

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.0 Math

### Understandings

- Fundamental units
- Derived SI units
- Significant figures
- Unit conversions
- Random and systematic errors
- Absolute, fractional, and percentage uncertainties
- Error bars
- Uncertainty of gradient and intercepts
- Vector and scalar quantities
- Adding and subtracting vectors
- Combination and resolution of vectors

If you are interested in learning more about mathematical physics then please read the books *Mathematical Methods in the Physical Sciences* by Mary L. Boas and *div grad curl and all that* by H.M. Schey.

**The solutions can be found on the YouTube channel: Go Physics Go**

**Part 1: State the fundamental units**

<b>Quantity</b>	<b>Unit</b>
Length	
Mass	
Time	
Current	
Temperature	
Quantity	
Light intensity	

Memorize these two acronyms to memorize the SI fundamental units:

**My Knuckles Grow Stronger And Kill More Creatures**  
=  
**Meters KiloGrams Seconds Amperes Kelvin Moles Candela**

**Part 2: Answer the following questions about fundamental units**

1. What is the meaning and what are the fundamental units of *perimeter*?
2. What is the meaning, equation, and the fundamental units of *circumference*?
3. What is the meaning and what are the fundamental units of *area*?
4. What is the meaning and what are the fundamental units of *volume*?
5. A.1: Use the equation  $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$  to solve for the fundamental units of *velocity*.
6. A.1: Use the equation  $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$  to solve for the fundamental units of *acceleration*.
7. A.2: What are the units of *force*? Use the equation  $\Sigma\vec{F} = m\vec{a}$  to solve for the fundamental units of *force*.
8. A.2: Use the equation  $\vec{F}_H = -k\vec{x}$  to solve for the fundamental units of the *spring constant*  $k$ .
9. A.2: Use the equation  $F_d = 6\pi\eta rv$  to solve for the fundamental units of the *fluid viscosity*  $\eta$ .
- 10.A.2: Use the equation  $\vec{p} = m\vec{v}$  to solve for the fundamental units of *momentum*  $\vec{p}$ .

11.A.3: What are the units of *energy*?

12.A.3: Use the equation  $E_k = \frac{1}{2}mv^2$  to solve for the fundamental units of *kinetic energy*.

13.A.3: Use the equation  $E_p = mg\Delta h$  to solve for the fundamental units of the *gravitational potential energy* near the surface of a planet.

14.A.3: What are the fundamental units of *energy*?

15.A.3: What are the units of *work*? Use the equation  $W = Fs \cos \theta$  to solve for the fundamental units of *work*  $W$ .

16.A.3: What is the relationship between the fundamental units of *work* and *energy*?

17.A.3: What are the units of *power*? Use the equation  $P = \frac{\text{Work}}{t}$  to solve for the fundamental units of *power*  $P$ .

18.A.4 HL: Use the equation  $\tau = rF \sin \theta$  to solve for the fundamental units of *torque*  $\tau$ . The variable  $r$  has units of meters and the variable  $F$  represents the external force acting on an object.

19.A.4: HL: Use the equation  $I = kMR^2$  to solve for the fundamental units of the *moment of inertia*  $I$ . The variable  $k$  is a unitless constant which depends on the physical dimensions of the object, the variable  $M$  is the mass of the object, and the variable  $R$  has units of meters.

20.A.4: HL: Use the equation  $\Delta L = \tau \Delta t$  to solve for the fundamental units of angular momentum  $L$ .

21.A.4: HL: Use the equation  $L = I\omega$  to solve for the fundamental units of angular speed  $\omega$ .

22.A.4: HL: Use the equation  $\tau = I\alpha$  to solve for the fundamental units of angular acceleration  $\alpha$ .

23.B.1: What is the equation and what are the fundamental units of density  $\rho$ ?

24.B.1: Use the equation  $\overline{E_k} = \frac{3}{2} k_B T$  to determine the fundamental units for the Boltzmann's constant  $k_B$ .

25.B.1: Use the equation  $Q = mc\Delta T$  to determine the fundamental units for the specific heat capacity  $c$ . The variable  $Q$  has the units of energy.

26.B.1: Use the equation  $Q = mL_f$  to determine the fundamental units for the latent heat of fusion  $L_f$ . The variable  $Q$  has the units of energy.

27.B.1: Use the equation  $\frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{\Delta x}$  to determine the fundamental units for the thermal conductivity  $k$ . The variable  $Q$  has the units of energy.

- 28.B.1: Use the equation  $L = \sigma AT^4$  to solve for the fundamental units of the *Stefan-Boltzmann constant*  $\sigma$ . The variable  $L$  represents the luminosity of an object and has units of Watts, the variable  $A$  represents the surface area of an object, and the variable  $T$  represents the temperature of an object.
- 29.B.1: Use the equation  $b = \frac{L}{4\pi d^2}$  to solve for the fundamental units of the apparent brightness  $b$ . The variable  $L$  represents the luminosity of an object and has units of Watts. The variable  $d$  represents the distance from a light source and has units of meters.
- 30.B.2: Use the equation  $e = \frac{P}{\sigma T^4}$  to solve for the fundamental units of emissivity  $e$ . The fundamental units for the Stefan-Boltzmann constant  $\sigma$  has been solved for earlier.
- 31.B.3: What are the units of *pressure*? Use the equation  $P = \frac{F}{A}$  to solve for the fundamental units of *pressure*  $P$ .
- 32.B.3: Use the equation  $PV = nRT$  to solve for the fundamental units of the *ideal gas constant*  $R$ .
- 33.B.4 HL: Use the equation  $S = \frac{\Delta Q}{\Delta T}$  to solve for the fundamental units of *entropy*  $S$ .
- 34.B.4 HL: Use the equation  $S = k_B \ln \Omega$  to determine the fundamental units for the *Boltzmann's constant*  $k_B$ .

35.B.5: What are the units of *current I*? Use the equation  $I = \frac{\Delta q}{\Delta t}$  to solve for the fundamental units of *current I*.

36.B.5: Use the equation  $I = \frac{\Delta q}{\Delta t}$  to solve for the fundamental units of *charge q*.

37.B.5: Use the equation  $V = W/q$  to solve for the fundamental units of *voltage V*.

38.B.5: What are the units for the *resistance* in a resistor *R*? Use the equation  $V = IR$  to solve for the fundamental units of *resistance R*.

39.C.1: What is the meaning and what are the fundamental units of *period T*?

40.C.1: What is the meaning and what are the fundamental units of *frequency f*?

41.C.1: What is the meaning and what are the fundamental units of *wavelength λ*?

42.C.2: Intensity is defined as power per unit area. What are the fundamental units of *intensity I*?

43.D.1: Use the equation  $F_{\text{gravity}} = \frac{Gm_1m_2}{r^2}$  to solve for the fundamental units of the *gravitational constant G*.

44.D.1 HL: Use the equation  $V_g = \frac{Gm}{r}$  to solve for the fundamental units of the *gravitational potential V<sub>g</sub>*.

45.D.2: Use the equation  $F_{\text{electric}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$  to solve for the fundamental units of the *permittivity of free space*  $\epsilon_0$ .

46.D.2: Use the equation  $F_{\text{electric}} = k \frac{q_1 q_2}{r^2}$  to solve for the fundamental units of the *Coulomb constant*  $k$ .

47.D.2: Use the equation  $\vec{F}_e = q\vec{E}_{\text{ext}}$  to solve for the fundamental units of the *electric field*  $E$ .

48.D.2 HL: Use the equation  $V_e = \frac{kQ}{r}$  to solve for the fundamental units of the *electric potential*  $V_e$ .

49.D.3: What are the units of *magnetic field*  $\vec{B}$ ? Use the equation  $\vec{F}_B = q\vec{v}\vec{B}_{\text{ext}}$  to solve for the fundamental units of the *magnetic field*  $\vec{B}$ .

50.D.3: Use the equation  $\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}$  to solve for the fundamental units of  $\mu_0$ .

51.D.4 HL: What are the units for the *magnetic flux*  $\Phi$ ? HL: Use the equation  $\Phi = BA \cos \theta$  to solve for the fundamental units for the *magnetic flux*  $\Phi$ .

52.E.1: Use the equation  $E = hf$  to solve for the fundamental units of *Planck's constant*  $h$ .

53.E.2: HL: Use the equation  $\lambda = \frac{h}{p}$  to solve for the fundamental units of the *de Broglie wavelength*  $\lambda$ .



**Part 3: Determine the number of significant figures**

1. 1,000	21.0.00020	41.100.00
2. 1,000.	22.0.0205	42.300.0000
3. 1,000.00	23.0.2	43.301
4. 1,020	24.8,000	44.301.001
5. 1020.	25.8,070	45.301.0010000
6. 1,020.0	26.8.0	46.8,670
7. 1,000.001	27.8.007	47.80,600
8. 1,200	28.800,700	48.8,670.00
9. 1,200.	29.800,700.00	49.1,000,000
10.1,200.00	30.4	50.1,200,000
11.1,200.03	31.4.0	51.1,205,000
12.1,200.0300	32.4.000	52.4,000
13.100200	33.1.2	53.4,300
14.100200.	34.1.25	54.4,300.
15.100200.00	35.1.250000	55.4,030
16.4,500	36.10	56.4003
17.4,050	37.10.	57.4,003.
18.405	38.100	58.0.00867
19.0.0000405	39.101	59.0.00086000753
20.0.0002	40.100.	60.0.0230076

