How does wind speed (2.0 km/h, 4.0 km/h, 6.0 km/h, 8.0 km/h, 10.0 km/h, 12.0 km/h, 14.0 km/h, ± 3.0%) affect the power output (mW) of a wind turbine?

Background

In the contemporary world, the increase in demand for resources and rapid economic development has led to climate change and the energy crisis of coal and gas, driving an urgent need to explore renewable energy. Wind energy is a powerful and efficient renewable energy. Compared with coal, which produces around 980 grams of CO_2 per kilowatt-hour, wind energy only generates about 11 grams per kilowatt-hour, only one-hundredth of coal's emissions (Office of Energy). It has become the main technology to mitigate pollution while fulfilling the demands of thousands.

A wind turbine converts the kinetic energy of wind to mechanical energy, and then to electricity by letting the wind spin the blades attached to the top of its tower ("WINDExchange"). The power output of the wind turbine is affected by several factors including the density of the air, the size of the rotor, with wind speed being the most significant factor. Understanding these variables is of great importance to better come up with an approach to designing wind turbines. Therefore, this experiment aims to understand the relationship between wind speed and the power output of a wind turbine.

The wind power equation

The power output extractable by a wind turbine from a free, undisturbed wind stream flowing over a cylindrical column of air is given in Equation 1:

$$P = \frac{1}{2}\rho\pi R^2 v^3 \text{……Eq.(1)}$$

Table 1 below lists the meaning and units of each variable in Equation 1:

Variable	Meaning	Unit
P	The power extractable by the wind turbine from the cylinder of the wind stream	Watts (W)
ρ	The density of air	kg m^{-3}
π	constant	no units
R	The radius of the cross-sectional area of the column of air swept by the blades of the wind turbine	m
v	The speed of the wind	ms^{-1}

Table 1.

Deriving the theoretical relationship between power and wind speed

To analyze the data collected later on with graph in a more straightforward way, it is useful to derive a linear relationship between the power output and wind speed.

1. $\frac{1}{2}\rho\pi R^2 v^3$ is made up of different constants, representing different factors of affecting the power

output. Detailed methods to control them will be explained later. I will simply these constants into one unknown constant m.

$$P = \frac{1}{2}\rho\pi R^2 v^3$$

$$P = m \cdot v^3 \text{……Eq. (2)}$$

2. Take the power of one-third of both sides of equation (Eq.(2)),

$$(P)^{\frac{1}{3}} = (m \cdot v^3)^{\frac{1}{3}}$$

$$P^{\frac{1}{3}} = (m)^{\frac{1}{3}} \cdot v$$

It is easy to deduce that theoretically, plotting wind speed against the power extractable $(P)^{\frac{1}{3}}$ will form a linear curve, because

$$P^{\frac{1}{3}} \propto v$$

In reality, the power output of a wind turbine cannot increase infinitely. The power output will achieve a maximum level once the wind speed reaches a limit, which is called *rated speed* in the wind power industry (Energy Education). After the rated speed, the power output no longer depends on the change in wind speed and will turn zero at a cut-out speed where the turbine must be turned off to avoid further damage.

Figure 1 shows the wind power curve:

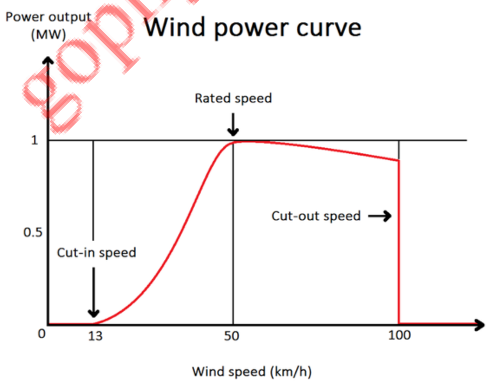


Fig 1. (Energy Education)

Hypothesis

It can be hypothesized that as the simulated wind speed (km/h) increases, the power output (W) of the model wind turbine will increase until the device reaches its maximum. There will be an increasing curve which tends to level off between the power output and wind speed. When plotting $P^{\frac{1}{3}}$ against the wind speed (km/h), an increasing linear line and a flat line once it achieves a certain wind speed would be expected.

