How will the recoil distance be affected by the mass of the projectile when the projectile launcher remains the same mass and medium?

Background information

The research question is to investigate the relationship between the recoil distance of a launcher, which is the instantaneous reaction force of the initial propulsive force, and the mass of the projectile.

According to Newton's Third Law of Motion, if one object exerts a force on another object, then the second object will exert an equal and opposite force on the first object(Creaco et al.). Recoil force is the force that the projectile exerts on the launcher. It is the instantaneous reaction force of the initial propulsive force of the projectile. High pressure gas will act on the bottom of the chamber, generating a force that drives the weapon to move backwards, which is called recoil force. Conservation of momentum states that if a system is not subjected to external forces, or if the vector sum of the external forces is 0 Newtons, then the total momentum of the system remains unchanged.(Chen)

The relationship between the recoil distance of the launcher and the recoil force and the mass of the projectile

A artillery vehicle parked on level ground at an elevation angle θ Launching the shell (Figure 1), the exit velocity of the shell relative to the artillery vehicle is u, and the masses of the artillery vehicle and shell are M and m, respectively. Ignoring ground friction, determine: (a) the recoil velocity of the artillery vehicle; (b) If the barrel length is l, the distance at which the artillery vehicle recoils during the firing process.

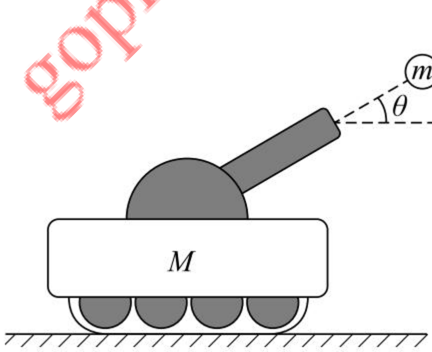

Figure 1 The artillery and shell

Taking the system composed of artillery vehicles and shells as the research object, the ground is selected as the reference frame. Ignoring ground friction, the system is subjected to a resultant force of 0 Newtons in the horizontal direction, resulting in conservation of momentum in the horizontal direction. (The first two alphabet of subscript represent vehicle, ground or cannonball. The first object is referenced by the second object. The third alphabet represents direction. e.g. v_{cgh} represents the velocity of the cannonball related to ground in horizontal direction)

$$Mv_{\mathrm{vg}} + mv_{\mathrm{cgh}} = 0$$

According to the Galilean velocity transformation relationship

$$v_{\mathrm{cg}} = v_{\mathrm{cv}} + v_{\mathrm{vg}}$$

The horizontal component is

$$v_{\mathrm{cgh}} = v_{\mathrm{vgh}} + v_{\mathrm{vg}}$$

$$v_{\mathrm{cgh}} = u_{\mathrm{cvh}} \cos\theta$$

The recoil speed of the artillery vehicle is

$$v_{\mathrm{vg}} = -\frac{m}{m+M} u_{\mathrm{cvh}} \cos\theta$$

During the process of launching a projectile, assuming that the velocity of the projectile relative to the artillery vehicle at time *t* is *u(t)* and the velocity of the artillery vehicle relative to the ground at time *t* is v*(t)*, then the recoil velocity of the artillery vehicle is

$$v(t) = -\frac{m}{m+M} u(t) \cos\theta$$

The recoil distance of the artillery vehicle during the entire launch process is

$$\Delta x = \int_0^t v(t) = -\frac{m\cos\theta}{m+M} \int_0^t u(t) dt$$

$\int_0^t u(t) dt$ is the distance relative to the movement of the artillery vehicle before the projectile exits the barrel, the length *l* of the barrel. The recoil distance is

$$\Delta x = -\frac{ml\cos\theta}{m+M}$$

(Li et al.)

According to this equation, the recoil distance by the recoil forces is positively correlated to the mass of projectile.

Hypothesis

The increase of the mass of the projectile causes the increase of the recoil distance. The increase of the mass will bring the increase of the kinetic energy of the object and the recoil force, which is the reaction force of the propulsive force of the object.