**Part 4: Unit conversions. Give all your solutions to three significant figures.**

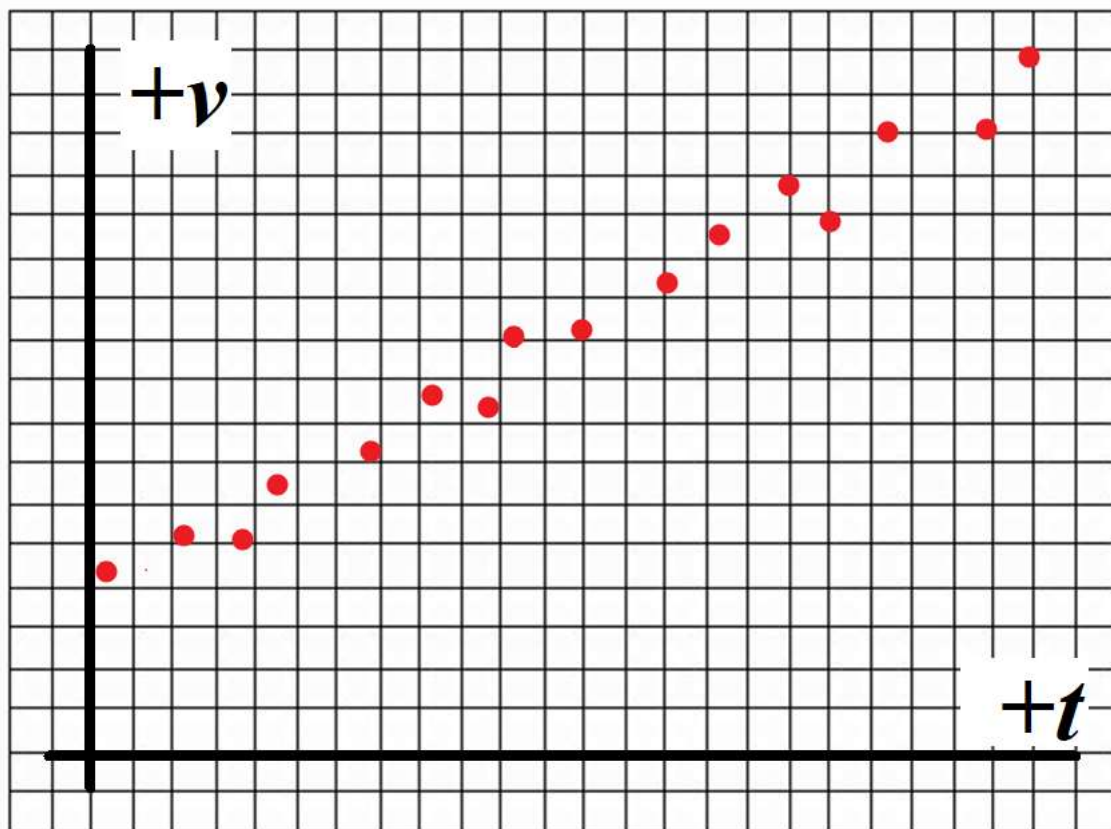
1. A man has a mass of 80.0 kg. What is the mass of the man in pounds? Show all your work and place a box around your answer.
2. How many seconds are in 80.0 years? Show all your work and place a box around your answer.
3. In 2009 Usain Bolt ran 100. m in a record time of 9.58 s. If he continues to run at this constant rate then how many meters will he run in one day? Show all your work and place a box around your answer.
4. In 2018 Eliud Kipchoge ran a marathon (42.195 km) in a record time of 2:01:39. If he continues to run at this constant rate then how many meters will he run in one day? Show all your work and place a box around your answer.
5. The circumference of the Earth is about 40,075.017 km from the Equator. What is the circumference of the Earth in inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.
6. The surface area of Earth is about 510,064,472 square kilometers. What is the surface area of the Earth in square inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.

7. The volume of Earth is about 1,083,206,916,846 cubic kilometers. What is the volume of Earth in cubic inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.
8. The speed of light is 299,792,458 m/s. What is the distance, in kilometers, light travels in one year? Show all your work and place a box around your answer.
9. The density of gold is 19.32 grams per cubic centimeters. What is the density of gold in kilograms per cubic meters? Show all your work and place a box around your answer.
10. The density of gold is 19.32 grams per cubic centimeters. What is the density of gold in pounds per cubic feet? There is approximately 0.454 kg in one pound. There is approximately 0.305 meters in one foot. Show all your work and place a box around your answer.
11. A man drinks 60.0 liters of water in a 30.0 day month. On average how many cubic meters of water does he drink per hour? Show all your work and place a box around your answer.

**Part 5: Answer the following questions**

1. Define *random error* and give two examples.
2. Define *systematic error* and give two examples.
3. Define *accuracy* and give an example of high accuracy and low accuracy.
4. Define *precision* and give an example of high precision and low precision.
5. List some rules with regards to uncertainties in measurements.
6. State the equation and give the meaning of *standard deviation*  $\sigma$ .

7. Calculate the *absolute uncertainty*, *fractional uncertainty*, and *percent uncertainty* for a measured length of  $87.65 \pm 0.43$  m.
8. **Use a pencil and ruler!** A *displacement vs. time* graph of an object moving with a constant speed in a straight line is given below. Draw a best fit line, estimate the slope and y-intercept, and finally give an equation of the speed of the object in the form  $y = mx + b$  which is equivalent to  $v(t) = mt + x_i$ . The horizontal blocks have units of 1 m and the vertical blocks have units of 1 m/s.



**Part 6: Learn how to add, subtract, multiply, and divide uncertainties**

$$1. \quad \begin{array}{r} 3.14 \pm 0.15 \\ + \quad 9.26 \pm 0.53 \end{array}$$

$$2. \quad \begin{array}{r} 6.26 \pm 0.43 \\ + \quad 3.8 \pm 0.27 \end{array}$$

$$3. \quad \begin{array}{r} 1.69 \pm 0.39 \\ + \quad 9.37 \pm 0.51 \end{array}$$

$$4. \quad \begin{array}{r} 5.89 \pm 0.79 \\ - \quad 3.23 \pm 0.84 \end{array}$$

$$5. \quad \begin{array}{r} 9.50 \pm 0.28 \\ - \quad 8.4 \pm 0.97 \end{array}$$

$$6. \quad \begin{array}{r} 5.82 \pm 0.09 \\ - \quad 4.94 \pm 0.45 \end{array}$$

$$7. \quad \begin{array}{l} 3.14 \pm 0.15 \\ \times \quad 9.26 \pm 0.53 \end{array}$$

$$8. \quad \begin{array}{l} 6.26 \pm 0.43 \\ \times \quad 3.8 \pm 0.27 \end{array}$$

$$9. \quad \begin{array}{l} 1.69 \pm 0.39 \\ \times \quad 9.37 \pm 0.51 \end{array}$$

$$10. \quad \begin{array}{l} 5.89 \pm 0.79 \\ \div \quad 3.23 \pm 0.84 \end{array}$$

$$11. \begin{array}{r} 9.50 \pm 0.28 \\ \div 8.4 \pm 0.97 \end{array}$$

$$12. \begin{array}{r} 5.82 \pm 0.09 \\ \div 4.94 \pm 0.45 \end{array}$$

$$13. (3.14 \pm 0.15)^2$$

$$14. (9.26 \pm 0.53)^3$$

$$15. (6.26 \pm 0.43)^4$$

$$16. \sqrt{(3.14 \pm 0.15)}$$



17.  $\sqrt[3]{(9.26 \pm 0.53)}$

18.  $\sqrt[4]{(6.26 \pm 0.43)}$

19. What is the percent uncertainty of the perimeter of a rectangle if it has a length of  $2.45 \pm 0.3$  m and a width of  $3.56 \pm 0.4$  m?

20. What is the percent uncertainty of the area of a rectangle if its length is uncertain by 3% and its width is uncertain by 4%?

21. What is the percent uncertainty of the volume of a box if its length is uncertain by 3%, its width is uncertain by 4%, and its height is uncertain by 5%?

22. What is the percent uncertainty of the perimeter/circumference of a circle if its radius is uncertain by 7%?

23. What is the percent uncertainty of the area of a circle if its radius is uncertain by 7%?

24. What is the percent uncertainty of the volume of a sphere if its radius is uncertain by 7%?

25. Mustafa has a height of  $(172 \pm 0.2)$  cm. Nour has a height of  $(167 \pm 0.35)$  cm. How much taller, including uncertainty, is Mustafa taller than Nour?

26. Twelve identical square tiles each have a length of 45.62 cm with an uncertainty of 0.2 cm. What is the total length, including uncertainty, of the 12 tiles if they are each placed side-by-side?

27. What is the perimeter, including uncertainty, of a rectangle with a length of  $(3.14 \pm 0.15)$  cm and a width of  $(9.26 \pm 0.53)$  cm?

28. What is the area, including uncertainty, of a rectangle with a length of  $(3.14 \pm 0.15)$  cm and a width of  $(9.26 \pm 0.53)$  cm?

29. What is the volume, including uncertainty, of a box with a length of  $(3.14 \pm 0.15)$  cm, a width of  $(9.26 \pm 0.53)$  cm, and a height of  $(6.26 \pm 0.43)$  cm?

31. What is the area, including uncertainty, of a circle with radius of  $(3.83 \pm 0.27)$  cm?

32. What is the volume, including uncertainty, of a sphere with radius of  $(3.83 \pm 0.27)$  cm?

33. What is the speed, including uncertainty, of a boat which travels  $(31.41 \pm 0.59)$  m in  $(2.65 \pm 0.35)$  s?

**Part 7: Define the following terms**

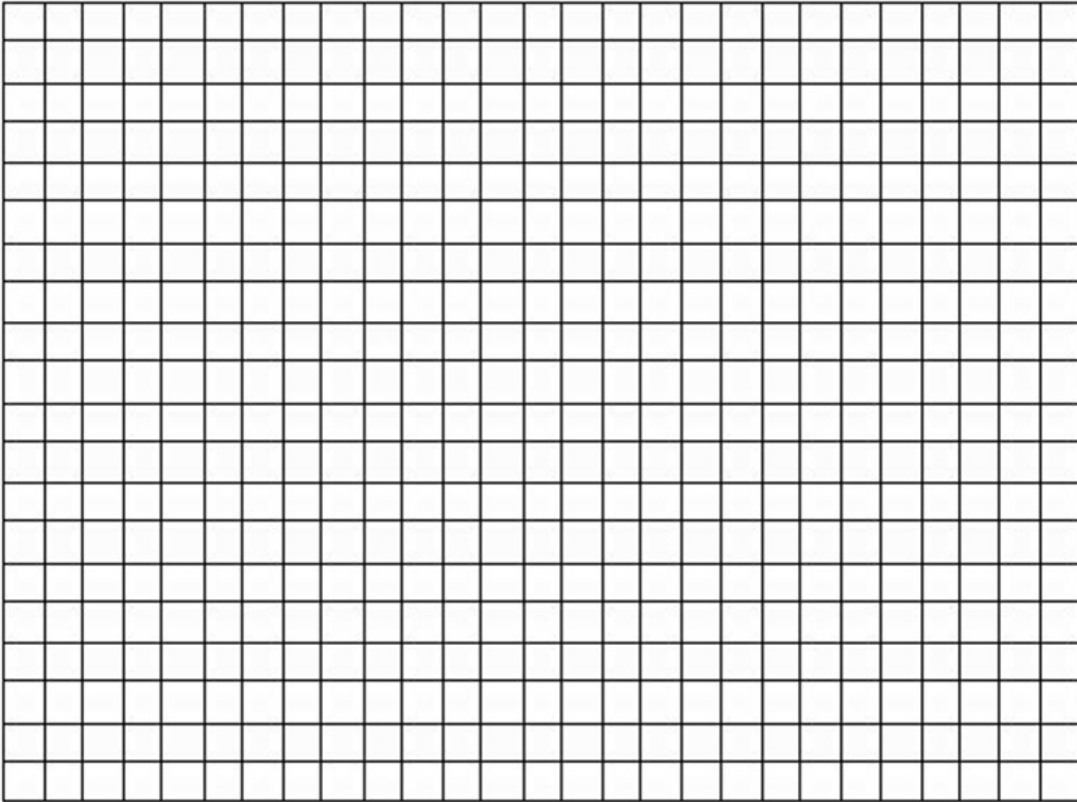
1. magnitude
2. scalar
3. vector (What is the symbol for a vector?)