Methodology

The circuit power equation to calculate the actual power output

In the experiment, the actual power output of the model wind turbine will be calculated through the current and voltage measured by multimeters. The circuit equation is given in Equation 2:

$$P_{\text{act}} = IV \text{…… Eq.(3)}$$

Table 2 below shows the meaning and unit of each variable in Equation 3:

Variable	Meaning	Unit
P_{act}	the actual power output of the model wind turbine	Watts (W)
I	Current of the circuit	Ampere (A)
V	Voltage of the circuit	Volt (V)

Table 2.

Table 3 below lists the dependent variable and the independent variable:

Variables	Details
Independent	The wind speed (2.0 km/h, 4.0 km/h, 6.0 km/h, 8.0 km/h, 10.0 km/h, 12.0 km/h, 14.0 km/h ± 3.0%). This will be manipulated by measuring the desired speed using an anemometer.
Dependent	The power output of the wind turbine. This will be calculated by the currents (± 0.5 mA) and voltages (± 0.05 v) recorded through two multimeters.

Table 3.

Table 4 below lists the control variables:

Control Variables	Why and how to control the variable
Air density	According to the Background, the air density of the wind stream affects the power output of the wind turbine. It must be controlled so that the change in power output is solely due to the change in the wind speed. It will be kept constant by completing the experiment in the same room and keeping the device at the same height, ensuring constant pressure and temperature.
The length of blades	The radius of the cross-sectional area of the wind stream, which is equal to the length of blades, affects

	the power output of the wind turbine. It will be kept constant by using the same blades throughout the experiment.
The number of blades	The number of blades must be controlled as it will impact the value of the efficiency by generating more torque, air resistance, and turbulence, affecting the power output. It will be controlled by using a fixed number of blades throughout the experiment.
Room temperature	The temperature affects the power output of the wind turbine. As temperature increases, the air particles gain more kinetic energy, thus leading to a higher power output. It can be controlled by completing the experiment in the same room in a relatively short time interval.

Table 4.

Note the experiment will be performed in an empty room with a windless environment to avoid wind turbulence. After the anemometer is used to measure wind speed, it must be removed immediately to allow an unobstructed wind stream.

Apparatus

Table 5 below lists the apparatus of the experiment：

Apparatus	Quantity	Uncertainty
Model wind turbine	1	/
Blades	6	/
Multimeter for current	1	± 0.5 mA
Multimeter for voltage	1	± 0.05 v
Anemometer	1	± 3.0 %
Torque motor	1	/
Clip cords	2 sets	/
Resistor	1	/
Electric fan	1	/

Table 5.

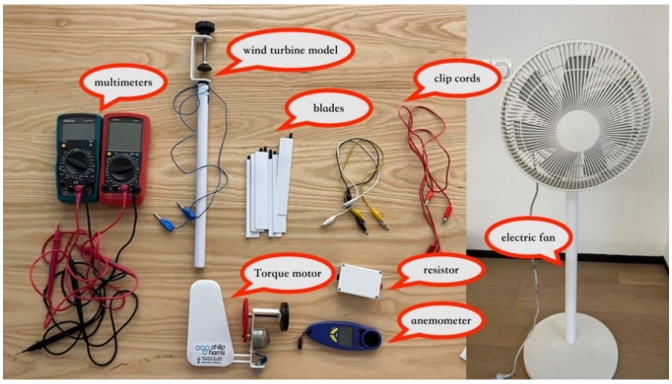


Figure 2. A display of the apparatus

Procedure

1. Construct the circuit by connecting the model wind turbine, a 2-ohm resistor and the multimeters with clip cords.

2. Place the electric fan parallel and in front of the model turbine. The blades of the model should be within the diameter of the electric fan to avoid additional drag force.

3. Turn on the electric fan and the model wind turbine.

4. Use the anemometer to record the wind speed of 2.0 km/h flowing through the blades on the data table. After recording, remove the anemometer immediately.

5. Use multimeters to record the current (I) and voltage (V) of the system on the data table.

6. Repeat steps 2, 4, and 7 five times with more than three concordant results.

7. Repeat steps 2 and 4-6 for 2.0 km/h, 4.0 km/h, 6.0 km/h, 8.0 km/h, 10.0 km/h, 12.0 km/h, 14.0 km/h, ± 3.0% wind speed, respectively. The different wind speeds are achieved by placing the electric fan at varying distances from the model wind turbine.

