How does the surface area of a flat circular canopy ($0.10^2\pi$ m^2, $0.15^2\pi$ m^2, $0.20^2\pi$ m^2, $0.25^2\pi$ m^2, $0.30^2\pi$ m^2) of a parachute toy hanging a 20g object affect its terminal velocity (m/s) when falling from the third floor of a building (9.5 meters)?

Exploration

Introduction

Parachutes can be used to decelerate a person or object that is experiencing a fall by increasing the drag force. It is applied in different aspects such as military static-line jumping and emergency cases (New World Encyclopedia, 2022). In the current investigation, I would like to focus on parachute toys with a flat circular canopy as a simplified version of parachutes in practical life. A free body diagram of a falling parachute is shown below in figure 1.

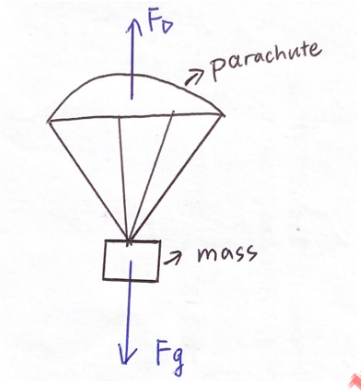

Fig.1 Parachute hanging a mass with forces labeled

Research Question

How does the surface area of a flat circular canopy ($0.10^2\pi\ \text{m}^2$, $0.15^2\pi\ \text{m}^2$, $0.20^2\pi\ \text{m}^2$, $0.25^2\pi\ \text{m}^2$, $0.30^2\pi\ \text{m}^2$) of a parachute toy hanging a 20g object affect its terminal velocity (m/s) when falling from the third floor of a building (9.5 meters)?

Background

Drag force

As mentioned, the increase in drag force is critical to the function of parachutes. The drag force exerted on an object is the force which acts opposed to the movement of the object when it flows through a certain fluid. The drag equation was initially proposed by Lord Rayleigh, now commonly expressed in eq. 1 below:

$$F_{\mathrm{d}} = \frac{1}{2} C_{\mathrm{d}} \rho u^2 A \qquad (1)$$

(Wegener, 1991)

where F_{d} (N) is the drag force,
C_{d} is the drag coefficient, with no units
ρ (kg/m^3) is the density of fluid,
u (m/s) is the flow velocity, which is the velocity of fluid relative to the moving object,
A (m^2) is the surface area with a dimension of L^2.

For Eq. 1, C_d is not constant in all cases. It has been claimed that Reynolds number is approximately inversely correlated with the drag coefficient (Lind and Sanders, 2004). The equation for Reynolds number is defined in eq.2 below.

$$Re = \frac{\rho u L}{\mu} \tag{2}$$

(Lind and Sanders, 2004)

where Re is the Reynolds number, with no units
ρ (kg/m^3) is the density of fluid,
u (m/s) is the flow speed,
L (m) is the linear dimension of object, and
μ (Pa·s) is the dynamic viscosity of fluid.
A large Re means the object experiences greater turbulent flow, while a small Re means that the object experiences greater laminar flow.

Terminal velocity

The terminal velocity is a constant velocity an object reaches when its net force becomes zero (Moebs et al., 2016). In the current investigation, this occurs when

$$F_\mathrm{g} + F_\mathrm{d} = 0N$$

$$F_\mathrm{d} = m|g| \tag{3}$$

where m is the mass of the object hung under the parachute, g is the gravitational constant $9.81\mathrm{m/s^2}$ down, and F_d is the drag force exerted on the parachute canopy. In this case, the mass of the parachute canopy is negligible, and the drag force of the object is also negligible compared to the drag force of the parachute canopy.

Determining the relationship between the surface area and terminal velocity

1. Since there is an inverse correlation between Reynolds number and drag coefficient, I assumed that their relationship is

$$C_d = \frac{c}{Re} \tag{4}$$

where c is an unknown constant without units.

2. After substituting Eq. (2), Eq. (3) and Eq. (4) to Eq. (1) and substituting linear dimension L by area A, I was able to obtain the relationship between the surface area and flow speed when the object reaches terminal velocity:

$$m|g| = \frac{1}{2} c \times \frac{\mu}{\rho u \sqrt{A}} \times \rho u^2 A$$

Thus,

$$m|g| = \frac{c\mu u\sqrt{A}}{2}$$

$$u = \frac{2m|g|}{c\mu\sqrt{A}}$$

3. In ideal conditions, the object's terminal velocity v_{T} should be equal to the flow speed of air as the object reaches zero net force. Hence, I deduced that

$$v_{\mathrm{T}} = \frac{2m|g|}{c\mu} \times A^{-1/2}$$

$$v_{\mathrm{T}}{}^2 = \left(\frac{2m|g|}{c\mu}\right)^2 \times \frac{1}{A} \tag{5}$$

As $\left(\frac{2m|g|}{c\mu}\right)^2$ is a constant in my investigation, I substituted it by a single constant M.

$$(v_{\mathrm{T}})^2 = M \times \frac{1}{A} \tag{6}$$

Hypothesis

It is hypothesized that as the surface area of a flat circular canopy increases, the terminal velocity of a parachute toy hanging a 20g object decreases when falling from the third floor (9.5 meters high) of a building. When plotting the square of terminal velocity against the multiplicative inverse of canopy area, a linear relationship is expected.

Methodology

Variables

Independent variable:

The independent variable is the area of a flat circular canopy ($0.10^2\pi$ m^2, $0.15^2\pi$ m^2, $0.20^2\pi$ m^2, $0.25^2\pi$ m^2, $0.30^2\pi$ m^2) of a parachute toy. This is manipulated by drawing and cutting circles of different radii to make the canopy.

Dependent variable:

The dependent variable is the terminal velocity of parachute toy (m/s). This is measured by analyzing the recorded video of parachute trajectory using the software Logger Pro.

Table 1 below shows the variables controlled in the experiment and why and how to control them:

Controlled variables	Why and how to control
Mass of object hung under the parachute toy	According to the background section, the mass of the object hung under the parachute toy determines the drag force it receives when reaching its terminal velocity. Since the terminal velocity is dependent of drag force, when the mass of the object increases, the terminal velocity would also increase. This variable is controlled by using the same object with a mass of 20g for all trials.
Density of fluid	According to the background section, the square of fluid density is proportional to the drag force. When fluid density increases, the terminal speed will decrease. This variable is controlled by dropping the parachute toy in the air in all trials.
Dynamic viscosity of fluid	According to the background section, the dynamic viscosity of fluid is inversely proportional to the drag force. When fluid density increases, the terminal speed will increase. This variable is also be controlled by using the air as the fluid in all trials.
Shape of canopy on the parachute toy	The drag coefficient is determined by the shape of canopy as the surface area varies for different shapes. This variable is controlled by using a flat circular canopy for all trials.
Number of strings of the parachute toy	When there are more strings on a parachute toy, the shape of canopy alters and affects the drag force. This variable is controlled by using four strings while making the parachute toy for all trials.

Table 1: List of controlled variables

Materials

Table 2 below shows the materials used, their quantities and uncertainties.

Material name	Quantity	Uncertainty
Woven nylon fabric	1 reel	/

Tape measure	1	±0.0005 m
String	1 reel	/
Scissor	1	/
Pen	1	/
Hole puncher	1	/
Phone camera	1	/
20g weight	1	/
Logger pro software	1	±0.001 m

Table 2: Material list

The materials used in the experiment are displayed and labelled in figure 2 below.

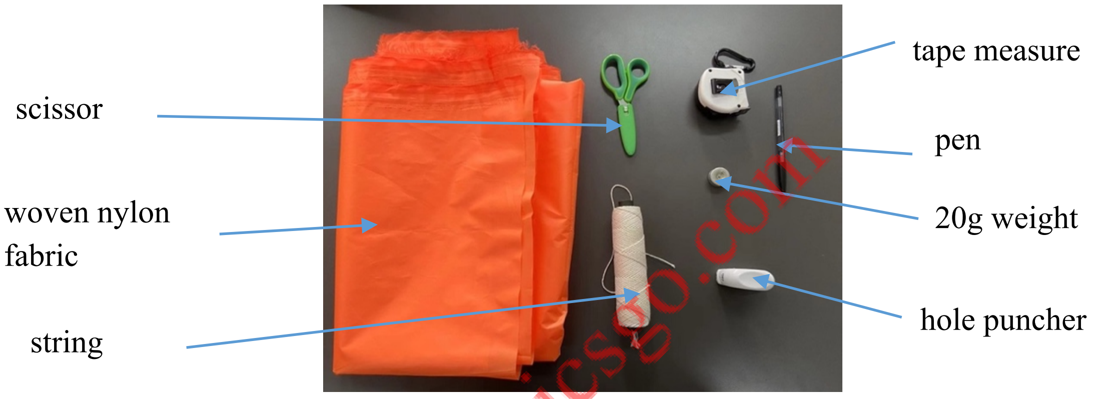

Fig. 2 Material used in the experiment

Procedure

1. Tie the reel of string on the end of a pen. Place the tape measure on the knot of the string and mark on the position of the string where the reading on the tape measure shows 10cm.
2. Cut the string at the mark. Press the end of string on the woven nylon fabric and draw a circle (radius is 10cm).
3. Cut down the circle and draw four lines from the center to divide the circle into four equal parts.
4. Use the hole puncher to make four holes at the end of each line. Measure four strings each with a length of 30cm and tie them to the holes.
5. Tie the four strings to the 20g weight to finalize the making of parachute toy. Figure 3 below shows an image of the parachute made.
6. Bring the parachute toy to the third floor of a building with a height of around 9.5 meters and drop it from the window. Meanwhile, the process of falling should be recorded by a phone camera.
7. Import the video to the Logger Pro software to obtain the terminal speed of the parachute.
8. Repeat steps 1-7 for canopies with a radius of 0.15m, 0.20m, 0.25m, and 0.30m.