**Part 8: Determine if the following quantities are *scalars* or *vectors*.**

1. Money	30. Specific heat capacity
2. Perimeter	31. Latent heat of fusion
3. Circumference	32. Luminosity
4. Area	33. Brightness
5. Volume	34. Emissivity
6. Angle	35. Albedo
7. Time	36. Pressure
8. Length	37. Moles
9. Distance	38. Entropy (HL)
10. Displacement	39. Charge
11. Speed	40. Current
12. Velocity	41. Voltage
13. Acceleration	42. Resistance
14. Force	43. Electromotive force
15. Linear momentum	44. Wavelength
16. Impulse	45. Period
17. Work	46. Frequency
18. Calories	47. Gravitational field strength
19. Energy	48. Gravitational potential (HL)
20. Kinetic energy	49. Electric field strength
21. Potential energy	50. Electric Potential (HL)
22. Power	51. Magnetic field strength
23. Angular speed (HL)	52. Magnetic flux (HL)
24. Angular acceleration (HL)	53. Activity
25. Moment of inertia (HL)	54. Half-life
26. Torque (HL)	
27. Angular momentum (HL)	
28. Density	
29. Temperature	

**Part 9: Drawing vectors. Use a pencil and ruler!**

1. Let the vectors  $\vec{A} = (x_1, y_1) = (3, -2)$  and  $\vec{B} = (x_2, y_2) = (-1, 4)$ 
  - a. Draw a horizontal and vertical axis on the graph below. Label the horizontal axis x and the vertical axis y.
  - b. Draw  $\vec{A}$  on the graph below.
  - c. What is the magnitude of the horizontal component of  $\vec{A}$ ?
  - d. What is the magnitude of the vertical component of  $\vec{A}$ ?
  - e. What is the magnitude of  $\vec{A}$ ?
  - f. Draw  $\vec{B}$  on the graph below.
  - g. What is  $\vec{A} + \vec{B}$ ? Draw it on the graph below.
  - h. What is the magnitude of the horizontal component of  $\vec{A} + \vec{B}$ ?
  - i. What is the magnitude of the vertical component of  $\vec{A} + \vec{B}$ ?
  - j. What is the magnitude of  $\vec{A} + \vec{B}$ ?
  - k. What is  $\vec{B} + \vec{A}$ ? Draw it on the graph below.
  - l. What is  $\vec{A} - \vec{B}$ ? Draw it on the graph below.
  - m. What is  $\vec{B} - \vec{A}$ ? Draw it on the graph below.
  - n. What is  $-\vec{A} - \vec{B}$ ? Draw it on the graph below.
  - o. What is  $-\vec{B} - \vec{A}$ ? Draw it on the graph below.





**Part 10: The Classic “Boat Crossing a River” Problem**

1. Adam is on a boat. It is moving from south to north on a river at a speed of 9.00 m/s. The water in the river is moving from east to west with a speed of 4.00 m/s. The river is 81.0 m wide.
  - a. Draw a figure.
  - b. How long will it take for the boat to reach the other side?
  - c. How many meters will the boat have traveled westward?
  - d. What will be the total displacement of the boat?

2. Enoch is on a boat. It is moving from north to south on a river at a speed of 6.00 m/s. The water in the river is moving from west to east with a speed of 3.00 m/s. The river is 99.0 m wide.
- Draw a figure.
  - How long will it take for the boat to reach the other side?
  - How many meters will the boat have traveled westward?
  - What will be the total displacement of the boat?

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.1 Kinematics

### Understandings

- The motion of bodies through space and time can be described and analyzed in terms of position, velocity, and acceleration.
- The velocity is the rate of change of position and acceleration is the rate of change of velocity.
- The change in position is the displacement.
- The difference between distance and displacement.
- The difference between the instantaneous and average values of velocity, speed, and acceleration, and how to describe them.
- The equations of motion for solving problems with uniformly accelerated motion as given by
  - $s = \frac{u+v}{2}t$
  - $v = u + at$
  - $s = ut + \frac{1}{2}at^2$
  - $v^2 = u^2 + 2as$
- Motion with uniform and non-uniform acceleration.
- The behavior of projectiles in the absence of fluid resistance, and the application of the equations of motion resolved into vertical and horizontal components.
- The qualitative effect of fluid resistance on projectiles, including time of flight, trajectory, velocity, acceleration, range, and terminal speed.

## Equations

$$s = \frac{u+v}{2} t$$

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as$$

## Interesting facts

- The record for the tallest person in the world is Robert Wadlow who measured 272 cm. He died at the age of 22.
- The record for the tallest building in the world is the Burj Khalifa in the United Arab Emirates which is almost 830 m tall.
- The record for the tallest mountain above sea level is Mount Everest which is located between China and Nepal. It is measured to be about 8,848 m above sea level.
- The record for the lowest depth below sea level is the Mariana Trench which is about 10,984 m below sea level. Surprisingly both life and pollution is found near the bottom of the Marina Trench.
- The fastest baseball pitch ever recorded is from Aroldis Chapman at which was about 169.1 km/h or 46.97 m/s.

## Super Ultimate Graphing Challenge

<http://theuniverseandmore.com/>

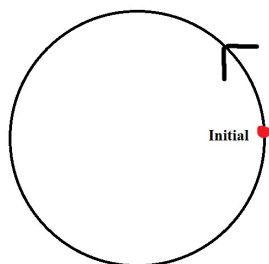
The solutions can be found on the YouTube channel Go Physics Go:

<https://www.youtube.com/@gophysicsgo/playlists>

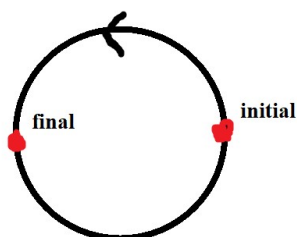
**Part 1: Use your favorite sources to answer the following questions**

1. C: What is the meaning of *zero dimensions*? *One dimension*? *Two dimensions*? *Three dimensions*? *Four dimensions*? If possible draw a figure for each.
2. C: Define *position*.
3. C: Define *distance*. Scalar or vector? Units? Example? Can *distance* be negative?
4. C: Define *displacement*. Scalar or vector? Units? Example? Can *displacement* be negative?
5. C: Define *speed*. Scalar or vector? Equation? Units? Example? Can *speed* be negative?
6. C: Define *velocity*. Scalar or vector? Equation? Units? Example? Can *velocity* be negative?
7. C: Define *average speed*. Scalar or vector? Equation? Units?
8. C: Define *average velocity*. Scalar or vector? Equation? Units?

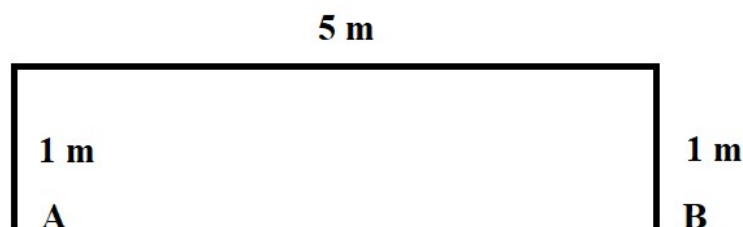
9. E: An object moves in a circle with a radius of 3.00 m. It takes the object 4.00 s to complete one revolution.



- a. What is the average speed and the average velocity of the object after it completes **one** cycle/revolution?
- b. What is the average speed and the average velocity of the object after it completes **one-half** cycle/revolution?



10.E: An object starts from rest at point A and then travels to point B by moving north 1.00 m, then east 5.00 m, and finally south 1.00 m in a total time of 14.0 s. What is the average speed and the average velocity of the object when it moves from point A to point B?



11.C: Define *instantaneous speed*. Example?

12.C: Define *instantaneous velocity*. Example?

13.C: Define *acceleration*. Equation? Units? Example? Can *acceleration* be negative?

14.C: What is the magnitude of the acceleration of free fall  $\vec{g}$  near the surface of the Earth? Which direction/way does it point? Is it positive or negative?

15.C: Define *projectile motion*.

16.C: What does the slope of a *displacement vs. time* graph tell us? Equation? Units?

17.C: What does the slope of a *velocity vs. time* graph tell us? Equation? Units?

18.C: What does the slope of an *acceleration vs. time* graph tell us? Equation? Units?

19.C: What does the area under a *displacement vs. time* graph tell us? Units?

20.C: What does the area under a *velocity vs. time* graph tell us? Units?

21.C: What does the area under an *acceleration vs. time* graph tell us? Units?

22.C: How would you go about determining the acceleration due to gravity near the surface of the Earth? Which equation will you use? Which instruments do you need? What will you do?



23.C: **Use a pencil and ruler!** Define *terminal velocity*. What is the relationship between speed and the force of friction? Draw a *distance vs. time* graph, a *speed vs. time* graph, and an *acceleration vs. time* graph of an object being dropped from rest from a very high height above the surface of the Earth with both the force of friction and the force of gravity acting on it.

24.C: **Use a pencil and ruler!** Draw a *speed vs. time* graph of a skydiver first jumping out of an airplane, then reaching terminal velocity, then opening his parachute, then reaching a second terminal velocity, and finally hitting the ground.

25.C: A ball/projectile is thrown with an initial angle of  $50.0^\circ$  north of east. Draw its trajectory with no air friction and with air friction.