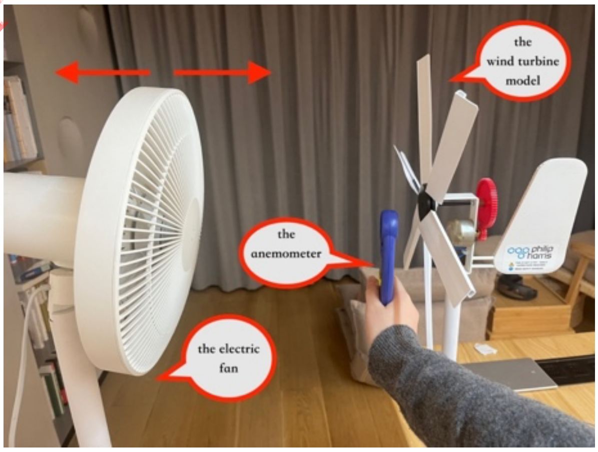


Figure 3. A display of the setup

Figure 4. The method to record the wind speed with the anemometer

Safety, environmental and ethical issues

As the experiment involves an electric fan and constructing circuit, it might cause electric shock accidents. I will pay careful attention and turn off the model and the fan whenever I am not recording data. Moreover, the high speed of blades could also lead to injuries, so I will ensure to conduct experiments with a safe and relatively empty space under supervision.

Raw Data

Qualitative Data

In the experiment, when the electric fan was on, the blades of the wind turbine started to rotate, and both multimeters showed numbers for current and voltage. When wind speed was increased, both the current and voltage in the circuit increased. The increase tends to be slower as the wind speed achieved 10.0 km/h.

Quantitative Data- Raw Data Table

Table 6. measured wind speed, measured current and voltage by multimeters:

* Please see the rest in Appendix

Trial #	Wind speed (km/h) ± 3.0%	Current measured by the multimeter (mA) ± 0.5 mA	Voltage measured by the multimeter (V) ± 0.05 V
1	2.0	33.5	0.07
2		32.4	0.08
3		28.9	0.07
4		34.5	0.06
5		32.3	0.07
1	4.0	180.3	0.35
2		102.1	0.19
3		264.0	0.50
4		176.8	0.35
5		95.4	0.22

Table 6.

Sample Calculation

According to the previous exploration, the power output needs to be processed by using the current and voltage measured in the experiment. It is needed to calculate the average value and uncertainty of the current and voltage.

A sample calculation based on trial 1 of wind speed 10.0 km/h is given:

The average value of the measured current is:

$$\text{Current measured}(I) = \frac{375.6 + 400.2 + 372.1 + 383.2 + 359.8}{5} = 378.18\text{ mA}$$

The uncertainty of the current is estimated using half range method:

$$\Delta I = \frac{\mathrm{MAX}(I) - \mathrm{MIN}(I)}{2} = \frac{400.2 - 372.1}{2} = 14.05\text{ mA} \approx 10\text{ mA}$$

Similarly, the average value of the measured voltage is calculated:

$$\text{Voltage measured}(V) = \frac{0.84 + 0.9 + 0.83 + 0.87 + 0.84}{5} = 0.86\text{ V}$$

As the uncertainty of the voltage calculated using the half range method is smaller compared to the uncertainty of the anemometer itself, the uncertainty of the device ± 0.05 V will be used.

In my investigation, the power output of the wind turbine is defined as

$$\text{Power output }(P_{\text{act}}) = \text{Current measured }(I) \cdot \text{Voltage measured }(V)$$

$$\text{Power output }(P_{\text{act}}) = IV = 380 \cdot 0.86 = 326.8\text{ mW}$$

$$\frac{\Delta P_{\text{act}}}{P_{\text{act}}} = \frac{\Delta I}{I} + \frac{\Delta V}{V}$$

$$\Delta P_{\text{act}} = (\frac{\Delta I}{I} + \frac{\Delta V}{V}) \times P_{\text{act}}$$

$$\Delta P_{\text{act}} = \left(\frac{10}{380} + \frac{0.05}{0.86}\right) \times 326.8 = 27.6\text{ mW} \approx 30\text{ mW}$$