Figure 3 below shows a completed toy parachute with a radius of 0.15m.

Fig. 3 An image of one finalized parachute

Safety, ethical and environmental issues

When releasing the parachute from the building, it is important to make sure no that one walks by downstairs to prevent the parachute from hitting others.

Analysis

Raw data collection

Qualitative data

During the experiment, it is observed that the parachute falls at a slower speed when the canopy of parachute has a larger surface area.

Quantitative data

Videos of the falling trajectory of each parachute are recorded and inputted into Logger Pro. Using video analysis, data for the travel displacement of the parachute can be collected. Table 3 below demonstrates the data for time and the corresponding vertical position of the parachute with a surface area of $0.10^2\pi\ \mathrm{m}^2$. Full raw data is shown in the appendix section.

Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
Time (s)	Vertical position (m)	Time (s)	Vertical position (m)	Time (s)	Vertical position (m)	Time (s)	Vertical position (m)	Time (s)	Vertical position (m)
2.233	9.117	2.533	9.128	1.167	8.523	2.767	8.739	3.433	9.020
2.367	8.693	2.667	8.884	1.300	8.357	2.900	8.544	3.567	8.724
2.500	8.338	2.800	8.453	1.433	8.011	3.033	8.187	3.700	8.331
2.633	7.913	2.933	7.898	1.567	7.590	3.167	7.726	3.833	7.927
2.767	7.350	3.067	7.311	1.700	7.193	3.300	7.256	3.967	7.357
2.900	6.697	3.200	6.725	1.833	6.625	3.433	6.653	4.100	6.755
3.033	6.115	3.333	6.033	1.967	6.036	3.567	5.998	4.233	6.010
3.166	5.595	3.467	5.404	2.100	5.542	3.700	5.341	4.367	5.478
3.300	5.032	3.600	4.762	2.233	4.934	3.833	4.689	4.500	4.881
3.433	4.458	3.733	4.155	2.367	4.318	3.967	4.010	4.633	4.313
3.567	3.904	3.867	3.537	2.500	3.677	4.100	3.400	4.767	3.735

Table 3: Time and vertical position of parachute with a surface area of $0.10^2\pi\ \mathrm{m}^2$

Determining the terminal velocity

Using the raw data, the graph for vertical displacement of parachute against time can be plotted. Since the terminal velocity is defined as the constant velocity an object reaches when falling, the terminal velocity can be obtained by determining the slope of the linear section in the vertical displacement versus time graph.

An example for this calculation is shown in figure 4 below. This example is the data for the first trial in the case when the surface area of the parachute is $0.10^2\pi\ m^2$. In figure 4, the final five points fit into a linear function, with a correlation coefficient of -0.999. The terminal velocity is the slope of this linear function, which is -4.168 m/s.

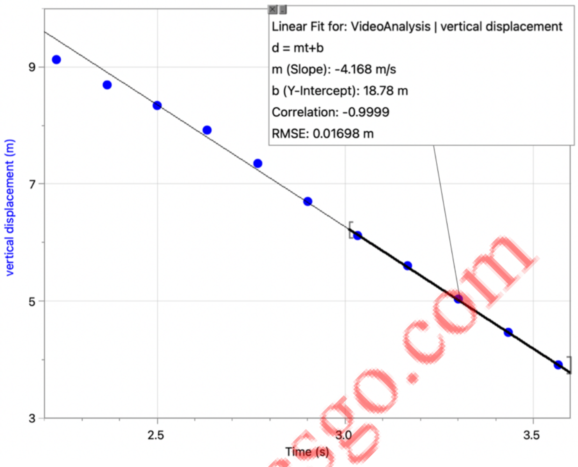

Fig. 4 Example of vertical displacement of parachute against time graph

The same method is used to determine the terminal velocity for the remaining trials. The vertical displacement graphs for the other trials are listed in the appendix. Table 4 below shows the terminal velocity for each condition.

Area (m^2), ± 0.0005	Terminal velocity (m/s), ± 0.001				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
0.0314	-4.168	-4.681	-4.660	-4.895	-4.348
0.0707	-3.835	-3.124	-3.290	-3.656	-3.754
0.1257	-2.241	-2.977	-3.133	-2.623	-2.913
0.1963	-2.025	-2.106	-1.577	-2.330	-2.272
0.2827	-1.376	-1.668	-2.279	-1.453	-2.218

Table 4: Data for surface area and terminal velocity of parachute

Sample calculation

In this section, the condition when the surface area of the parachute is 0.0314 m^2 ($0.10^2\pi\ m^2$) is used for a sample calculation. As mentioned in the exploration section, a linear relationship is expected between the square of the terminal velocity ($v_T{}^2$) and the multiplicative inverse of the canopy surface area (A^{-1}). The mean terminal velocity is calculated below:

$$v_T = \frac{(-4.168) + (-4.681) + (-4.660) + (-4.895) + (-4.348)}{5}$$

$$v_T = -4.550\ \frac{\mathrm{m}}{\mathrm{s}} \approx -4.6\ \frac{\mathrm{m}}{\mathrm{s}}$$

Thus, the square of the terminal velocity is:

$$v_T{}^2 = \left(-4.550\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 20.706\ \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

The absolute uncertainty of the terminal velocity is calculated by dividing the difference between the maximum and minimum value by 2:

$$\Delta v_\mathrm{T} = \frac{(-4.168) - (-4.895)}{2}$$

$$\Delta v_\mathrm{T} = 0.3635 \approx 0.4\ \frac{m}{s}$$

Thus, the uncertainty for the square of the terminal velocity can be obtained:

$$\frac{\Delta v_\mathrm{T}{}^2}{v_\mathrm{T}{}^2} = 2 \times \frac{\Delta v_\mathrm{T}}{v_\mathrm{T}}$$

$$\frac{\Delta v_\mathrm{T}{}^2}{20.706} = 2 \times \frac{0.4}{-4.550}$$

$$\Delta v_\mathrm{T}{}^2 \approx 3.601\ \frac{\mathrm{m}^2}{\mathrm{s}^2} \approx 4\ \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

The multiplicative inverse of canopy surface area is calculated below.

$$\frac{1}{A} = \frac{1}{0.0314}\ \frac{\mathrm{m}^2}{\mathrm{s}^2} \approx 31.8471\ \frac{1}{\mathrm{m}^2}$$

Since the uncertainty of the surface area is half of the smallest division, which is ± 0.0005 m^2, the uncertainty of the multiplicative inverse of the surface area is:

$$\frac{\Delta A^{-1}}{A^{-1}} = \left|-1 \times \frac{\Delta A}{A}\right|$$

$$\frac{\Delta A^{-1}}{31.8471} = \left|-1 \times \frac{0.0005}{0.0314}\right|$$

$$\Delta A^{-1} \approx 0.5071\ \frac{1}{\mathrm{m}^2} \approx 0.5\ \frac{1}{\mathrm{m}^2}$$

Processed data analysis

Using the method from the sample calculation above, the data for all trials is processed. The processed data is demonstrated in table 5 below:

Area (m), ± 0.0005	1/Area (1/m)	Uncertainty of 1/Area (1/m)	Terminal velocity, v_T (m/s)	Uncertainty of v_T (m/s)	${v_T}^2$ (m^2/s^2)	Uncertainty of ${v_T}^2$ (m^2/s^2)
0.0314	31.8	0.5	4.6	0.4	21	4
0.0707	14.1	0.1	3.6	0.4	13	3
0.1257	7.96	0.03	2.8	0.4	8	2
0.1963	5.09	0.01	2.0	0.4	4	2
0.2827	3.537	0.006	1.8	0.5	3	2

Table 5: Processed data

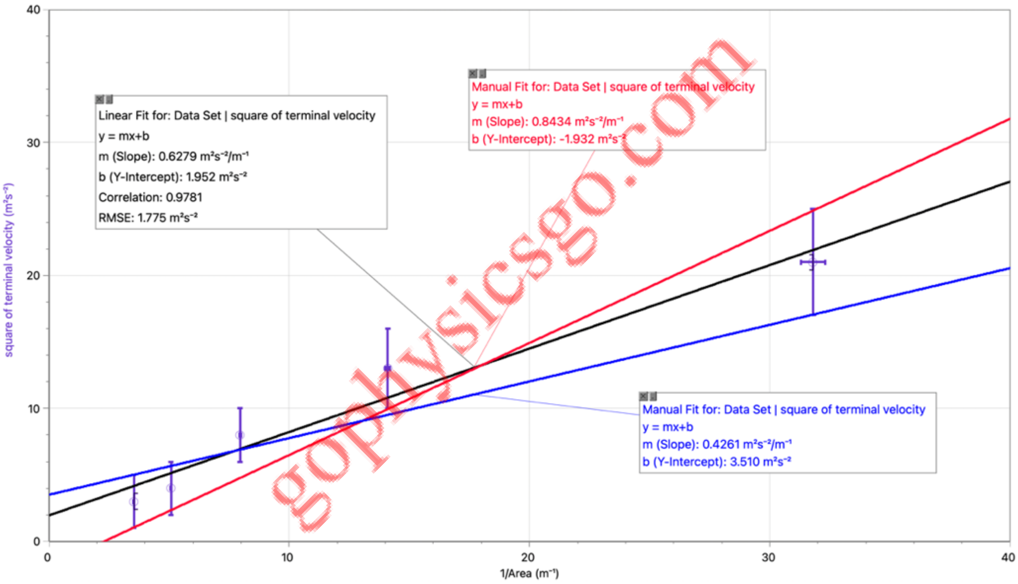

Fig. 5 Graph of square of terminal velocity against 1/Area

Figure 5 above shows the relationship between the square of the terminal velocity and the multiplicative inverse of canopy surface area. The black line represents for the best fit line, while the red and blue lines represent the worst fit lines.