26.E: A ball is thrown vertically upwards with an initial velocity of 40.0 m/s in the absence of air friction. For this problem let the acceleration due to gravity be  $10.0 \text{ m/s}^2$  down.

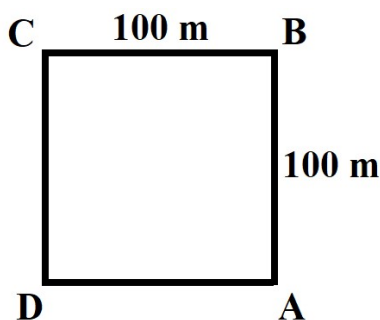
a. Complete the table below:

Time (s)	Acceleration $\left[\frac{\text{m}}{\text{s}^2}\right]$	Velocity $\left[\frac{\text{m}}{\text{s}}\right]$ $v_f = at + v_i$	Displacement [m] $y_f = \frac{1}{2}at^2 + v_i t + y_i$	Total distance traveled [m]
0				
1				
2				
3				
4				
5				
6				
7				
8				

- b. **Use a pencil and ruler!** Draw an *acceleration vs. time* graph, a *velocity vs. time* graph, a *speed vs. time* graph, a *displacement vs. time* graph, and a *distance vs. time* graph for the ball.

### Part 2: Distance, Displacement, Speed, and Velocity

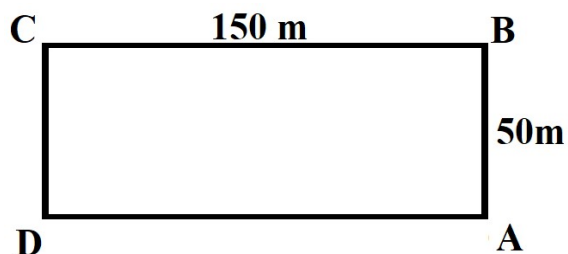
1. E: Wayde Van Niekerk from Russia runs 400. meters at a constant speed around a square track in a time of 43.03 s beginning at point A in a counterclockwise direction as shown below.



Complete the following table by determining the distance, displacement, speed, and velocity of Wayde Van Kiekerk at the following points. State both the magnitude and direction for the displacement and velocity of Wayde Van Kiekerk.

	Point B	Point C	Point D	Point A
Total Distance				
Total Displacement				
Average Speed				
Average Velocity				

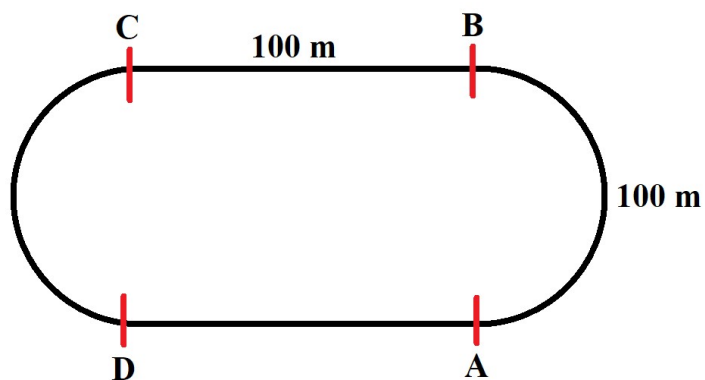
2. E: Wayde Van Niekerk from Russia runs 400. meters at a constant speed around a rectangular track in a time of 43.03 s beginning at point A in a counterclockwise direction as shown below.



Complete the following table by determining the distance, displacement, speed, and velocity of Wayde Van Kiekerk at the following points. State both the magnitude and direction for the displacement and velocity of Wayde Van Kiekerk.

	Point B	Point C	Point D	Point A
Total Distance	50.0 m	200. m	250. m	400. m
Total Displacement				
Average Speed				
Average Velocity				

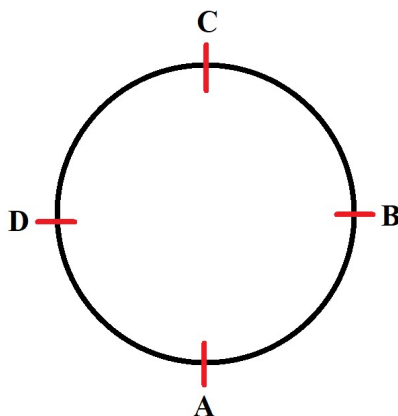
3. E: Wayde Van Niekerk from Russia runs 400. meters at a constant speed around an Olympic track in a time of 43.03 s beginning at point A in a counterclockwise direction as shown below. Each semicircle has an outer perimeter of 100. meters.



Complete the following table by determining the distance, displacement, speed, and velocity of Wayde Van Kiekerk at the following points. State both the magnitude and direction for the displacement and velocity of Wayde Van Kiekerk.

	Point B	Point C	Point D	Point A
Total Distance				
Total Displacement				
Average Speed				
Average Velocity				

4. E: Wayde Van Niekerk from Russia runs 400. meters at a constant speed around a circular track in a time of 43.03 s beginning at point A in a counterclockwise direction as shown below.

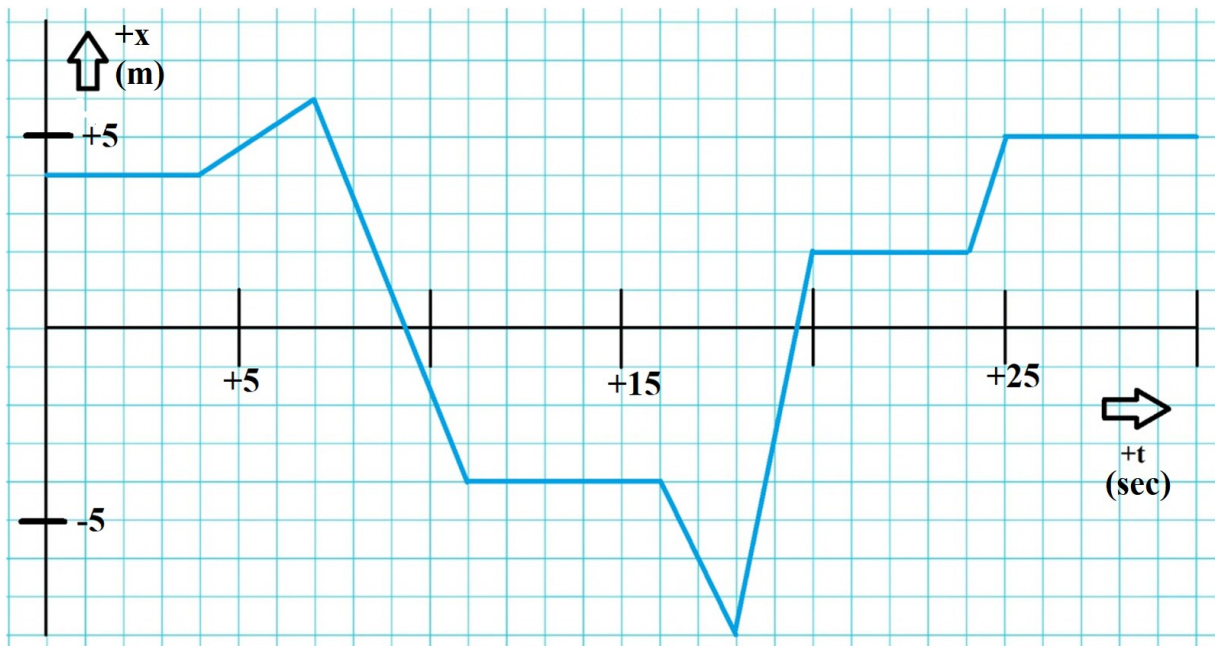


Complete the following table by determining the distance, displacement, speed, and velocity of Wayde Van Kiekerk at the following points. State both the magnitude and direction for the displacement and velocity of Wayde Van Kiekerk.

	Point B	Point C	Point D	Point A
Total Distance				
Total Displacement				
Average Speed				
Average Velocity				

### Part 3: Motion Graphs

1. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *displacement vs. time* graph is shown below.



a. What does the slope of a *displacement vs. time* graph tell us?

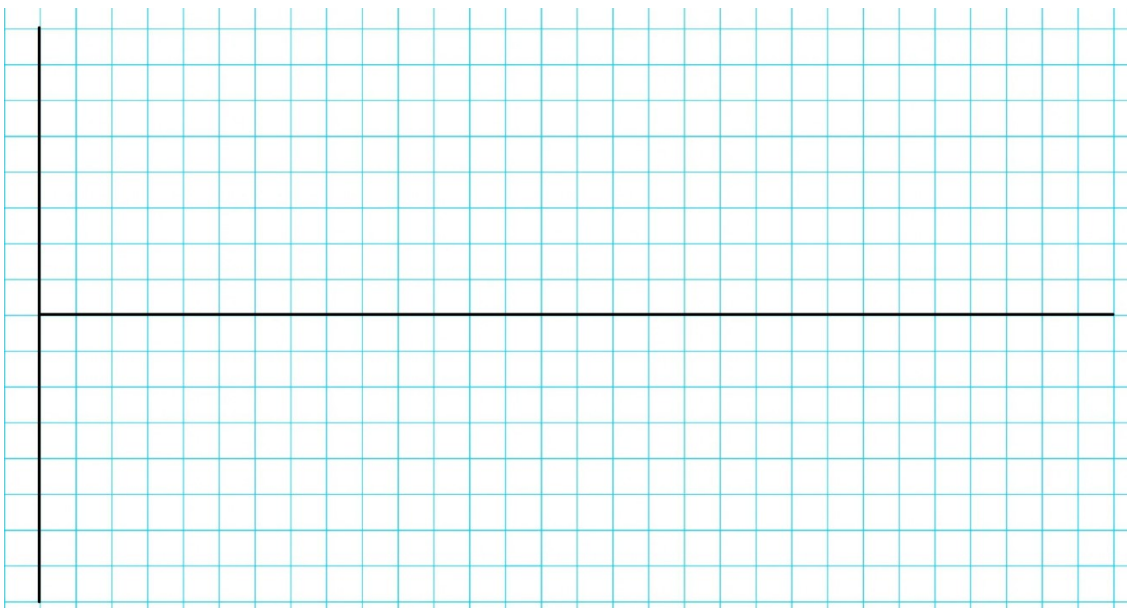
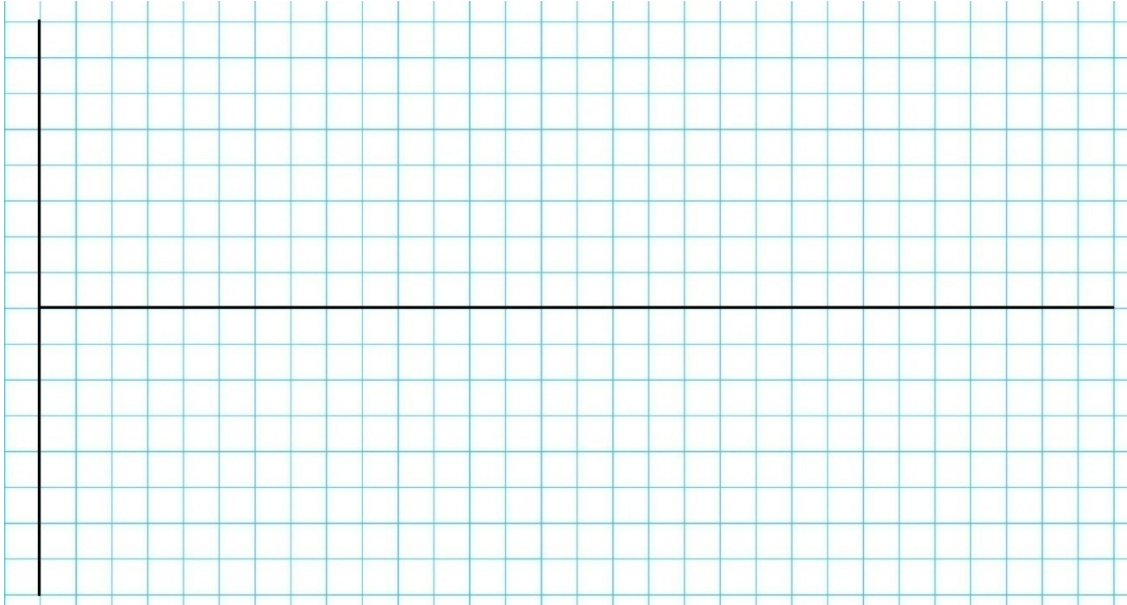