Methodology

Independent Variable and Dependent Variable

Table 1 below shows the variables and method of changing independent variable and measuring dependent variable.

Independent Variable and how to change	The mass of the projectile. It will be manipulated by the mass of mixture of white vinegar solution and baking soda in the bottle(100, 200, 300, 400, 500ml).
Dependent Variable and how to measure	The distance of the launcher repelled by recoil force. It will be measured by a ruler.

Table 1

Controlled Variable

Table 2 below lists the control variable, why to control and how to control

Control Variable	Why to control	How to control
The medium of launcher and the contact face between launcher and supporter of launcher	The difference of medium will cause the difference of frictional coefficient. This will affect the distance repel by the recoil force differently with the same recoil force.	All the trials will be done with the same launcher and supporter.
The mass of launcher	The mass of launcher will cause the difference of frictional force. This will affect the distance repel by the recoil force differently with the same recoil force.	All the trials will be done with the same launcher.
The mass of empty bottle	Difference of mass of empty bottle will cause the difference of the mass of projectile with the same mass of mixture of white vinegar solution and baking soda.	All the trials will be done with five identical bottles.

Table 2

Procedure

Material and equipment list

Material or equipment	Quantity	Uncertainty
Measuring cylinder (500mL)	1	±5mL
Measuring cylinder(50mL)	1	±0.5mL
Water	/	/
White vinegar(50mL)	25	±5mL
Baking soda(5g)	25	±0.1g
plastic bottles(550mL)	5	/
stoppers	at least 1	/
metal tubing	1	/
wood blocks	2	/
napkins	25	/
Ruler(1m)	1	±0.05m

Building the launcher

1. Record the mass of steel tubing.
2. Place two pieces of wood together horizontally
3. Place the steel tubing on the wood, adjust it until the system is in equilibrium.
4. Place a ruler on the tail of tubing, and adjust the position of the steel tubing until it is at the same position of one end of wood block and the 0 tick mark.

The production of projectile(bottle rockets)

Making bottle rocket

1. Measure 5g of baking soda and pour it onto a napkin, then knead the napkins into a ball.
2. Measure 50ml of white vinegar, and pour it into the empty bottle
3. Pour n*100+50ml of water into the bottle. n is the extra time of repeats of the manipulation of Independent Variable
4. Repeat step 1-3 for 4 times