According to the analysis using Logger Pro, the best fit line reveals the relationship:

$$ {v_T}^2 = 0.6279 \times \frac{1}{A} + 1.952 $$

The worst fit lines reveal the relationship:

$$ {v_T}^2 = 0.8434 \times \frac{1}{A} - 1.932, \ \ {v_T}^2 = 0.4261 \times \frac{1}{A} + 3.510 $$

The uncertainty of the slope is

$$\frac{0.8434 - 0.4261}{2} = 0.20865\ \mathrm{m^3s^{-2}} \approx 0.2\ \mathrm{m^3s^{-2}}$$

The uncertainty of the y-intercept is

$$\frac{3.510 - (-1.932)}{2} = 2.721\ \mathrm{m^2s^{-2}} \approx 3\ \mathrm{m^2s^{-2}}$$

Hence, the relationship between the square of the terminal velocity and the multiplicative inverse of the surface area considering uncertainty is

$${v_\mathrm{T}}^2 = (0.6\ \mathrm{m^3s^{-2}} \pm 0.2)\frac{1}{A} - (2\ \mathrm{m^2s^{-2}} \pm 3)$$

Conclusion

The aim of the current research is to investigate how the surface area of a flat circular canopy in a parachute toy affects its terminal velocity. It was hypothesized that there is a linear relationship between the square of the terminal velocity and the multiplicative inverse of canopy surface area.

The processed data is plotted on a graph, which revealed the relationship that ${v_\mathrm{T}}^2 = (0.6\ \mathrm{m^3s^{-2}} \pm 0.2)\,1/A - (2\ \mathrm{m^2s^{-2}} \pm 3)$. This means that when the multiplicative inverse of the canopy surface area increases by 1 m^{3}s^{-2}, the square of the terminal velocity increases by $0.6\ \mathrm{m^3s^{-2}} \pm 0.2$. This result supports the hypothesis that a linear relationship is expected between the square of the terminal velocity and the multiplicative inverse of the canopy surface area. This is because an increase in surface area increases drag force exerted on the parachute, which decreases the terminal velocity of the parachute based on the Newton's second law.

Despite a linear relationship, the results fail to maintain low random and systematic errors. As calculated in the analysis section, the uncertainty of the slope is $\pm 0.2\ \mathrm{m^3s^{-2}}$. The percentage uncertainty of the slope is $(0.2/0.6) \times 100\% \approx 33.3\%$. Since this value is larger than 10%, it can be indicated that the result is not highly precise and reliable. In addition, the correlation coefficient shows a value of 0.9781, which implies that there are some random errors present.

Since the theoretical linear relationship is $(v_\mathrm{T})^2 = M \times (1/A)$, it is expected that the linear graph would pass through the origin. However, from the graph I plotted, the best fit line does not pass the origin. This suggests that there are also some systematic errors present.

Literature value and percentage error

According to eq. 6 derived in the exploration section, the relationship$(v_\mathrm{T})^2 = M \times (1/A)$ has a constant value M, where $M = \left(\frac{2m|g|}{c\mu}\right)^2$. This means that the value of M shall be the same if the object hung under the parachute is the same, and if the parachute falls through the same fluid, which in this case is the air. However, since *c* is an unknown constant, the theoretical value cannot be obtained through direct formula calculation. Thus, my results are compared to the results from a similar study conducted by Tripathy. Based on the processed data from the literature, it can be calculated that the slope between the square of terminal velocity and the multiplicative inverse of the circular canopy surface area is approximately 0.0378 (Tripathy, 2023). The object hung under the parachute in the literature has a mass of 4 grams, which is five times smaller than the mass in my study. Since constant *M* is proportional to the square of object mass, the literature value can be deduced:

$$\text{literature value} = 0.0378 \times 5^2 = 0.945$$

While in my investigation, the slope appears to be around 0.628.

Thus, the percentage error can be calculated using the formula:

$$\text{percentage error} = \frac{|\text{experiment value} - \text{literature value}|}{\text{literature value}} \times 100\%$$

$$\text{percentage error} = \frac{0.945 - 0.628}{0.945} \times 100\% \approx 33.5\%$$

Since the percentage error is larger than 10%, the result fails to reach high accuracy.

Evaluation

As mentioned, the results have 33.3% random error, some systematic error, and 33.5% inaccuracy. This means that there are certain limitations in the current investigation. Since the relationship between the independent variable and dependent variable manages to meet the hypothesis, strengths are also present.

Weaknesses

- Firstly, there are variables which are difficult to be controlled. For example, wind may be present during the fall. When the direction of parachute motion is the same as the direction of the wind, the wind exerts a force which increases the net force on the parachute, thus making the terminal velocity measured larger. Conversely, when the direction of parachute motion is opposite to the direction of wind, the terminal velocity measured appears to be smaller. Hence, the uncontrolled wind could lead to inaccuracy.
- Secondly, since nylon woven fabric has relatively high flexibility, the shape of parachute canopy changes in shape due to the tension from the strings. This could decrease the surface area provided by the parachute as some parts of the fabric may be folded inwards.

Therefore, the actual surface area is smaller than the value I use for data analysis. This could lead to an increase in systematic errors.

- Thirdly, while dropping the parachute, I may unintentionally add a force to it. An extraneous force increases or decreases the net force of the parachute, affecting its acceleration. The change in acceleration can lead to a change in terminal velocity, which is a possible reason for random errors.
- Fourthly, for some trials the parachute fails to fall in a straight line. One reason could be that the string used in the experiment sometimes twist together during the fall, causing a slightly curved trajectory.

Improvements

In response to the weaknesses mentioned in the section above, I proposed the following improvements.

- To minimize the effect of wind, one improvement could be conducting an indoor experiment. For example, I can go to a gym or an empty place to drop the parachute.
- Dealing with the second weakness, one alternative is using fabric with greater density and rigidity. This makes the canopy less likely to be deformed due to the string. Another possible approach could be building a frame with materials such as wood stripes and wires under the canopy for fixation purposes. However, in this case, the mass of canopy should be considered.
- Regarding to the issue of unintentionally creating extra force, I can be more careful with my way of dropping the parachute. For example, I can hold the two sides of the canopy between the fingers so that the parachute can be dropped simply by opening my fingers. This cause much less extraneous forces compared to dropping using the entire hand.

Strengths

Despite limitations, there are also strengths in the study:

- For each variation in the value of independent variable (which is the surface area of parachute), five trials are conducted, and the average terminal velocity is determined. This helps decreases random errors and makes the results more accurate.
- In the analysis part, video analysis is used to determine the terminal velocity of the parachute. This method is more accurate compared to calculation of terminal velocity based on falling duration. This is because video analysis provides a specific trajectory of falling in forms of *position versus time* graphs. Using the graph, I can identify which part is linear, which indicates when the object reaches terminate velocity.

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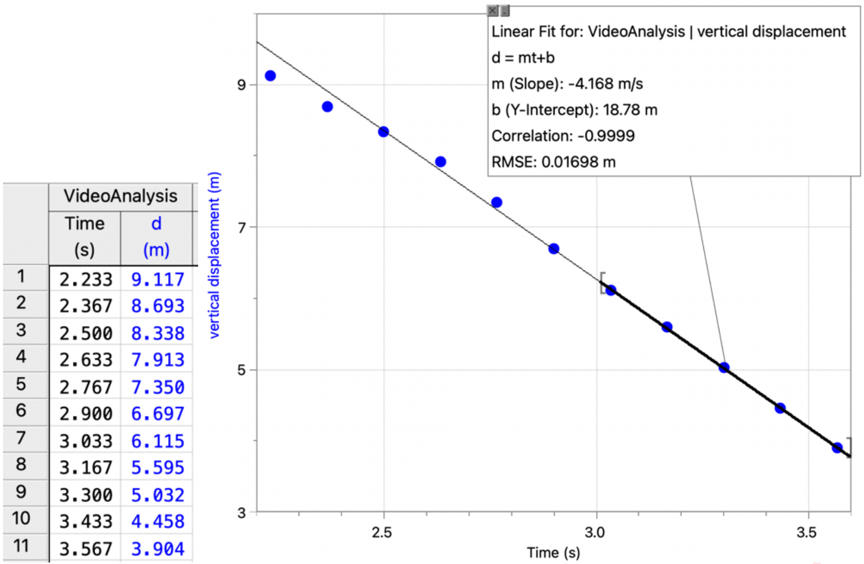
Appendix

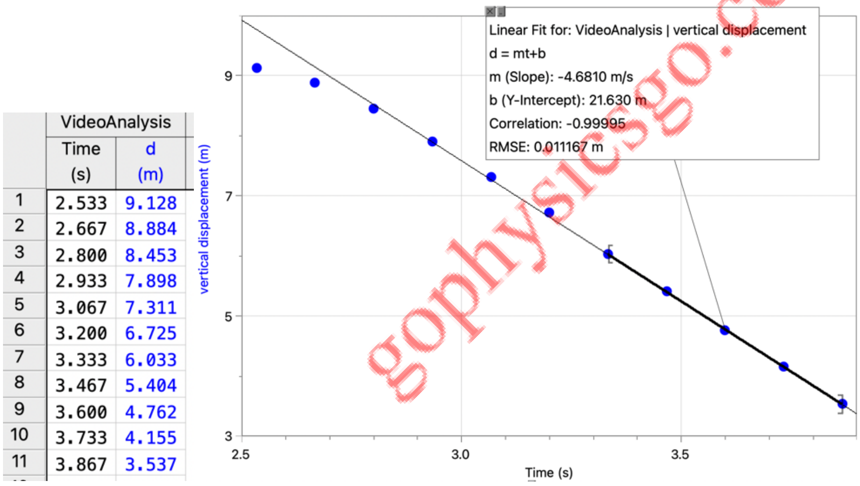
Condition 1: surface area = $0.10^2\pi$ m^2



	VideoAnalysis Time (s)	VideoAnalysis d (m)
1	2.233	9.117
2	2.367	8.693
3	2.500	8.338
4	2.633	7.913
5	2.767	7.350
6	2.900	6.697
7	3.033	6.115
8	3.167	5.595
9	3.300	5.032
10	3.433	4.458
11	3.567	3.904