Thus, for wind speed of 2.0 km/h, the average power output and its uncertainty is:

$$p_{\text{act}} = 330\text{ mW} \pm 30\text{ mW}$$

Therefore,

$$P_{\text{act}}^{\frac{1}{3}} = 6.9\ (\text{mW})^{\frac{1}{3}}$$

$$\frac{\Delta P_{\text{act}}^{\frac{1}{3}}}{P_{\text{act}}^{\frac{1}{3}}} = |\frac{1}{3} \times \frac{\Delta P_{\text{act}}}{P_{\text{act}}}|$$

$$\Delta P_{\text{act}}^{\frac{1}{3}} = |\frac{1}{3} \times \frac{\Delta P_{\text{act}}}{P_{\text{act}}}| \times P_{\text{act}}^{\frac{1}{3}}$$

$$\Delta P_{\text{act}}^{\frac{1}{3}} = \left|\frac{1}{3} \times \left(\frac{10}{380} + \frac{0.05}{0.86}\right)\right| \times 6.91 = 0.2\text{ mW}$$

Processed Data

Following a similar procedure, the processed data for other values of $P_{act}^{\frac{1}{3}}$ is summarized in Table 7:

Wind speed (km/h) ± 3.0%	Average $P_{\text{act}}^{\frac{1}{3}}$ $((\text{mW})^{\frac{1}{3}})$	Uncertainty $P_{\text{act}}^{\frac{1}{3}}$ $(\pm\ (\text{mW})^{\frac{1}{3}})$
2.0	1.3	0.4
4.0	3.7	0.8
6.0	4.3	0.2

8.0	5.5	0.2
10.0	6.9	0.2
12.0	7.0	0.3
14.0	7.1	0.3

Table 7.

*The average $P_{\text{act}}^{\frac{1}{3}}$ is adjusted according to the significant figures in uncertainty.

The power output of the wind turbine was graphed against wind speed. A general shape which combines a upgoing curve and a flat trend since 10 km/h can be seen in Figure 5, generated by the software Logger Pro. However, to analyze the data more accurately, further graphs that separate the time into two intervals are needed.

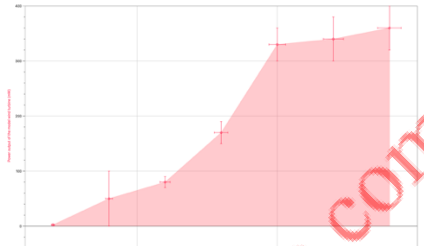
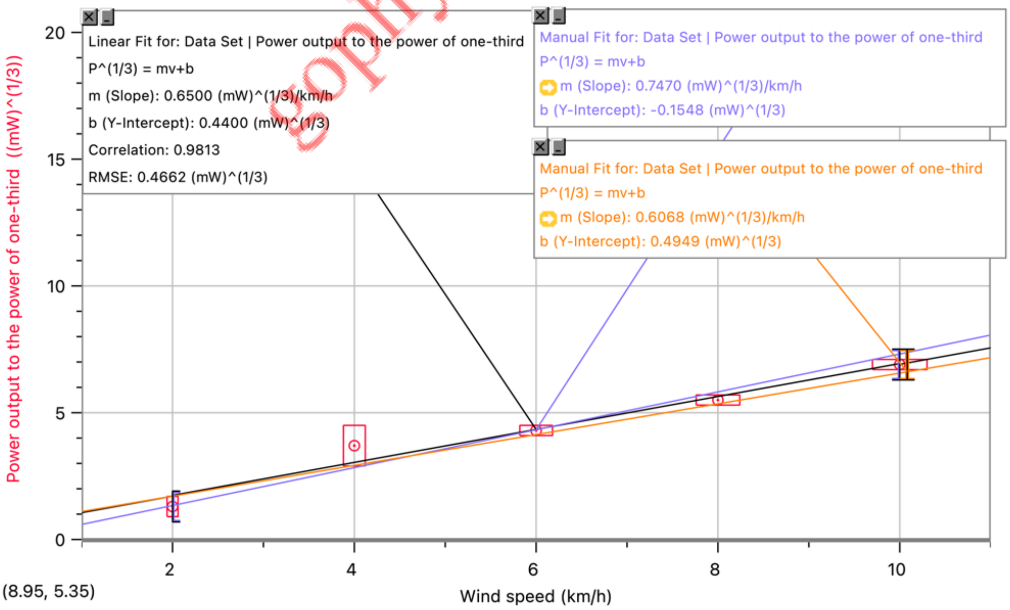