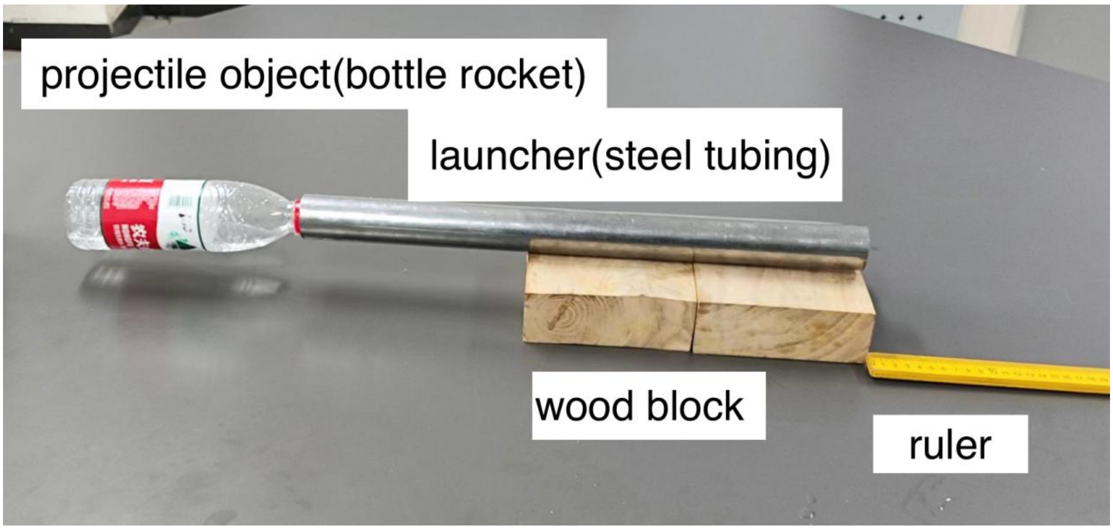

Figure 2. Set up

Data collection

1. Measure the mass of the bottle rocket(include the bottle, water, vinegar, stopper, napkin and baking soda powder).
2. Push the napkin into the liquid of water and cover the bottle with stopper immediately to stop the gas run out of the bottle. Begin to record the video at the same time to prevent the lost of data.
3. When the bottle rocket launched, record the final position of the tail of steel tubing.
4. Use the ruler to measure the distance between two positions.
5. Repeat steps all above for other 4 times except the first two step of building the launcher.

Safety, environmental and ethical issues

The projectile will have a great kinetic energy because its velocity is high during its movement. It is necessary to stay away from the expected route of the bottle's movement.

Data collection

Qualitative data

In the beginning of the experiment, the bottle contains transparent liquid and it is connected with the steel tubing. During the experiment, the white powder and the liquid is reacting and there are small bobbles in the bottle. When the volume of carbon dioxide exceed the volume of air in the bottle, the bottle is detached with the steel tubing and it move toward the direction of the bottom of the bottle. The steel tubing is recoiled by some distance.

Quantitative data

Table 4 below lists the mass of bottle and steel tubing

Volume of water+vinegar(±5ml)	Mass of bottle(*m*)(±0.1g)	Mass of bottle+steel tubing(*M*+*m*)(±0.1g)
100	146.5	2381.8
200	246.5	2481.8
300	346.5	2581.8
400	446.5	2681.8
500	556.5	2781.8

Table 4

Raw data table

Table 5 below lists the raw data

Amount of water and white vinegar(±5ml)/Distance of tubing recoiled(±0.05cm)	Trial1	Trial2	Trial3	Trial4	Trial5
100	2.80	3.00	3.20	3.10	2.80
200	4.90	4.80	5.00	5.20	4.80
300	6.70	6.60	7.00	6.70	6.50
400	7.90	8.10	7.80	8.40	8.50
500	10.10	9.70	9.50	9.80	10.00

Table 5

5. Data propagation

Average and uncertainty propagation

The average distance of steel tubing recoiled is

$$\bar{\Delta \mathrm{x}} = \frac{2.70 + 3.00 + 3.20 + 3.10 + 2.80}{5} = 2.96cm$$

The absolute uncertainty is

$$\Delta_{\Delta x} = \frac{3.20 - 2.80}{2} = 0.20$$

The percentage uncertainty of mass of projectile(m) is

$$\Delta m\,\% = \frac{0.1}{146.5} \times 100\% = 0.068\%$$

The percentage uncertainty of mass of bottle, water, vinegar and steel tubing(*M*) is

$$\Delta M\,\% = \frac{0.2}{2381.8} = 0.017\%$$

The percentage uncertainty of length of steel tubing(l) is

$$\Delta l\% = \frac{0.1}{50} = 0.200\%$$

The percentage uncertainty of distance recoiled(Δx) is

$$\frac{\Delta m\%}{m} + \frac{\Delta M\%}{M} + \frac{\Delta l\%}{l} = 0.285\%$$

The absolute uncertainty is

$$\Delta x = 0.285\% \times 2.96 + 0.2 + 0.05 = 0.26$$

The percentage uncertainty is

$$\Delta x\% = \frac{0.26}{2.96} = 8.8\%$$

Table 6 below lists the data after error propagation

Mass of bottle	Average distance recoiled	Percentage uncertainty of distance recoiled
146.5	2.96cm	8.7%
246.5	4.94cm	5.3%
346.5	6.70cm	4.0%
446.5	8.14cm	3.3%
546.5	9.82cm	2.8%

Table 6

The data of average distance recoiled are enough precise.