- b. Give all your solutions to two significant figures. Determine the *displacement* and *velocity* of the object at

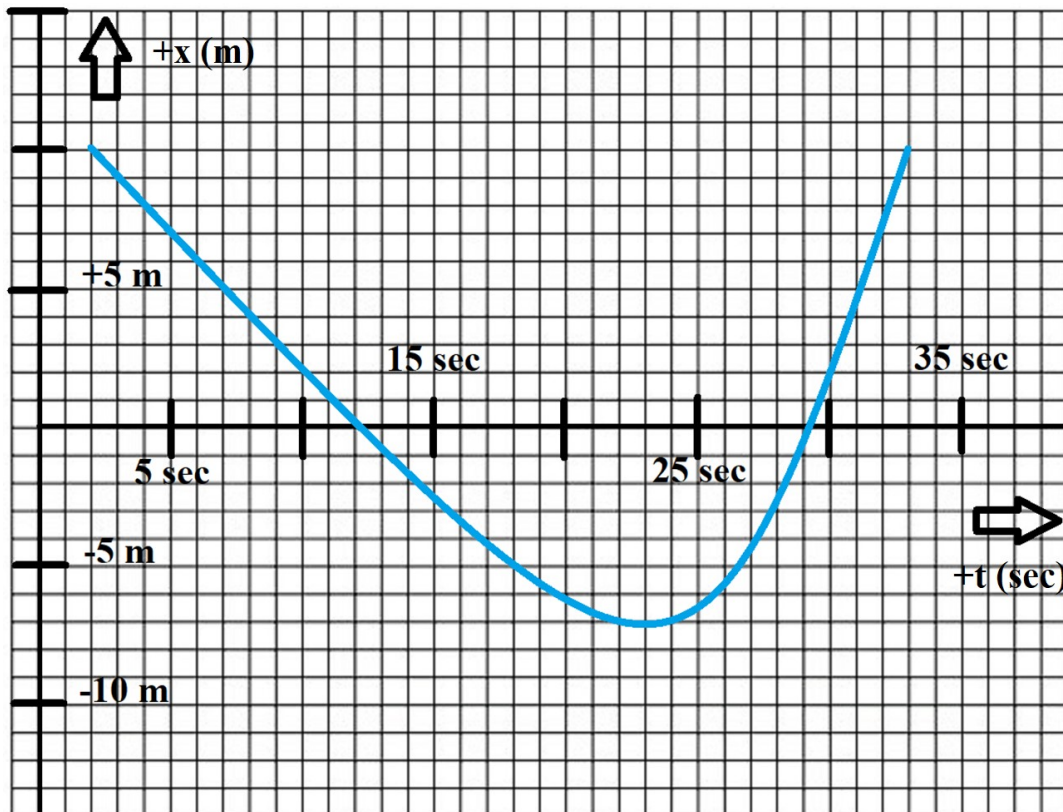
Time (s)	Displacement (m)	Velocity (m/s)
3.0		
5.0		
9.0		
13.0		
17.0		
19.0		
23.0		
24.5		
28		

- c. What is the *total distance* the object travels from  $t = 0$  s to  $t = 30$  s?
- d. What is the *displacement* of the object from  $t = 0$  s to  $t = 30$  s?
- e. What does the slope of a *velocity vs. time* graph tell us?

- f. **Use a pencil and ruler!** Draw a *velocity vs. time* graph and an *acceleration vs. time* graph. Label your axes!



2. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *displacement vs. time* graph is shown below.



a. Is the object moving to the left or the right? Is it speeding up or slowing down?

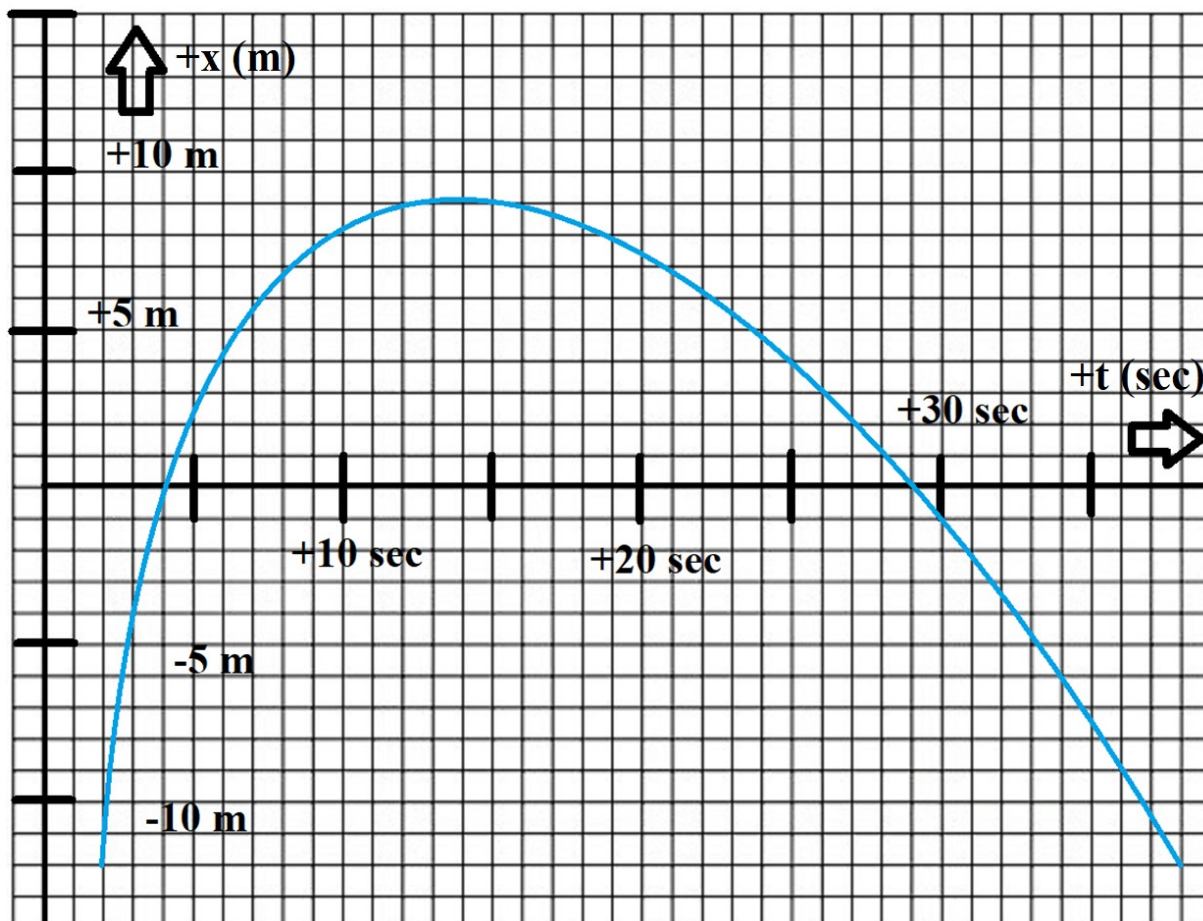
From $t = 2 \text{ s}$ to $t = 12 \text{ s}$	
From $t = 12 \text{ s}$ to $t = 23 \text{ s}$	
From $t = 23 \text{ s}$ to $t = 29 \text{ s}$	
From $t = 29 \text{ s}$ to $t = 33 \text{ s}$	

- b. Give all your solutions to two significant figures. Determine the *displacement* and *velocity* of the object at

Time (s)	Displacement (m)	Velocity (m/s)
12.		
23.		
29.		

- c. What is the *total distance* the object travels from  $t = 2$  s to  $t = 33$  s?
- d. What is the *displacement* of the object from  $t = 2$  s to  $t = 33$  s?

3. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *displacement vs. time* graph is shown below.



a. Is the object moving to the left or the right? Is it speeding up or slowing down?