Fig 5. The general relationship between power output (P_{act}) and wind speed (v)

Figure 6 shows $P_{\text{act}}^{\frac{1}{3}}$ of the wind turbine vs. wind speed between 2 km/h to 10km/h:



Linear Fit for: Data Set | Power output to the power of one-third
P^(1/3) = mv+b
m (Slope): 0.6500 (mW)^(1/3)/km/h
b (Y-Intercept): 0.4400 (mW)^(1/3)
Correlation: 0.9813
RMSE: 0.4662 (mW)^(1/3)

Manual Fit for: Data Set | Power output to the power of one-third
P^(1/3) = mv+b
m (Slope): 0.7470 (mW)^(1/3)/km/h
b (Y-Intercept): -0.1548 (mW)^(1/3)

Manual Fit for: Data Set | Power output to the power of one-third
P^(1/3) = mv+b
m (Slope): 0.6068 (mW)^(1/3)/km/h
b (Y-Intercept): 0.4949 (mW)^(1/3)

Fig 6.

The best fit line reveals a relationship:

$$P_{\mathrm{act}}{}^{\frac{1}{3}} = 0.6500\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}}\ v + 0.4400\ (\mathrm{mW})^{\frac{1}{3}}$$

The worst fit line reveals relationships:

$$P_{\mathrm{act}}{}^{\frac{1}{3}} = 0.7470\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}}\ v - 0.1548(\mathrm{mW})^{\frac{1}{3}},\ P_{\mathrm{act}}{}^{\frac{1}{3}} = 0.6068\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}}\ v + 0.4949(\mathrm{mW})^{\frac{1}{3}}$$

The uncertainty of the gradient is:

$$\frac{0.7470\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}} - 0.6868\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}}}{2} = 0.03\ (\frac{\mathrm{kg}}{\mathrm{km}})^{\frac{1}{3}}$$

The uncertainty of the Y-intercept is:

$$\frac{0.4949\ (\mathrm{mW})^{\frac{1}{3}} - (-0.1548)\ (\mathrm{mW})^{\frac{1}{3}}}{2} = 0.3\ (\mathrm{mW})^{\frac{1}{3}}$$

Therefore,

$$P_{\mathrm{act}}{}^{\frac{1}{3}} = (0.65\ \pm 0.03\ )\left(\frac{\mathrm{kg}}{\mathrm{km}}\right)^{\frac{1}{3}} v + (0.4\ \pm 0.3)(\mathrm{mW})^{\frac{1}{3}}$$

The relationship reveals that during 2 km/h to 10 km/h, $P_{\mathrm{act}}{}^{\frac{1}{3}}$ has a proportional linear relation with the wind speed. This proves that the power output of the model wind turbine follows a cubic correspondence with the wind speed.

Figure 8 shows $P_{\mathrm{act}}{}^{\frac{1}{3}}$ of the wind turbine vs. wind speed between 10 km/h to 14 km/h:

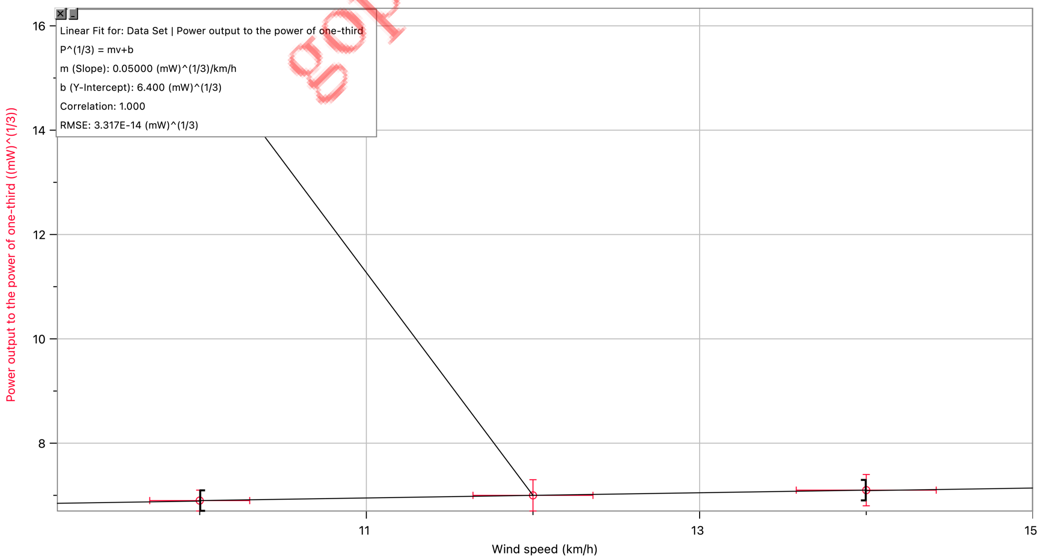


Fig 8.

The graph shows that from 10 km/h to 14km/h, there is almost a flat line where the gradient is only 0.05000 $(\frac{\text{kg}}{\text{km}})^{\frac{1}{3}}$. It reveals that the model wind turbine has reached its maximum output. Since the power output no longer increases with the wind speed, uncertainties are irrelevant and hence will not be calculated.

Conclusion

This investigation aims to explore the effect of the change in wind speed (km/h) on the power output (mW) of the wind turbine in experiments. The power output is calculated by measuring the current and voltage in the circuit of the model wind turbine. It was hypothesized that there is a linear relationship between the power output to the power of one-third $P_{\text{act}}^{\frac{1}{3}}$ and wind speed until the wind turbine has reached its maximum output.

Between 2.0 km/h-10.0 km/h, the processed data plotted shows a linear relationship $P_{\text{act}}^{\frac{1}{3}}=(0.65 \pm 0.03)\left(\frac{\text{kg}}{\text{km}}\right)^{\frac{1}{3}} \text{v}+(0.4 \pm 0.3)(\text{mW})^{\frac{1}{3}}$, meaning as wind speed increases by 1.0 km/h, the power output $^{1/3}$ will increase by $(0.65 \pm 0.03)(\text{mW})^{\frac{1}{3}}$. The calculation supports the hypothesis of the linear relationship. More importantly, it indirectly proves the positive cubic relationship between the power output and the wind speed, since an increase in wind speed allows the blades to rotate faster, translating more mechanical energy which is converted to the higher power output of the wind turbine.

Between 10.0 km/h-14.0 km/h, the processed data showcases that the wind speed no longer increases the power output of the model wind turbine, where uncertainties are somehow irrelevant. This also supports the hypothesis that once the turbine achieves its maximum output, the power output tends to level off as the conversion of energy is not efficient.

The line of best fit does not go through the origin, but it is generally acceptable. The y-intercept shows $(0.4 \pm 0.3)(\text{mW})^{\frac{1}{3}}$, where the uncertainty is close to the value, meaning there is a small systematic error in the experiment.

The slope of the linear relationship has an absolute uncertainty of $(\pm 0.03)\left(\frac{\text{kg}}{\text{km}}\right)^{\frac{1}{3}}$, which is equivalent to the percentage uncertainty of $\frac{0.03}{0.65} \times 100\% = 5\%$, showing that there is not a lot of random error in the experiment, which proves the precision and reliability of my results.

Literature Values and Percentage Error

The theoretical slope $(m)^{\frac{1}{3}}$, where m is equal to $\frac{1}{2}\rho\pi R^2$ as defined in previous section, can be calculated. Here, I will use the approximate value of air density 1.29 kg/m^3 and the length of blades of 0.45 m. $\frac{1000}{3600}$ needs to be added to ensure the value due to inconsistency in units of the speed km/h.

$$(m)^{\frac{1}{3}} = (\frac{1}{2}\rho\pi R^2)^{\frac{1}{3}} = (\frac{1}{2}\cdot 1.29\cdot \pi \cdot 0.45^2 \cdot \frac{1000}{3600})^{\frac{1}{3}} = 0.49\left(\frac{\mathrm{kg}}{\mathrm{km}}\right)^{\frac{1}{3}}$$

The percentage error is calculate using this formula:

$$\text{Pecentage error} = \left|\frac{\text{literature value} - \text{experimental value}}{\text{literature value}}\right| \times 100\%$$

Substituting the theoretical and experimental value of $(\mathrm{m})^{\frac{1}{3}}$:

$$\text{Pecentage error} = \left|\frac{0.49 - 0.65}{0.49}\right| \times 100\% = 33\%$$

The percentage error suggests the results of my investigation are not highly accurate, but understandable since the equipment in the equipment is rather primitive.