Data interpretation

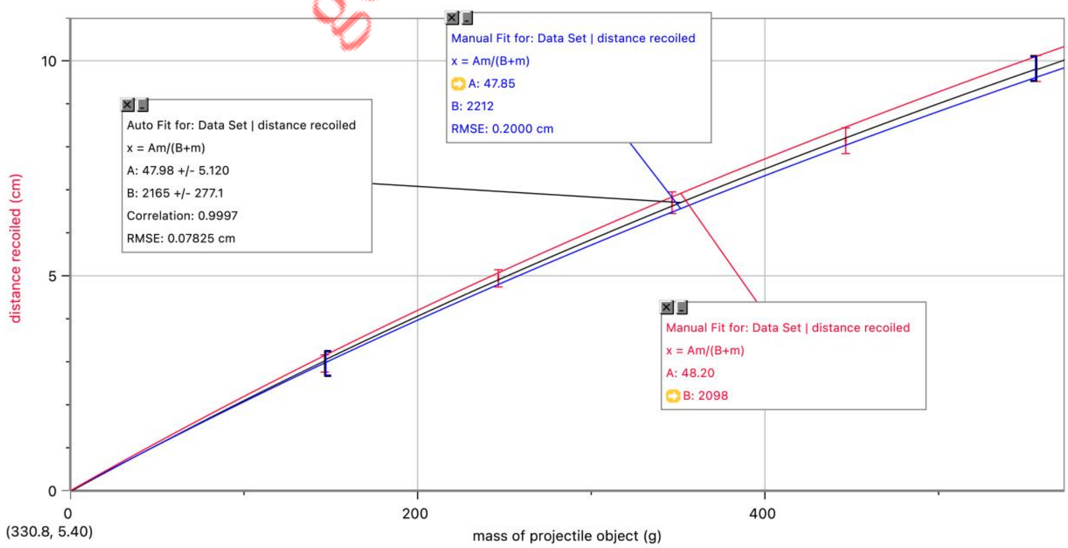

Figure 3 The relationship between the mass of projectile and distance recoiled

The equation of the best fit line of relationship between the mass of projectile and distance recoiled is

$$x = \frac{47.98m}{m + 2165}$$

The worst fit lines reveals the relationship

$$x = \frac{47.85m}{m + 2212}$$

$$x = \frac{48.20m}{m + 2098}$$

The uncertainty of length and mass of steel tubing are

$$\frac{48.20 - 47.85}{2} = 0.18m$$

$$\frac{2212 - 2098}{2} = 57g$$

The equation of relationship between th mass of projectile and distance recoiled is

$$x = \frac{(47.98 \pm 0.18)m}{m + (2165 \pm 57)}$$

The percentage uncertainty are

$$\Delta l = \frac{0.18}{47.98} \times 100\% = 0.4\%$$

$$\Delta M = \frac{57}{2165} \times 100\% = 2.6\%$$

Theoretical value and percentage error calculation

With the measurement before the experiment, the value of l and M are

$$l = 50cm \ M = 2235.3g$$

The theoretical equation of the best fit line of the equation is

$$x = \frac{50m}{m + 2235.3}$$

The percentage error of l and M are

$$\frac{50 - 47.98}{50} \times 100\% = 4.0\%$$

$$\frac{2235.3 - 2165}{2235.3} = 3.1\%$$

6. Conclusion

The aim of the experiment is to investigate how the mass of a projectile affects the recoil distance by a recoil force. The hypothesis is that the greater the mass of a projectile, the greater the distance recoiled because The increase of the mass will bring the increase of the total momentum of the process, which the initial velocity of the launcher will increase when the projectile is launched, which will increase the distance recoiled when the acceleration of launcher causes by frictional force remains the same. In addition, the recoil force, which is the reaction force of the propulsive force of the object increases because of the mass of the projectile increases. With the experiment of measuring the distance of a steel tubing recoiled by launching the projectile of mass of changing total volume of water and white vinegar into 100, 200, 300, 400 and 500ml, add baking soda, which is $NaHCO_3$ to generate enough CO_2 to change the pressure of the internal component of bottle to launch the projectile. The data in raw data table is obtained and calculate the average value of distance recoiled and percentage uncertainty.