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -4.168 m/s
b (Y-Intercept): 18.78 m
Correlation: -0.9999
RMSE: 0.01698 m

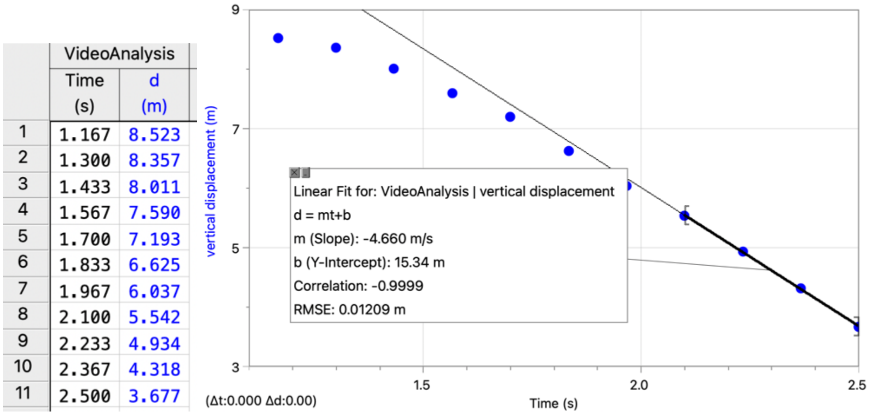
trial 1



	VideoAnalysis Time (s)	VideoAnalysis d (m)
1	2.533	9.128
2	2.667	8.884
3	2.800	8.453
4	2.933	7.898
5	3.067	7.311
6	3.200	6.725
7	3.333	6.033
8	3.467	5.404
9	3.600	4.762
10	3.733	4.155
11	3.867	3.537

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -4.6810 m/s
b (Y-Intercept): 21.630 m
Correlation: -0.99995
RMSE: 0.011167 m

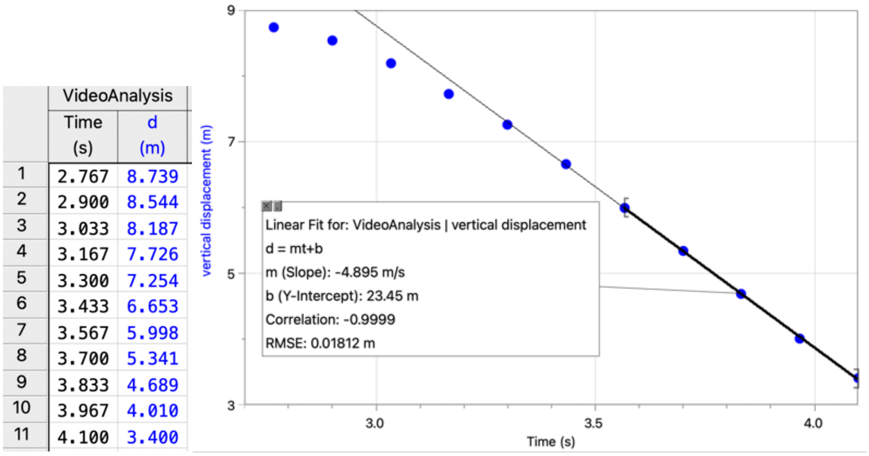
trial 2



	VideoAnalysis Time (s)	VideoAnalysis d (m)
1	1.167	8.523
2	1.300	8.357
3	1.433	8.011
4	1.567	7.590
5	1.700	7.193
6	1.833	6.625
7	1.967	6.037
8	2.100	5.542
9	2.233	4.934
10	2.367	4.318
11	2.500	3.677

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -4.660 m/s
b (Y-Intercept): 15.34 m
Correlation: -0.9999
RMSE: 0.01209 m

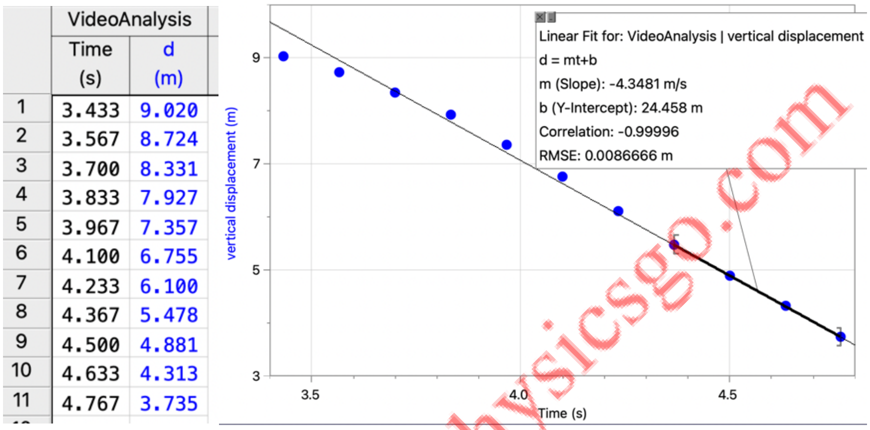
trial 3



VideoAnalysis		
	Time (s)	d (m)
1	2.767	8.739
2	2.900	8.544
3	3.033	8.187
4	3.167	7.726
5	3.300	7.254
6	3.433	6.653
7	3.567	5.998
8	3.700	5.341
9	3.833	4.689
10	3.967	4.010
11	4.100	3.400

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -4.895 m/s
b (Y-Intercept): 23.45 m
Correlation: -0.9999
RMSE: 0.01812 m

trial 4

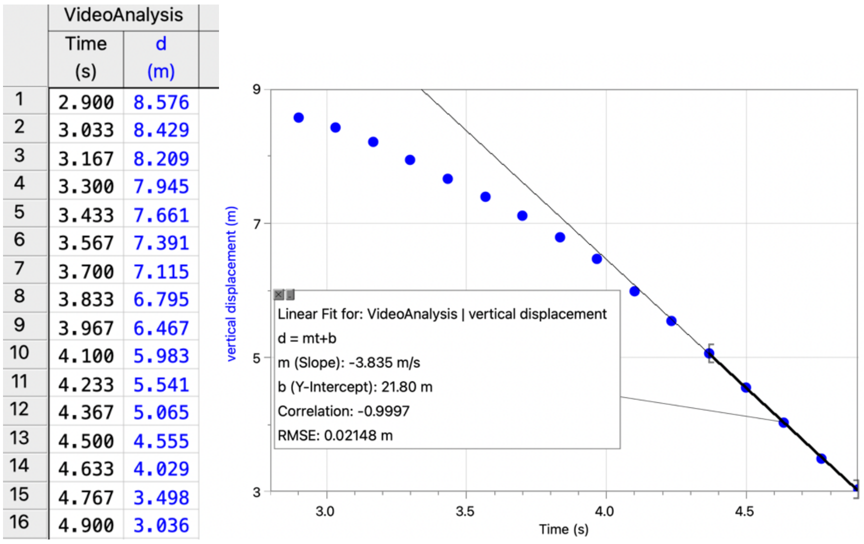


VideoAnalysis		
	Time (s)	d (m)
1	3.433	9.020
2	3.567	8.724
3	3.700	8.331
4	3.833	7.927
5	3.967	7.357
6	4.100	6.755
7	4.233	6.100
8	4.367	5.478
9	4.500	4.881
10	4.633	4.313
11	4.767	3.735

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -4.3481 m/s
b (Y-Intercept): 24.458 m
Correlation: -0.99996
RMSE: 0.0086666 m

trial 5

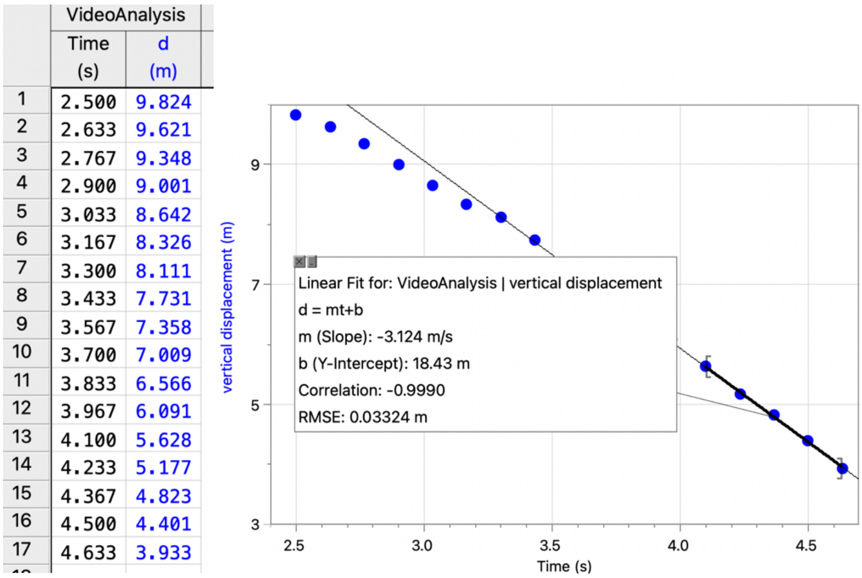
Condition 2: surface area = $0.15^2\pi$ m^2



VideoAnalysis		
	Time (s)	d (m)
1	2.900	8.576
2	3.033	8.429
3	3.167	8.209
4	3.300	7.945
5	3.433	7.661
6	3.567	7.391
7	3.700	7.115
8	3.833	6.795
9	3.967	6.467
10	4.100	5.983
11	4.233	5.541
12	4.367	5.065
13	4.500	4.555
14	4.633	4.029
15	4.767	3.498
16	4.900	3.036

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.835 m/s
b (Y-Intercept): 21.80 m
Correlation: -0.9997
RMSE: 0.02148 m