Evaluation

Although the hypothesis is scientifically proven by the wind power equation and the conventional wind power curve, the calculated gradient $(m)^{\frac{1}{3}}$ differs from the theoretical value, with a percentage error of 33%. The possible reasons are experimental uncertainties due to weaknesses in the experiment. However, there are also strengths which led to the overall success of the experiment. In the following session, the strengths and weaknesses will be reflected.

Table 8 lists the strengths of the investigation:

Strengths	Significance
The anemometer was removed immediately after the recording of the wind speed.	This makes sure that the wind speed was not blocked by other barriers like the anemometer, which adds accuracy to the power output calculated.
Five trials per wind speed and are mostly concordant, and seven wind speeds were experimented.	As the experiment was repeated for multiple times, random errors were controlled in the process, which contributes to the precision of the results.
Same set of apparatus was used throughout the experiment.	The wind turbine model, electric fan, resistor and other apparatus were kept the same throughout the experiment, which ensured that the data collected was precise and close with each other.
Blades were placed parallel to and within the diameter of the electric	This ensured a sufficient conversion of the wind from the fan to the mechanical energy of the

fan.	wind turbine through the blades, and avoided added drag force that would reduce accuracy.

Table 8.

Table 9 lists the weaknesses or limitations and improvements of the investigation:

Weaknesses or limitations and their impact	Improvements
Fluctuations in readings on multimeters existed constantly. This may be due to the sudden changes of the wind speed from the electric fan or the poor contact with the model wind turbine. Hence, there is estimation when recording the current and voltage as the multimeters did not show a consistent value, resulting in random error which reduces the precision of the experiment.	It would be better to have a better quality model of larger size and blades so that small changes in wind speed would not alter its power output that much. Also, I can use a wind speed monitor to ensure that the speed remains the same when recording.
When plugging in the multimeter, it added resistance to the circuit, decreasing the value of the current shown. This leads to a systematic error which reduces the accuracy of the results.	I can choose a multimeter with high input impedance, which draws minimal current from the circuit. I can also reduce the interference by using shorter clip cords to lower their contribution to the resistance in the circuit.
When using the anemometer to measure wind speed, the blades of the wind turbine also generated wind flow, interfering the recordings which also caused systematic error.	I can measure the wind speed beforehand with the model wind turbine turned off. I can record the distances with relation to each wind speed, then place the electric fan according to the distance measured.
The theoretical wind power equation is for a wind stream which strikes perpendicular to the blades of the wind turbine. However, there must be wind in my actual experiment which did not flow this way, reducing the efficiency of the wind turbine which would affect the power output.	I can construct and calibrate a wind tunnel to directly control the direction of the wind flow to ensure it strikes perpendicular to the surface of the turbine.

Table 9.

Works Cited

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“WINDExchange: Wind Energy Policies and Incentives.” *Windexchange.energy.gov*, U.S. Department of Energy, windexchange.energy.gov/policies-incentives.

Appendix

Table 6. measured wind speed, measured current and voltage by the multimeter

Trial #	Wind speed (kmh^{-1}) ± 3.0%	Current measured by the multimeter (mA) ± 0.5 mA	Voltage measured by the multimeter (V) ± 0.05 V
1	2.0	33.5	0.07
2		32.4	0.08
3		28.9	0.07
4		34.5	0.06
5		32.3	0.07
1	4.0	180.3	0.35
2		102.1	0.19
3		264.0	0.50
4		176.8	0.35
5		95.4	0.22
1	6.0	188.0	0.43
2		180.5	0.40
3		195.8	0.43
4		192.7	0.42
5		188.6	0.42
1	8.0	267.5	0.67
2		271.2	0.60
3		266.5	0.61
4		255.2	0.64
5		261.3	0.62
1	10.0	375.6	0.84
2		400.2	0.90
3		372.1	0.83
4		383.2	0.87
5		359.8	0.84
1	12.0	382.4	0.87
2		392.1	0.88
3		413.2	0.90
4		400.3	0.89
5		370.8	0.86
1	14.0	397.3	0.89
2		386.7	0.88
3		423.8	0.91
4		393.6	0.88
5		404.0	0.90