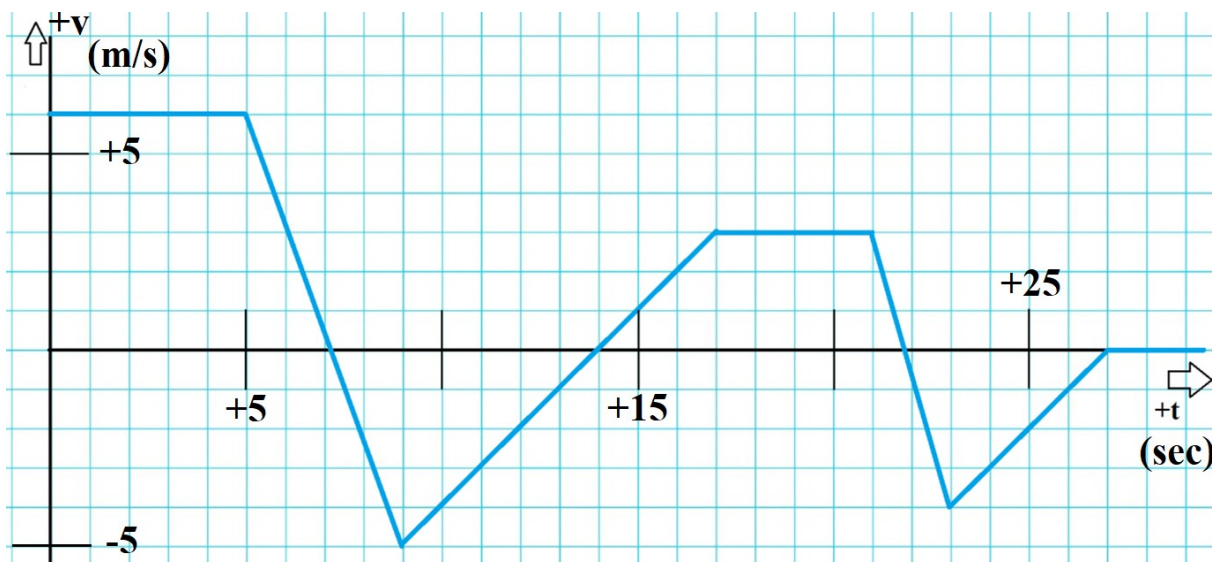
From $t = 2 \text{ s}$ to $t = 4 \text{ s}$	
From $t = 4 \text{ s}$ to $t = 14 \text{ s}$	
From $t = 14 \text{ s}$ to $t = 29 \text{ s}$	
From $t = 29 \text{ s}$ to $t = 38 \text{ s}$	

- b. Give all your solutions to two significant figures. Determine the *displacement* and *velocity* of the object at

Time (s)	Displacement (m)	Velocity (m/s)
4.0		
14.		
29.		

- c. What is the *total distance* the object travels from  $t = 2$  s to  $t = 38$  s?
- d. What is the *displacement* of the object from  $t = 2$  s to  $t = 38$  s?

4. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *velocity vs. time* graph is shown below.



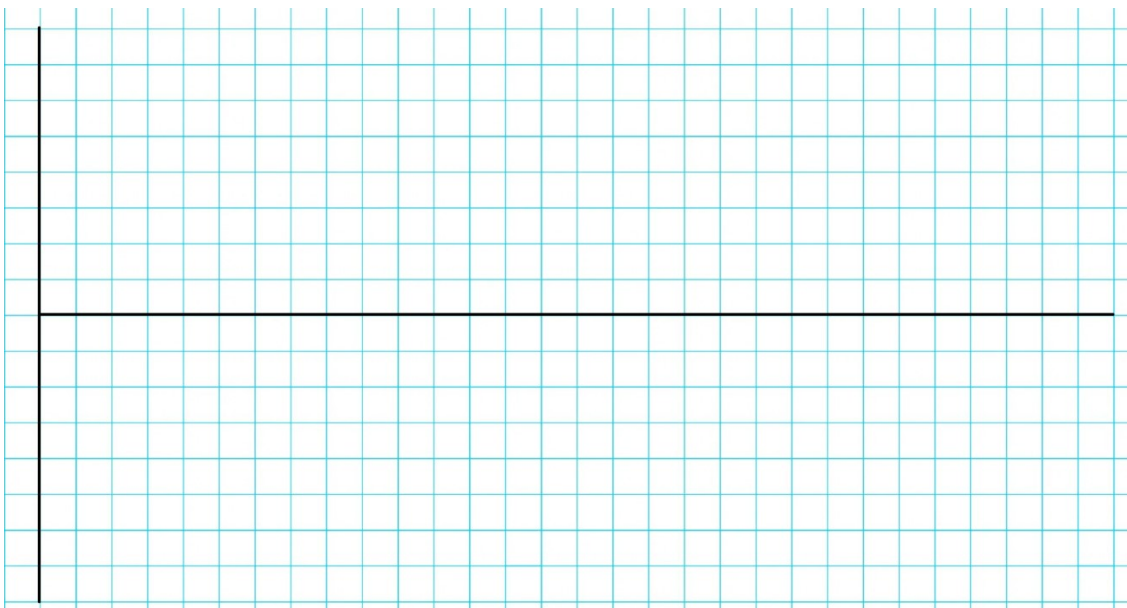
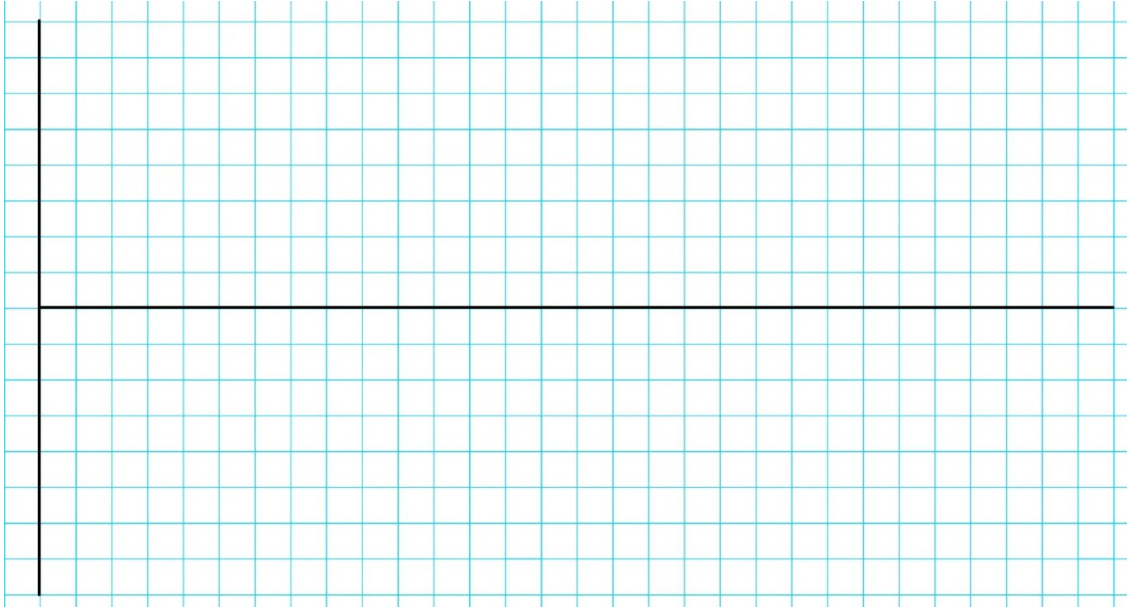
- What does the slope of a *velocity vs. time* graph tell us?
- Give all your solutions to two significant figures. Find the *velocity* and the *acceleration* of the object at

Time (s)	Velocity (m/s)	Acceleration (m/s <sup>2</sup> )
3.0		
7.0		
8.0		
10.		
14.		
18.		
22.		
29.		

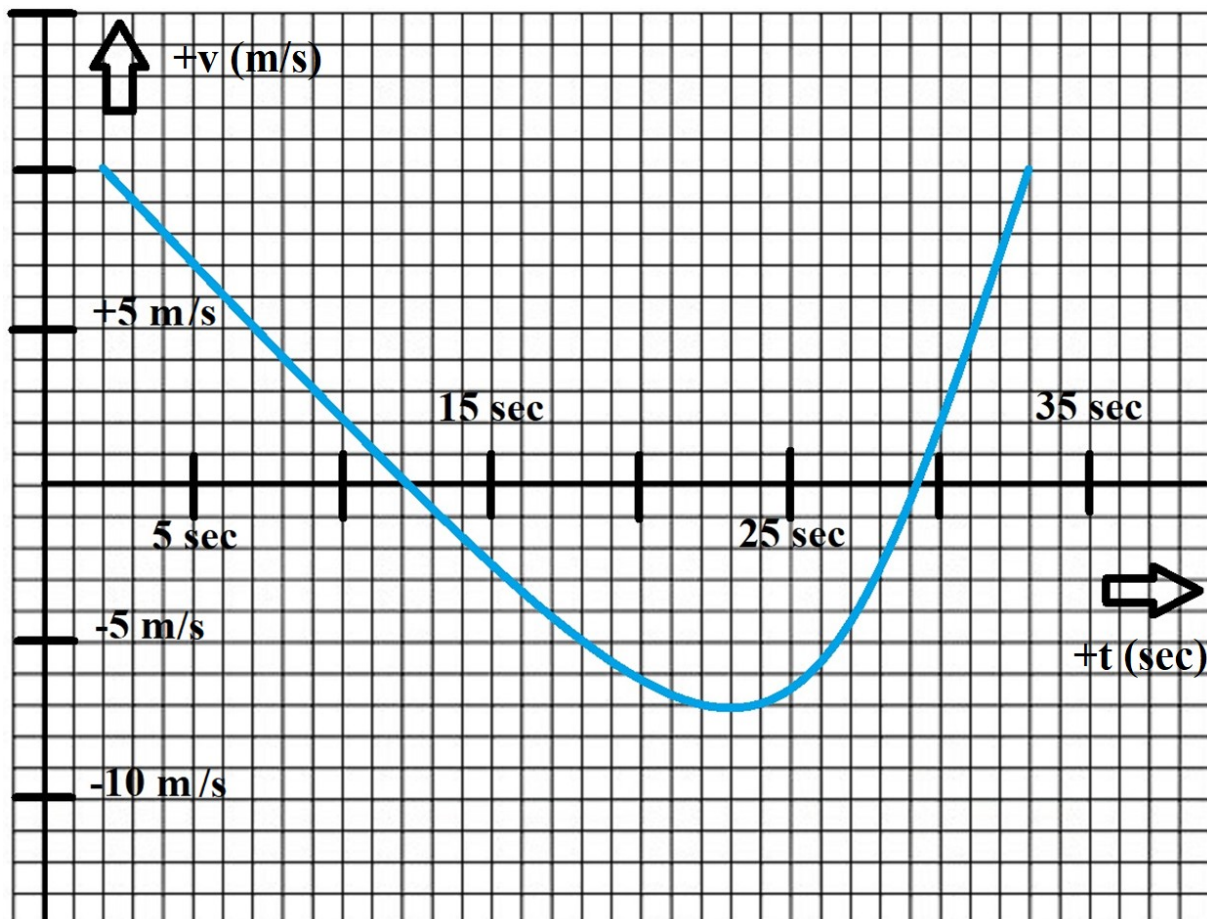
- c. What does the area under a *velocity vs. time* graph tell us?
- d. Find the *displacement* of the object from
- i.  $t = 0 \text{ s}$  to  $t = 5 \text{ s}$
  - ii.  $t = 5 \text{ s}$  to  $t = 9 \text{ s}$
  - iii.  $t = 9 \text{ s}$  to  $t = 17 \text{ s}$
  - iv.  $t = 17 \text{ s}$  to  $t = 27 \text{ s}$
- e. Determine the *total distance* the object travels from  $t = 0 \text{ s}$  to  $t = 29 \text{ s}$ .
- f. Determine the *displacement* of the object from  $t = 0 \text{ s}$  to  $t = 29 \text{ s}$ .