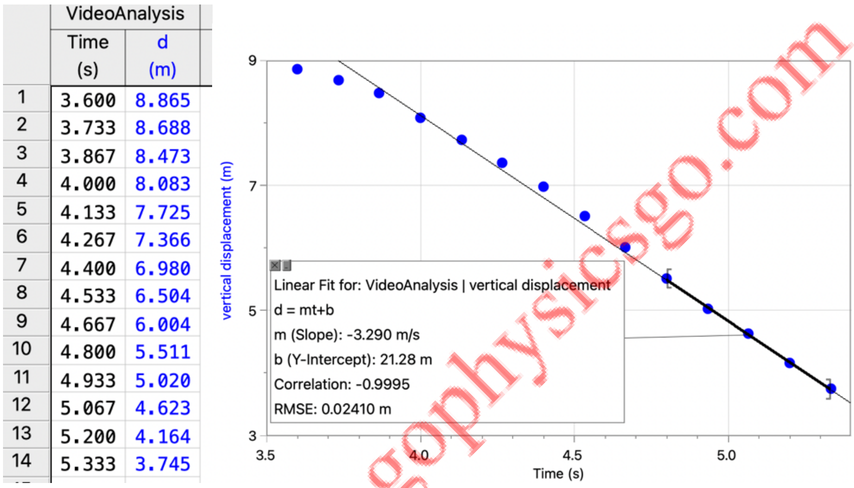
trial 1



	VideoAnalysis	
	Time (s)	d (m)
1	2.500	9.824
2	2.633	9.621
3	2.767	9.348
4	2.900	9.001
5	3.033	8.642
6	3.167	8.326
7	3.300	8.111
8	3.433	7.731
9	3.567	7.358
10	3.700	7.009
11	3.833	6.566
12	3.967	6.091
13	4.100	5.628
14	4.233	5.177
15	4.367	4.823
16	4.500	4.401
17	4.633	3.933

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.124 m/s
b (Y-Intercept): 18.43 m
Correlation: -0.9990
RMSE: 0.03324 m

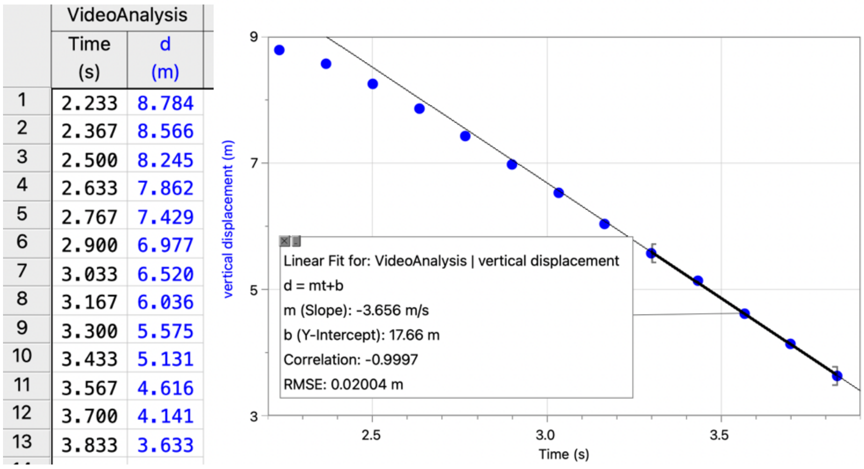
trial 2



	VideoAnalysis	
	Time (s)	d (m)
1	3.600	8.865
2	3.733	8.688
3	3.867	8.473
4	4.000	8.083
5	4.133	7.725
6	4.267	7.366
7	4.400	6.980
8	4.533	6.504
9	4.667	6.004
10	4.800	5.511
11	4.933	5.020
12	5.067	4.623
13	5.200	4.164
14	5.333	3.745

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.290 m/s
b (Y-Intercept): 21.28 m
Correlation: -0.9995
RMSE: 0.02410 m

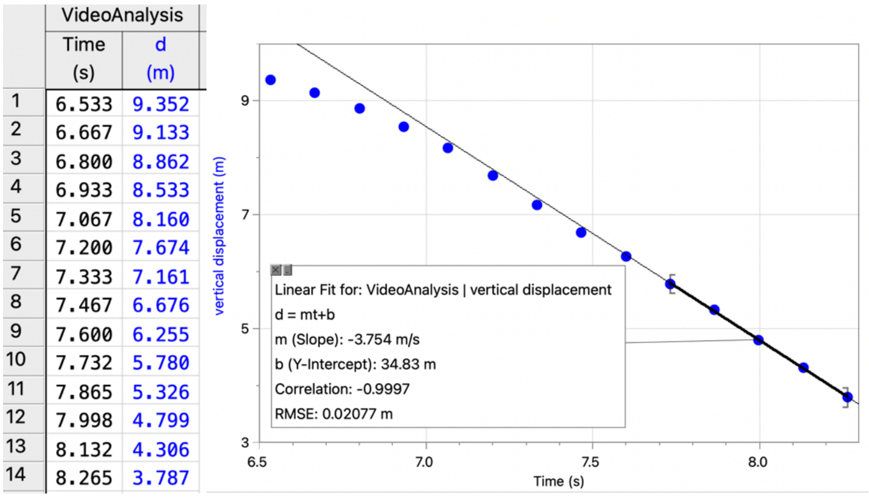
trial 3



	VideoAnalysis	
	Time (s)	d (m)
1	2.233	8.784
2	2.367	8.566
3	2.500	8.245
4	2.633	7.862
5	2.767	7.429
6	2.900	6.977
7	3.033	6.520
8	3.167	6.036
9	3.300	5.575
10	3.433	5.131
11	3.567	4.616
12	3.700	4.141
13	3.833	3.633

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.656 m/s
b (Y-Intercept): 17.66 m
Correlation: -0.9997
RMSE: 0.02004 m

trial 4

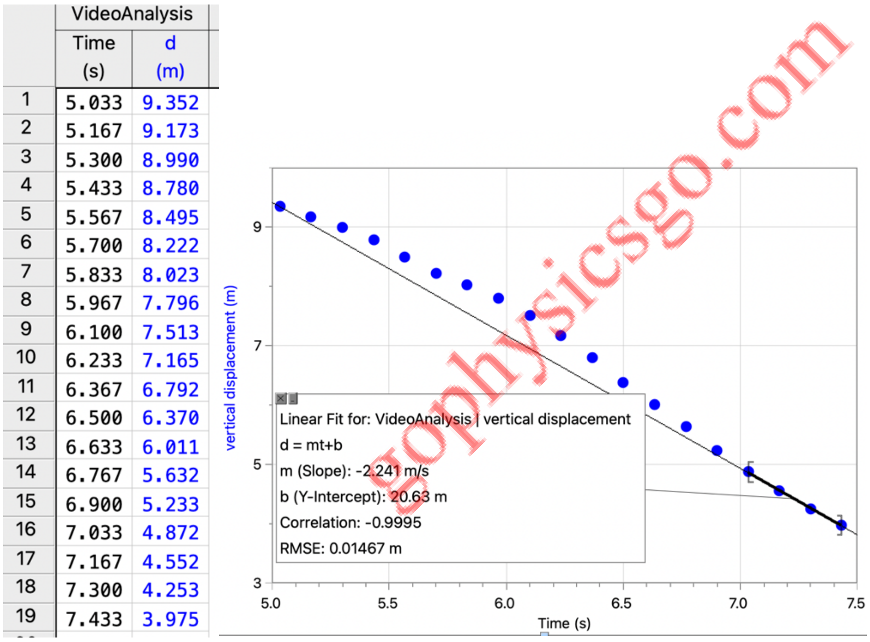


	VideoAnalysis	
	Time (s)	d (m)
1	6.533	9.352
2	6.667	9.133
3	6.800	8.862
4	6.933	8.533
5	7.067	8.160
6	7.200	7.674
7	7.333	7.161
8	7.467	6.676
9	7.600	6.255
10	7.732	5.780
11	7.865	5.326
12	7.998	4.799
13	8.132	4.306
14	8.265	3.787

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.754 m/s
b (Y-Intercept): 34.83 m
Correlation: -0.9997
RMSE: 0.02077 m

trial 5

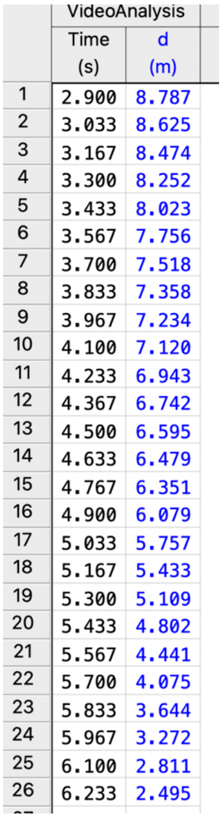
Condition 3: surface area = $0.20^2\pi$ m^2



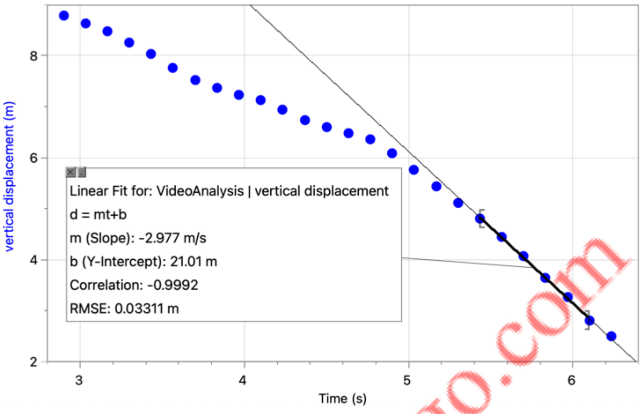
	VideoAnalysis	
	Time (s)	d (m)
1	5.033	9.352
2	5.167	9.173
3	5.300	8.990
4	5.433	8.780
5	5.567	8.495
6	5.700	8.222
7	5.833	8.023
8	5.967	7.796
9	6.100	7.513
10	6.233	7.165
11	6.367	6.792
12	6.500	6.370
13	6.633	6.011
14	6.767	5.632
15	6.900	5.233
16	7.033	4.872
17	7.167	4.552
18	7.300	4.253
19	7.433	3.975

