easier to detect it. This improvement reduces the random error of the measured primary frequency and increases its precision.
The instant of plucking the guitar string adding an external force to the string, may enlarge the value of the tension force displaying on the force gauge scale instantly, which reduces the accuracy and triggers out a systematic error of the value of the tension force.	A force gauge with tiny safety catches can be attached to prevent the value of the tension force from being accidentally enlarged by plucking the guitar string. This improvement eliminates one systematic error and makes the data of the tension force more accurate.
All trials throughout the experiment were conducted using one guitar string to easily obey control variables. However, the string after added the tension might distort or go through an expansion, decreasing the value of linear mass of the string, which made the data less accurate and caused a systematic error.	Prepare five guitar strings with identical properties before the experiment. Equip each one string on each trial to eliminate the effect of the distortion of strings on its measured linear mass. It reduces the systematic error and enhances the accuracy of the data.

Table 8. Weaknesses/limitations, impacts, improvements of the investigation

Extensions

There are plenty of possible variables which can significantly affect the primary frequency of the vibrating guitar string. The temperature is selected as the dependent variable to conduct the further investigation. Hence, a new research question is probable to be “what effect does the temperature of the guitar string have on the primary frequency of the string when plucked?”

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