- g. **Use a pencil and ruler!** Draw an *acceleration vs. time* graph and a *displacement vs. time* graph. Label your axes!



5. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *velocity vs. time* graph is shown below.



- a. Is the object moving to the left or the right? Is its acceleration increasing, decreasing, or constant?

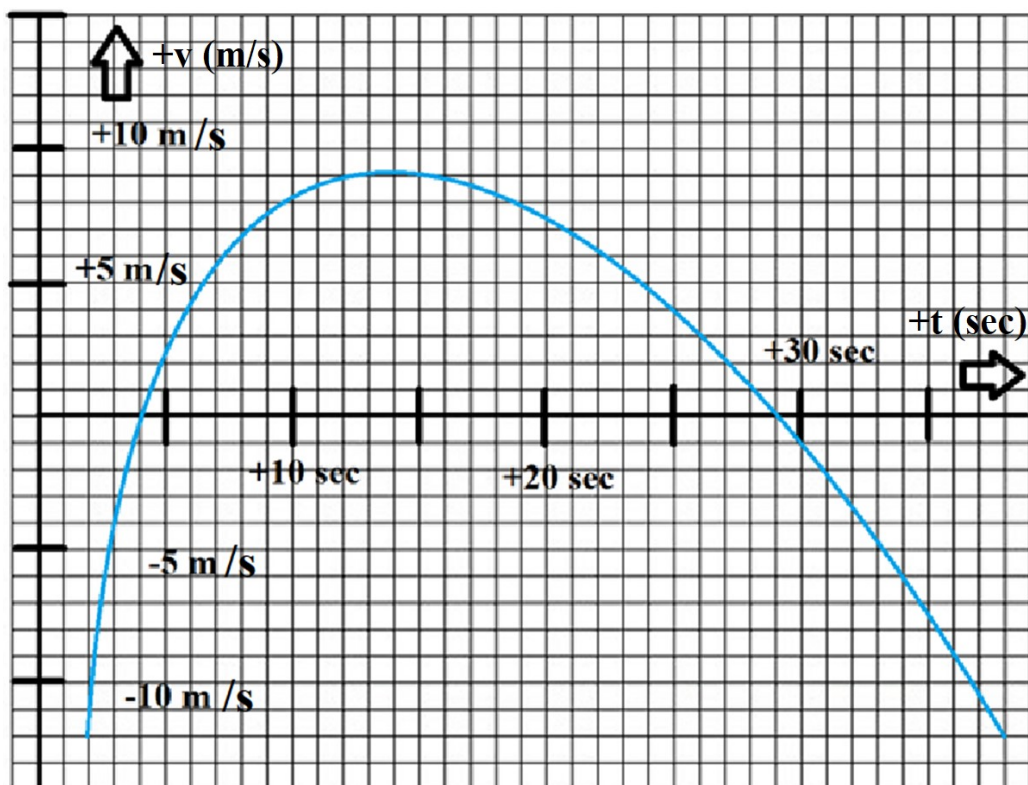
From $t = 2 \text{ s}$ to $t = 12 \text{ s}$	
From $t = 12 \text{ s}$ to $t = 23 \text{ s}$	
From $t = 23 \text{ s}$ to $t = 29 \text{ s}$	
From $t = 29 \text{ s}$ to $t = 33 \text{ s}$	

- b. Give all your solutions to two significant figures. Determine the *velocity* and *acceleration* of the object at

Time (s)	Velocity (m/s)	Acceleration (m/s <sup>2</sup> )
12		
23		
29		

- c. Determine the *total distance* the object travels from  $t = 2$  s to  $t = 33$  s.
- d. Determine the *displacement* of the object from  $t = 2$  s to  $t = 33$  s.

6. E: An object can move to the left or right in one dimension. Positive displacement is towards the right and negative displacement is towards the left. Its *velocity vs. time* graph is shown below.



- a. Is the object moving to the left or the right? Is its acceleration increasing, decreasing, or constant?

From $t = 2$ s to $t = 4$ s	
From $t = 4$ s to $t = 14$ s	
From $t = 14$ s to $t = 29$ s	
From $t = 29$ s to $t = 38$ s	

- b. Give all your solutions to two significant figures. Determine the *velocity* and *acceleration* of the object during the following times:

Time [s]	Velocity [m/s]	Acceleration [ $\text{m/s}^2$ ]
4		
14		
29		

- c. Determine the *total distance* the object travels from  $t = 2$  s to  $t = 38$  s.
- d. Determine the *displacement* of the object from  $t = 2$  s to  $t = 38$  s.

**Part 4: Motion equations**

There is no air friction for all the problems. The magnitude of the acceleration from gravity is  $9.81 \text{ m/s}^2$ .

1. E: A car starts from rest and speeds up to  $35.0 \text{ m/s}$  in  $12.0 \text{ s}$ .
  - a. What is the average acceleration of the car during these  $12.0 \text{ s}$ ?
  - b. What is the total distance traveled by the car during these  $12.0 \text{ s}$ ?

The car then travels at a constant speed of  $35.0 \text{ m/s}$  for  $900. \text{ m}$ .

- c. How long was the car travelling at this constant speed?

The car finally slows down from  $35.0 \text{ m/s}$  to  $15.0 \text{ m/s}$  in  $4.00 \text{ s}$ .

- d. What is the average acceleration (or deceleration) of the car during these  $4.00 \text{ s}$ ?
  - e. What is the total distance the car travels during these  $4.00 \text{ s}$ ?
  - f. What is the total distance the car travels since it started from rest?
  - g. What is the total time taken for the car to travel since it started from rest until it reaches a speed of  $15.0 \text{ m/s}$ ?

- h. **Use a pencil and ruler!** Draw a *displacement vs. time* graph, a *velocity vs. time* graph, and an *acceleration vs. time* graph for the car.

2. E: A car starts from rest and accelerates at a constant rate of  $4.00 \text{ m/s}^2$  for 8.00 s.

- a. What is the speed of the car after 8.00 s?
- b. How much distance did the car travel during these 8.00 s?

The car then moves at a constant speed for 12.0 s.

- c. How much distance did the car travel during these 12.0 s?

The car then slows to a stop at a rate of  $3.00 \text{ m/s}^2$ .

- d. How much time did it take for the car to decelerate and stop?
- e. How much distance did the car travel when it decelerates?

- f. What is the total time taken for the car to travel?
  - g. What is the total distance taken for the car to travel?
  - h. **Use a pencil and ruler!** Draw a *displacement vs. time* graph, a *velocity vs. time* graph, and an *acceleration vs. time* graph for the car.
3. E: Enoch throws a ball vertically upwards with an initial speed of 47.0 m/s at an elevation of 8,848 m above the surface of the Earth.
- a. What will be the acceleration of the ball (number and direction) at the moment after it is thrown upwards?
  - b. What will be the acceleration of the ball (number and direction) when it reaches its maximum height?
  - c. What will be the velocity of the ball when it reaches its maximum height?
  - d. How long will it take for the ball to reach its maximum height?



- e. How many meters above the surface of the Earth will the ball be when it reaches its maximum height?
- f. What will be the acceleration of the ball (number and direction) just before it strikes the surface of the Earth?
- g. What will be the velocity of the ball when it is 8,950 m above the surface of the Earth?
- h. How long will it take for the ball to be 8,950 m above the surface of the Earth?
- i. What will be the velocity of the ball just before it strikes the ground?
- j. What is the total distance the ball travels during the first 4.50 s?
- k. What is the total distance the ball travels?
- l. How long will it take for the ball to be 300. m above the surface of the Earth?

m. **Use a pencil and ruler!** Draw a *displacement vs. time* graph, a *velocity vs. time* graph, and an *acceleration vs. time* graph for the ball.

4. E: Noah drops a ball from rest at an elevation 830. m above the surface of the Earth.

- a. What will be the acceleration of the ball (number and direction) at the moment it is dropped?
- b. What will be the acceleration of the ball (number and direction) when it is 415. m above the surface of the Earth?
- c. What will be the acceleration of the ball (number and direction) just before it strikes the surface of the Earth?
- d. How long will it take for the ball to be 415 m above the surface of the Earth?
- e. What will be the velocity of the ball 415 m above the surface of the Earth?
- f. What will be the velocity of the ball just before it strikes the ground?

- g. What is the total distance the ball travels during the first 8.00 s?
  - h. How long will it take for the ball to be 300. m above the surface of the Earth?
  - i. What is the average speed of the ball?
  - j. **Use a pencil and ruler!** Draw a *displacement vs. time* graph, a *velocity vs. time* graph, and an *acceleration vs. time* graph for the ball.
5. E: Eber throws a ball vertically downwards with an initial speed of 22.0 m/s from a height of 8,848 m above the surface of the Earth.
- a. What will be the acceleration of the ball (number and direction) at the moment after it is thrown downwards?
  - b. What will be the acceleration of the ball (number and direction) just before it strikes the surface of the Earth?

- c. What will be the velocity of the ball just before it strikes the surface of the Earth?
- d. How long will it take for the ball to reach the surface of the Earth?
- e. What will be the velocity of the ball when it is 4,000 m above the surface of the Earth?
- f. How long will it take for the ball to reach 4,000. m above the surface of the Earth?
- g. What is the total distance the ball travels after 12.5 s?
- h. **Use a pencil and ruler!** Draw a *displacement vs. time* graph, a *velocity vs. time* graph, and an *acceleration vs. time* graph for the ball.