In the graph of figure 2, the general trend of the graph of the processed data is that the distance recoiled is gradually increasing while the mass of projectile increasing. The equation of the best fit line of the average distance recoiled and the mass of bottle is

$$x = \frac{47.98m}{m + 2165}$$

The R is namely 0.9978. The equation of best fit line is linear and R^2 is close to 1 so that the distance of steel tubing recoiled and the are very strongly correlated to the mass of projectile by the equation

$$x = \frac{lm}{M + m}$$

Because of the definition of the equation, the best fit line always passes through the origin. However, it does not mean that systematic errors do not exist.

Uncertainties cause the random error of the data and reduce its precision, and the error cause the systematic error of the data reduce its accuracy. The percentage error and percentage uncertainty is in the table.

Percentage uncertainty of length of launcher	Percentage error of length of launcher	Percentage uncertainty of mass of launcher	Percentage error of mass of launcher
0.4%	4.0%	2.6%	3.1%

The percentage uncertainty of the length and the mass of the launcher is lower than 10% so that the data is reliable and precise enough, and the percentage error of length and mass of launcher is lower than 10% so the data is accurate enough. The percentage error is higher than the percentage uncertainty so the systematic error is greater than the random error.

Evaluation

Strength, Weakness, Limitations, and Improvement

The data from the experiment indicates that there are random error and systematic errors, which means that the data is not precise and accurate perfectly so there are weakness, limitations and improvements. However, both the random error and systematic error are in an acceptable range so that there are strengths in the experiment.

Strengths

1. The range of the mass of bottle is enough, which control the random error at an acceptable range. The uncertainty is very small related to the difference of mass of bottle. This cause the random error is small and the data is enough precise.

2. The length of the steel tubing is appropriate, which reduce the percentage uncertainty of the apparatus. The length of the steel tubing is enough long so the percentage uncertainty of the length of the launcher is low. Additionally, the increase of the length of the steel tubing will increase the recoil force significantly, which reduced the random error and the data is precise enough.

Weaknesses, limitations

1. The systematic error indicates that the distance recoiled is always lower than the theoretical value. The systematic error is caused by the frictional force between steel tubing and wood pieces cause the recoil force measured is lower than the actual recoil force. Frictional force imposed the resistance of the moving of the launcher so the net force is the recoil force subtracted by the frictional force. The frictional force cause the actual distance recoiled always to be lower than the theoretical distance. However, the frictional force is inevitable during the experiment so the systematic error can not completely eliminate.

2. Another reason of the systematic error is the detach of the stopper cause the process of imposing the recoil force the bottle does not contains mass of stopper. When the bottle is launched, the stopper moved at an inverse direction toward the moving direction of bottle in the steel tubing. The recoil force generate is actually by the bottle without stopper so the actual distance recoiled is calculated by the mass of projectile subtract by the mass of stopper. This reason cause the actual distance recoiled measured is always lower than the theoretical distance.

3. The random error indicates that the distance recoiled of steel tubing reduce the impact by the change of mass of bottle. The uncertainty of the ruler is fixed, the lower the distance repelled, the percentage uncertainty will increase. The random errors of apparatus are amplified by the limited change of the distance recoiled. When the mass of launcher increase, the distance recoiled of launcher increase, which will reduce the percentage uncertainty of ruler when it is measuring the distance recoiled. This reason cause the significant uncertainty of the dependent variable.

Improvements

1. Measure the frictional coefficient before the experiment, then calculate the

frictional forces. When doing the data propagation, the frictional force will add on the raw data. This will transfer systematic error to random error proceduraly. The data will be more accurate.

2. Measure the mass of the bottle without the stopper, or measure the mass of stopper. This will eliminate the systematic error proceduraly and the data will be more accurate.

3. Replace the steel tubing with aluminium alloy tubing. Aluminium alloy has the lower density than steel. When the volume does not change, the mass of launcher will decrease significantly. The decrease of mass of launcher will cause the increase of the distance recoiled by the projectile when the mass of projectile is not change. This will reduce the impact of uncertainty of apparatus and reduce the random error, which will increase the precision of data.

References

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