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.241 m/s
b (Y-Intercept): 20.63 m
Correlation: -0.9995
RMSE: 0.01467 m

trial 1



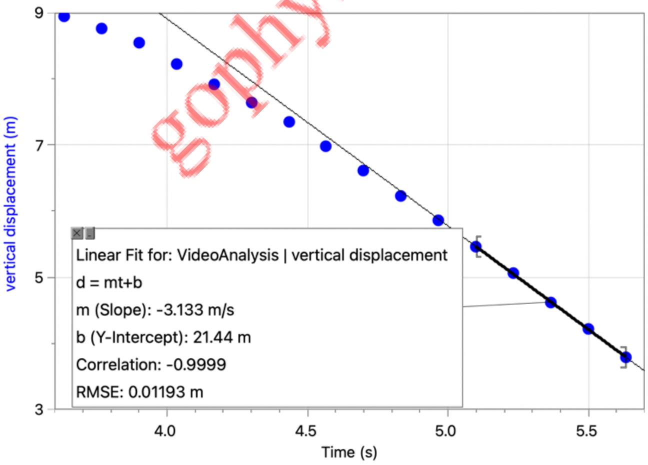
	VideoAnalysis	
	Time (s)	d (m)
1	2.900	8.787
2	3.033	8.625
3	3.167	8.474
4	3.300	8.252
5	3.433	8.023
6	3.567	7.756
7	3.700	7.518
8	3.833	7.358
9	3.967	7.234
10	4.100	7.120
11	4.233	6.943
12	4.367	6.742
13	4.500	6.595
14	4.633	6.479
15	4.767	6.351
16	4.900	6.079
17	5.033	5.757
18	5.167	5.433
19	5.300	5.109
20	5.433	4.802
21	5.567	4.441
22	5.700	4.075
23	5.833	3.644
24	5.967	3.272
25	6.100	2.811
26	6.233	2.495



Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.977 m/s
b (Y-Intercept): 21.01 m
Correlation: -0.9992
RMSE: 0.03311 m

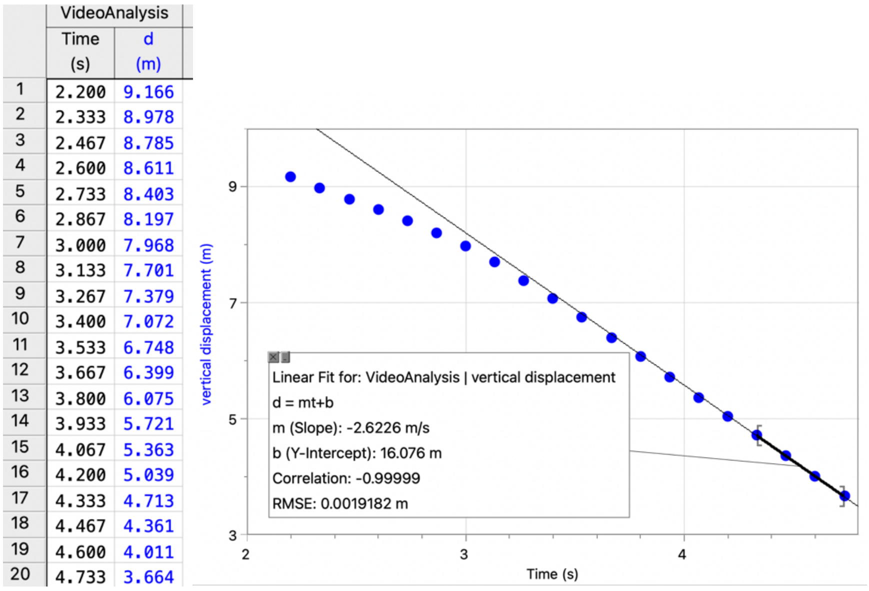
trial 2

	VideoAnalysis	
	Time (s)	d (m)
1	3.633	8.936
2	3.767	8.753
3	3.900	8.543
4	4.033	8.222
5	4.167	7.920
6	4.300	7.635
7	4.433	7.344
8	4.567	6.984
9	4.700	6.615
10	4.833	6.230
11	4.967	5.856
12	5.100	5.459
13	5.233	5.060
14	5.367	4.616
15	5.500	4.219
16	5.633	3.791



Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -3.133 m/s
b (Y-Intercept): 21.44 m
Correlation: -0.9999
RMSE: 0.01193 m

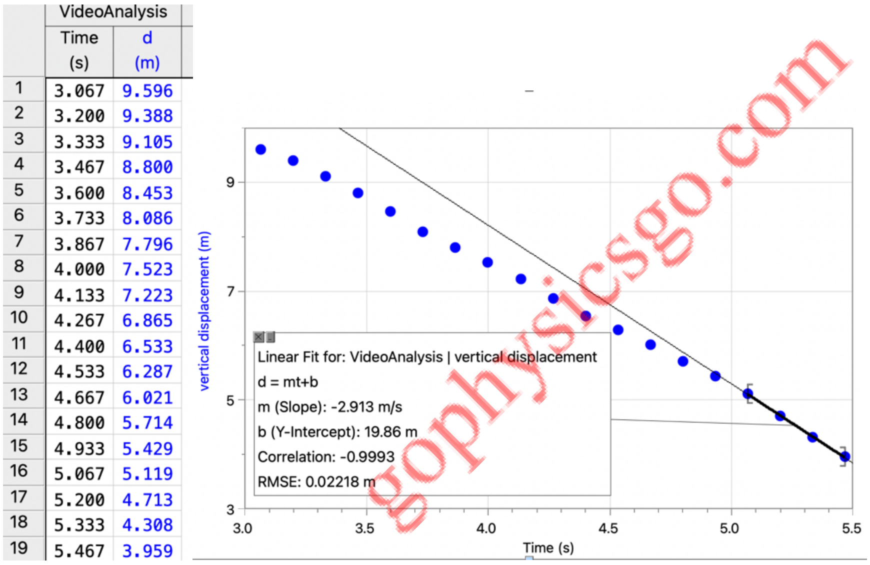
trial 3



	VideoAnalysis	
	Time (s)	d (m)
1	2.200	9.166
2	2.333	8.978
3	2.467	8.785
4	2.600	8.611
5	2.733	8.403
6	2.867	8.197
7	3.000	7.968
8	3.133	7.701
9	3.267	7.379
10	3.400	7.072
11	3.533	6.748
12	3.667	6.399
13	3.800	6.075
14	3.933	5.721
15	4.067	5.363
16	4.200	5.039
17	4.333	4.713
18	4.467	4.361
19	4.600	4.011
20	4.733	3.664

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.6226 m/s
b (Y-Intercept): 16.076 m
Correlation: -0.99999
RMSE: 0.0019182 m

trial 4

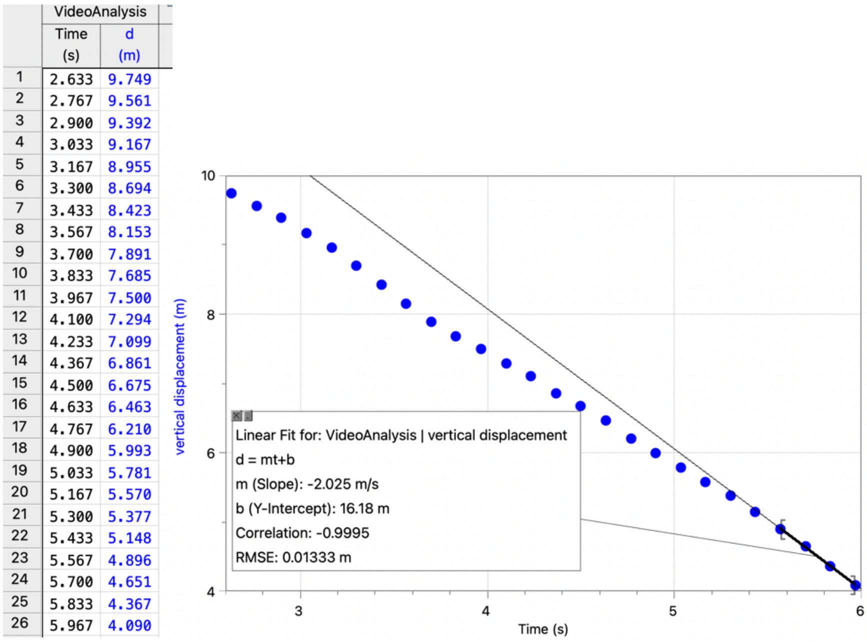


	VideoAnalysis	
	Time (s)	d (m)
1	3.067	9.596
2	3.200	9.388
3	3.333	9.105
4	3.467	8.800
5	3.600	8.453
6	3.733	8.086
7	3.867	7.796
8	4.000	7.523
9	4.133	7.223
10	4.267	6.865
11	4.400	6.533
12	4.533	6.287
13	4.667	6.021
14	4.800	5.714
15	4.933	5.429
16	5.067	5.119
17	5.200	4.713
18	5.333	4.308
19	5.467	3.959

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.913 m/s
b (Y-Intercept): 19.86 m
Correlation: -0.9993
RMSE: 0.02218 m

trial 5

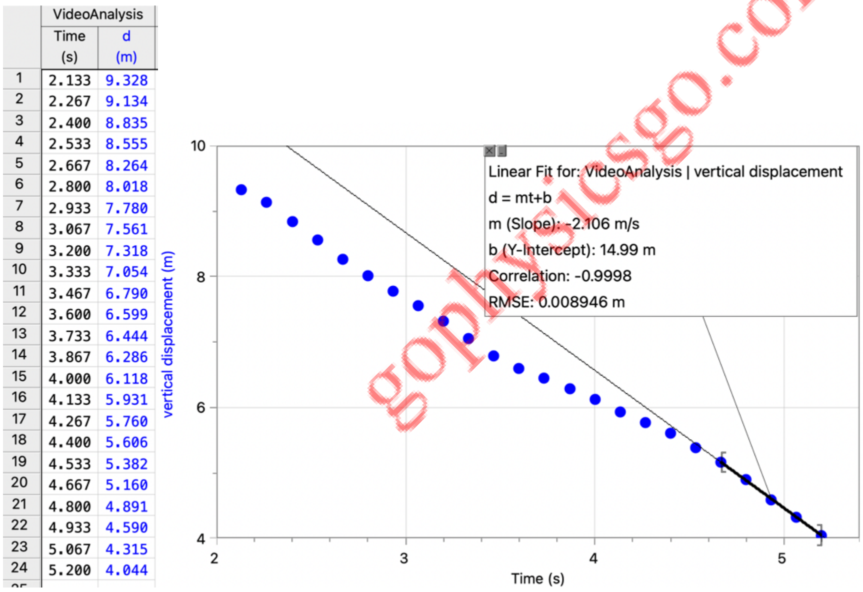
Condition 4: surface area = $0.25^2\pi\ \text{m}^2$



	VideoAnalysis	
	Time (s)	d (m)
1	2.633	9.749
2	2.767	9.561
3	2.900	9.392
4	3.033	9.167
5	3.167	8.955
6	3.300	8.694
7	3.433	8.423
8	3.567	8.153
9	3.700	7.891
10	3.833	7.685
11	3.967	7.500
12	4.100	7.294
13	4.233	7.099
14	4.367	6.861
15	4.500	6.675
16	4.633	6.463
17	4.767	6.210
18	4.900	5.993
19	5.033	5.781
20	5.167	5.570
21	5.300	5.377
22	5.433	5.148
23	5.567	4.896
24	5.700	4.651
25	5.833	4.367
26	5.967	4.090

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.025 m/s
b (Y-Intercept): 16.18 m
Correlation: -0.9995
RMSE: 0.01333 m

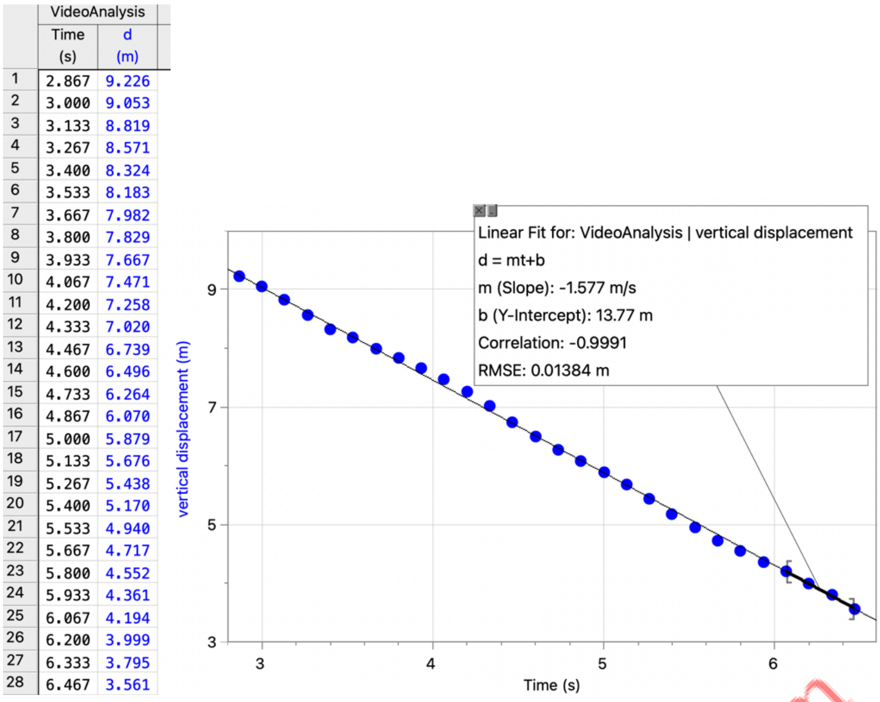
trial 1



	VideoAnalysis	
	Time (s)	d (m)
1	2.133	9.328
2	2.267	9.134
3	2.400	8.835
4	2.533	8.555
5	2.667	8.264
6	2.800	8.018
7	2.933	7.780
8	3.067	7.561
9	3.200	7.318
10	3.333	7.054
11	3.467	6.790
12	3.600	6.599
13	3.733	6.444
14	3.867	6.286
15	4.000	6.118
16	4.133	5.931
17	4.267	5.760
18	4.400	5.606
19	4.533	5.382
20	4.667	5.160
21	4.800	4.891
22	4.933	4.590
23	5.067	4.315
24	5.200	4.044

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.106 m/s
b (Y-Intercept): 14.99 m
Correlation: -0.9998
RMSE: 0.008946 m

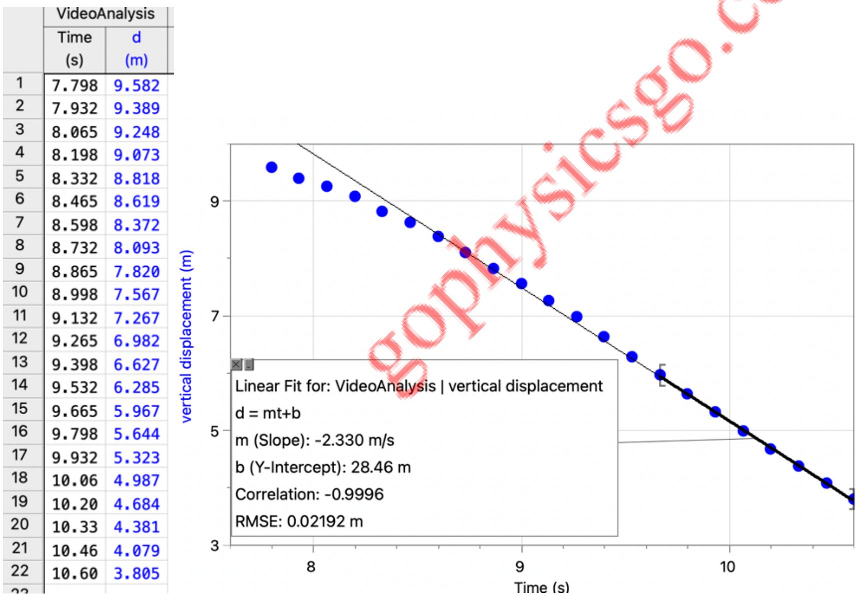
trial 2



	VideoAnalysis	
	Time (s)	d (m)
1	2.867	9.226
2	3.000	9.053
3	3.133	8.819
4	3.267	8.571
5	3.400	8.324
6	3.533	8.183
7	3.667	7.982
8	3.800	7.829
9	3.933	7.667
10	4.067	7.471
11	4.200	7.258
12	4.333	7.020
13	4.467	6.739
14	4.600	6.496
15	4.733	6.264
16	4.867	6.070
17	5.000	5.879
18	5.133	5.676
19	5.267	5.438
20	5.400	5.170
21	5.533	4.940
22	5.667	4.717
23	5.800	4.552
24	5.933	4.361
25	6.067	4.194
26	6.200	3.999
27	6.333	3.795
28	6.467	3.561

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -1.577 m/s
b (Y-Intercept): 13.77 m
Correlation: -0.9991
RMSE: 0.01384 m

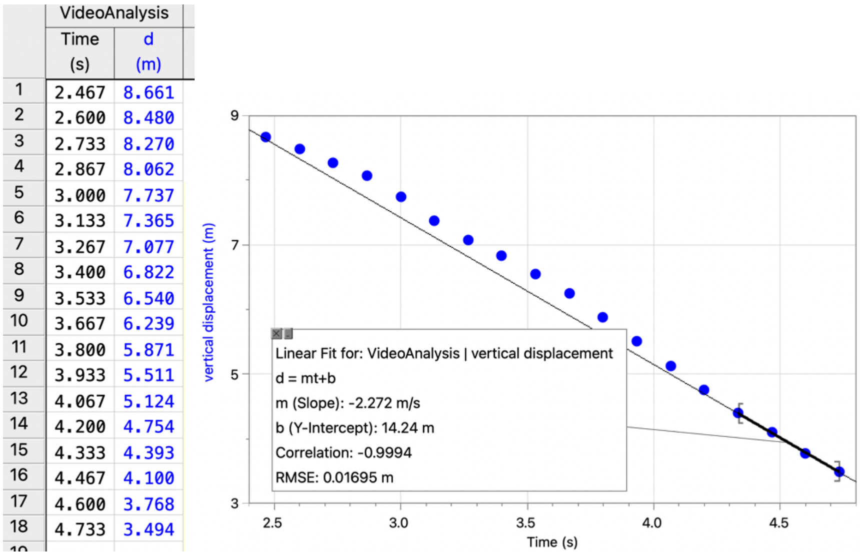
trial 3



	VideoAnalysis	
	Time (s)	d (m)
1	7.798	9.582
2	7.932	9.389
3	8.065	9.248
4	8.198	9.073
5	8.332	8.818
6	8.465	8.619
7	8.598	8.372
8	8.732	8.093
9	8.865	7.820
10	8.998	7.567
11	9.132	7.267
12	9.265	6.982
13	9.398	6.627
14	9.532	6.285
15	9.665	5.967
16	9.798	5.644
17	9.932	5.323
18	10.06	4.987
19	10.20	4.684
20	10.33	4.381
21	10.46	4.079
22	10.60	3.805

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.330 m/s
b (Y-Intercept): 28.46 m
Correlation: -0.9996
RMSE: 0.02192 m

trial 4



	VideoAnalysis	
	Time (s)	d (m)
1	2.467	8.661
2	2.600	8.480
3	2.733	8.270
4	2.867	8.062
5	3.000	7.737
6	3.133	7.365
7	3.267	7.077
8	3.400	6.822
9	3.533	6.540
10	3.667	6.239
11	3.800	5.871
12	3.933	5.511
13	4.067	5.124
14	4.200	4.754
15	4.333	4.393
16	4.467	4.100
17	4.600	3.768
18	4.733	3.494

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.272 m/s
b (Y-Intercept): 14.24 m
Correlation: -0.9994
RMSE: 0.01695 m

trial 5

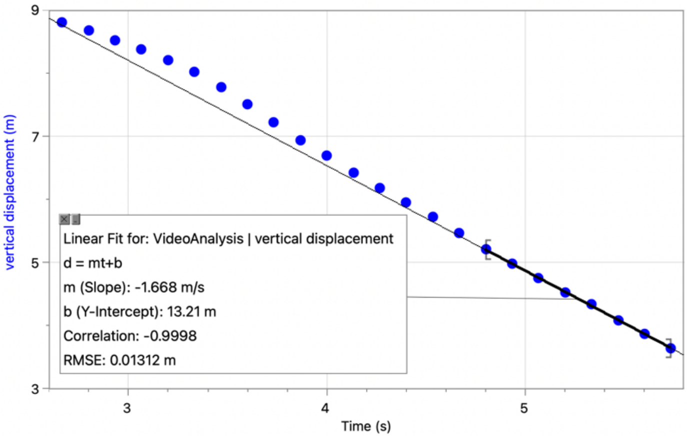
Condition 5: surface area = $0.30^2\pi\ \mathrm{m}^2$

	VideoAnalysis	
	Time (s)	d (m)
1	4.333	9.464
2	4.467	9.349
3	4.600	9.211
4	4.733	9.109
5	4.867	8.970
6	5.000	8.864
7	5.133	8.727
8	5.267	8.564
9	5.400	8.376
10	5.533	8.139
11	5.667	7.868
12	5.800	7.640
13	5.933	7.369
14	6.067	7.125
15	6.200	6.909
16	6.333	6.715
17	6.467	6.527
18	6.600	6.302
19	6.733	6.120
20	6.867	5.894
21	7.000	5.657
22	7.133	5.433
23	7.267	5.208
24	7.400	5.008
25	7.533	4.845
26	7.667	4.645
27	7.798	4.463
28	7.932	4.261
29	8.065	4.092
30	8.198	3.916

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -1.376 m/s
b (Y-Intercept): 15.19 m
Correlation: -0.9995
RMSE: 0.01046 m

trial 1

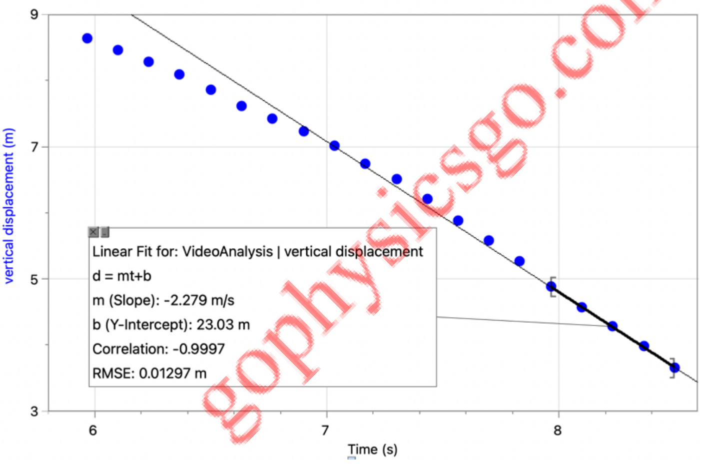
	VideoAnalysis	
	Time (s)	d (m)
1	2.667	8.810
2	2.800	8.675
3	2.933	8.517
4	3.067	8.374
5	3.200	8.206
6	3.333	8.014
7	3.467	7.780
8	3.600	7.512
9	3.733	7.217
10	3.867	6.940
11	4.000	6.691
12	4.133	6.422
13	4.267	6.181
14	4.400	5.957
15	4.533	5.730
16	4.667	5.473
17	4.800	5.207
18	4.933	4.980
19	5.067	4.748
20	5.200	4.530
21	5.333	4.343
22	5.467	4.087
23	5.600	3.867
24	5.733	3.644



Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -1.668 m/s
b (Y-Intercept): 13.21 m
Correlation: -0.9998
RMSE: 0.01312 m

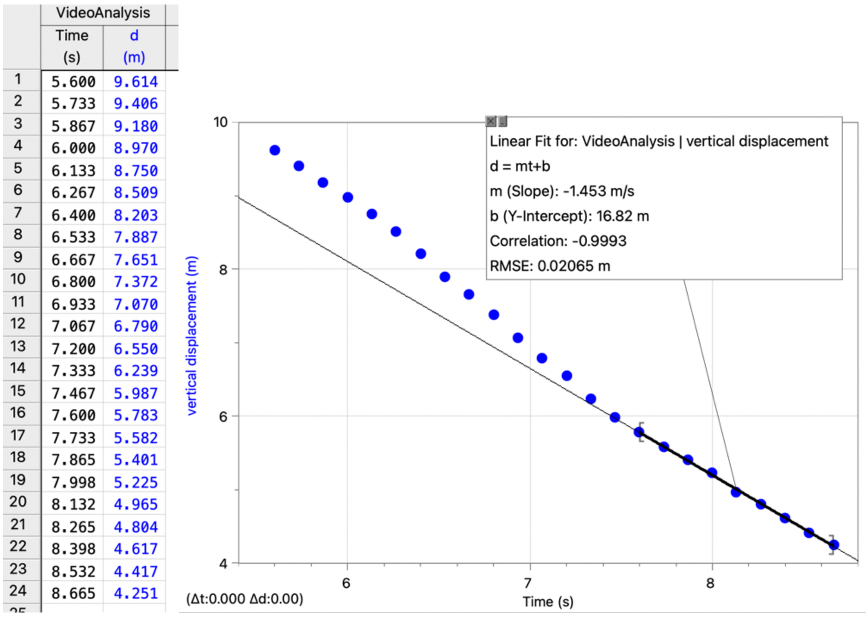
trial 2

	VideoAnalysis	
	Time (s)	d (m)
1	5.967	8.641
2	6.100	8.460
3	6.233	8.281
4	6.367	8.095
5	6.500	7.861
6	6.633	7.613
7	6.767	7.429
8	6.900	7.232
9	7.033	7.017
10	7.167	6.737
11	7.300	6.504
12	7.433	6.202
13	7.567	5.881
14	7.700	5.587
15	7.832	5.265
16	7.965	4.879
17	8.098	4.569
18	8.232	4.289
19	8.365	3.978
20	8.498	3.655



Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.279 m/s
b (Y-Intercept): 23.03 m
Correlation: -0.9997
RMSE: 0.01297 m

trial 3

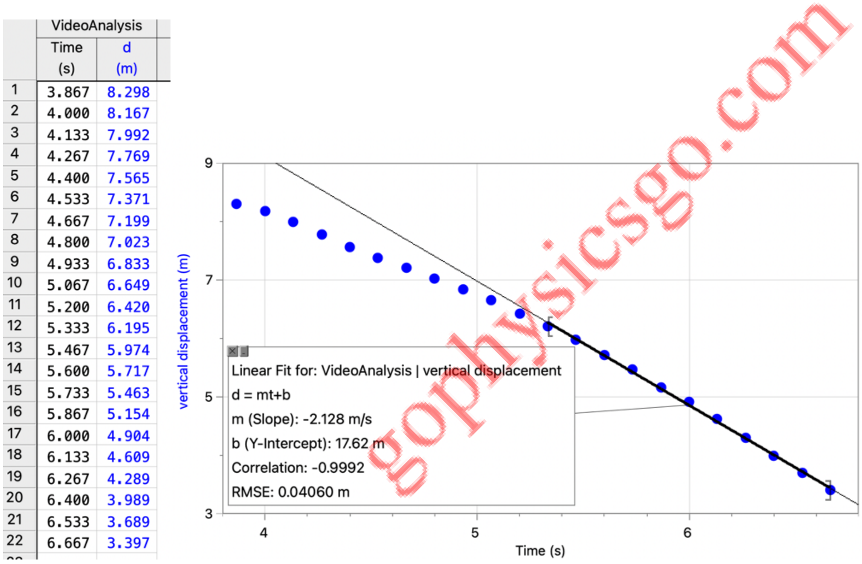


	VideoAnalysis	
	Time (s)	d (m)
1	5.600	9.614
2	5.733	9.406
3	5.867	9.180
4	6.000	8.970
5	6.133	8.750
6	6.267	8.509
7	6.400	8.203
8	6.533	7.887
9	6.667	7.651
10	6.800	7.372
11	6.933	7.070
12	7.067	6.790
13	7.200	6.550
14	7.333	6.239
15	7.467	5.987
16	7.600	5.783
17	7.733	5.582
18	7.865	5.401
19	7.998	5.225
20	8.132	4.965
21	8.265	4.804
22	8.398	4.617
23	8.532	4.417
24	8.665	4.251

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -1.453 m/s
b (Y-Intercept): 16.82 m
Correlation: -0.9993
RMSE: 0.02065 m

(Δt:0.000 Δd:0.00)

trial 4



	VideoAnalysis	
	Time (s)	d (m)
1	3.867	8.298
2	4.000	8.167
3	4.133	7.992
4	4.267	7.769
5	4.400	7.565
6	4.533	7.371
7	4.667	7.199
8	4.800	7.023
9	4.933	6.833
10	5.067	6.649
11	5.200	6.420
12	5.333	6.195
13	5.467	5.974
14	5.600	5.717
15	5.733	5.463
16	5.867	5.154
17	6.000	4.904
18	6.133	4.609
19	6.267	4.289
20	6.400	3.989
21	6.533	3.689
22	6.667	3.397

Linear Fit for: VideoAnalysis | vertical displacement
d = mt+b
m (Slope): -2.128 m/s
b (Y-Intercept): 17.62 m
Correlation: -0.9992
RMSE: 0.04060 m

trial 5