6. E: You are standing on the top of a building 135 m tall. You throw a ball upward with a velocity of 22.0 m/s. At the exact same moment a friend throws a second ball upward from the ground with a velocity of 46.0 m/s. These two balls then collide at some later time.
- How long after these two balls are released will they collide?
  - Where will these two balls be when they collide?
  - What will be the velocity of each ball just as they collide?
  - What will be the relative velocity between these two balls at the moment they collide?
7. E: From the top of a building 85.0 m tall a ball is dropped. At the same time another ball is thrown upward from the ground with a speed of 46.0 m/s.
- How long after the balls are released will they hit?

- b. How long above the ground will these two balls hit?
8. E: You are on the top of a building 44.2 m tall. The adjacent building is 98.1 m tall. You throw the ball upward so that the ball lands on the roof of the adjacent building 4.15 s after the ball is thrown. What will be the speed of the ball when it lands on the roof?
9. E: You are rushing to the train station to catch your morning commute. The train leaves the train station from rest with an acceleration of  $0.600 \frac{\text{m}}{\text{s}^2}$ . You arrive at the station exactly 4.00 s after the train leaves and you immediately start running after the train with a constant velocity of 8.50 m/s.
- a. How long after the train leaves the station do you catch up with the train?
- b. How far from the train station do you catch up with the train?

- c. With what minimum speed would you have to run in order to catch up with the train?

10.E: A marble is rolling along a horizontal tabletop, which is 94.0 cm above the floor, when the marble reaches the edge of the table and then falls to the floor. How long will it take for the marble to strike the floor?

11.E: A rifle bullet, which has a mass of 57.0 g, is fired horizontally from a rifle which is held 94.0 cm above the floor, with a velocity of 385 m/s. How long will it take for the bullet to strike the floor?

12.E: A marble is fired horizontally from a launching device attached to the edge of a tabletop which is 94.0 cm above the floor. The marble then strikes the floor 2.35 m from the edge of the table.

- a. How long will it take for the marble to reach the floor?
- b. What is the initial velocity of the marble as it leaves the launching device?
- c. What will be the horizontal velocity of the marble as it reaches the floor?

- d. What will be the vertical velocity of the marble as it reaches the floor?
- e. What will be the direction and magnitude of the velocity of the marble as it reaches the floor?

13.E: Salah throws a ball with an initial speed of 47.0 m/s at an angle of  $30.0^\circ$  north of east 830. meters above the surface of the Earth.

- a. Complete the table:

$x_i =$	$y_i =$
$v_{i,x} =$	$v_{i,y} =$
$a_x =$	$a_y =$

- b. What will be the horizontal velocity and horizontal acceleration of the ball (number and direction) when it reaches its maximum height?
- c. What will be the vertical velocity and vertical acceleration of the ball (number and direction) when it reaches its maximum height?
- d. How long will the ball be in the air for?



- e. What will be the range (horizontal distance) of the ball?
- f. What will be the maximum height of the ball from the surface of the Earth after it is thrown?
- g. How long will it take for the ball to reach its maximum height after it is thrown?
- h. How long does it take for the ball to reach 400. m above the surface of the Earth after it is thrown?
- i. How high above the surface of the Earth will the ball be 8.00 s after it is thrown?
- j. How far horizontally does the ball travel during the first 8.00 s after it is thrown?
- k. What will be the velocity of the ball (number and direction) 8.00 s after it is thrown?

1. What will be the displacement of the ball (number and direction) 8.00 s after it is thrown?

m. **Use a pencil and ruler!** Draw an *acceleration vs. time* graph, a *velocity vs. time* graph, a *speed vs. time* graph, a *displacement vs. time* graph, and a *distance vs. time* graph for the ball for both the horizontal direction and the vertical direction.

14.E: A projectile, which has a mass of 5.50 kg, is fired from the ground with an initial velocity of 169. m/s at an angle of  $23.0^\circ$  above the horizontal.

a. Complete the following table:

$x_i =$	$y_i =$
$v_{i,x} =$	$v_{i,y} =$
$a_x =$	$a_y =$

- b. What will be the velocity of this projectile at the highest point of its projectile?
- c. What will be the total flight time of this projectile?
- d. What will be the height of this projectile at the highest point of its trajectory?
- e. What will be the range of this projectile?
- f. What will be the vertical velocity of this projectile 3.50 s after it has been fired?
- g. What will be the horizontal velocity of this projectile 3.50 s after it has been launched?

- h. What will be the direction and magnitude of the projectile's velocity 3.50 s after it has been fired?
  
  
  
  
  
  
  
  
  
  
- i. What will be the height of this projectile 3.50 s after it has been fired?
  
  
  
  
  
  
  
  
  
  
- j. How far downrange will the projectile be 3.50 s after it has been fired?
  
  
  
  
  
  
  
  
  
  
- k. What will be the final displacement of the projectile 3.50 s after the projectile has been fired?

15.E: A Spanish Galleon enters a harbor defended by cannon placed on top of a castle wall which is 135 m above the water level. The cannon have a known muzzle velocity of 323 m/s and are aimed  $28.0^\circ$  above the horizontal. How far from the base of the castle wall will the Galleon be within the range of the cannon?





18.E: Lot throws a ball at an initial speed of 12.0 m/s at an angle of  $30.0^\circ$  south of east from 830. m above the surface of the Earth.

a. Complete the table:

$x_i =$	$y_i =$
$v_{i,x} =$	$v_{i,y} =$
$a_x =$	$a_y =$

- b. How long will the ball be in the air for?
- c. What will be the range of the ball?
- d. How long after the ball is thrown does it take to reach 400. m above the surface of the Earth?
- e. How high above the surface of the Earth will the ball be 4.00 s after it is thrown?
- f. How far horizontally does the ball travel during the first 4.00 s after it is thrown?



- g. What will be the velocity of the ball (number and direction) 4.00 s after it is thrown?
- h. What will be the displacement of the ball (number and direction) 4.00 s after it is thrown?

19.E: A 25.0 kg ball is thrown from the edge of a very tall building with an initial speed of 20.0 m/s at an angle of  $60.0^\circ$  north of east. There is an infinitely tall vertical wall 120. m from the building.

a. Draw a figure.

b. Complete the table:

$x_i =$	$y_i =$
$v_{i,x} =$	$v_{i,y} =$
$a_x =$	$a_y =$

c. How much time does it take for the ball to hit the wall?

d. At which height above or below the original position where the ball is thrown will the ball hit the wall?

e. What will be the velocity of the ball (number and direction) when it hits the wall?

f. What will be the displacement of the ball (number and direction) when it hits the wall?

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.2 – Forces and Momentum

### Understandings

- Newton's three laws of motion.
- Forces as interactions between bodies.
- Forces acting on a body can be represented in a free-body diagram.
- Free-body diagrams can be analyzed to find the resultant force on a system.
- The nature and use of the following contact forces:
  - The normal force  $F_N$  is the component of the contact force acting perpendicular to the surface that counteracts the body
  - The surface frictional force  $F_f$  acting in a direction parallel to the plane of contact between a body and a surface, on a stationary body as given by  $F_f \leq \mu_s F_N$  or a body in motion as given by  $F_f = \mu_d F_N$  where  $\mu_s$  and  $\mu_d$  are the coefficients of static and dynamic friction respectively.
  - The elastic restoring force  $F_H$  following Hooke's law as given by  $F_H = -kx$  where  $k$  is the spring constant.
  - The viscous drag force  $F_d$  acting on a small sphere opposing its motion through a fluid as given by  $F_d = 6\pi\eta rv$  where  $\eta$  is the fluid viscosity,  $r$  is the radius of the sphere, and  $v$  is the velocity of the sphere through the fluid.
  - The buoyancy  $F_b$  acting on a body due to the displacement of the fluid as given by  $F_b = \rho Vg$  where  $V$  is the volume of the fluid displaced.
- The nature and use of the following field forces:
  - The gravitational force  $F_g$  as the weight of the body and calculated as given by  $F_g = mg$ .
  - The electric force  $F_e$ .
  - The magnetic force  $F_m$ .
- Linear momentum is given by  $p = mv$  remains constant unless the system is acted upon by a resultant external force.
- A resultant external force applied to a system constitutes an impulse  $J$  as given by  $J = F\Delta t$  where  $F$  is the average resultant force and  $\Delta t$  is the time of contact.
- The applied external impulse equals the change in momentum of the system.

- Newton's second law in the form  $F = ma$  assumes mass is constant whereas  $F = \frac{\Delta p}{\Delta t}$  allows for situations where mass is changing.
- The elastic and inelastic collisions of two bodies.
- Explosions.
- Energy considerations in elastic collisions, inelastic collisions, and explosions.
- Bodies moving along a circular trajectory at a constant speed experience an acceleration that is directed radially towards the center of the circle – known as centripetal acceleration as given by  $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$ .
- Circular motion is caused by a centripetal force acting perpendicular to the velocity.
- A centripetal force causes the body to change direction even if its magnitude of velocity may remain constant.
- The motion along a circular trajectory can be described in terms of the angular velocity  $\omega$  which is related to the linear speed  $v$  by the equation as given by  $v = \frac{2\pi r}{T} = \omega r$ .

### Equations

$$F_f \leq \mu_s F_N$$

$$F_f = \mu_d F_N$$

$$F_H = -kx$$

$$F_d = 6\pi\eta r v$$

$$F_b = \rho V g$$

$$F_g = mg$$

$$p = mv$$

$$J = F\Delta t$$

$$F = ma = \frac{\Delta p}{\Delta t}$$

$$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

$$v = \frac{2\pi r}{T} = \omega r$$

The solutions can be found on the YouTube channel Go Physics Go:

<https://www.youtube.com/@gophysicsgo/playlists>

**Part 1: Use your favorite sources to answer the following questions**

1. C: What is the meaning and equation of *directly proportional*? *Inversely proportional*? Give an example of each.
  
  
  
  
  
  
  
  
  
  
2. C: What is *mass*? What are its units? Is it a scalar or vector?
  
  
  
  
  
  
  
  
  
  
3. C: What is a *force*? What are its units? Is it a scalar or vector? How many objects are needed for a *force*?
  
  
  
  
  
  
  
  
  
  
4. C: What is the *force of gravity*? This is also called *weight*.
  
  
  
  
  
  
  
  
  
  
5. C: What are the equations for the *force of gravity*
  - a. if we are near the surface of a planet?
  
  
  
  
  
  
  
  
  
  
  - b. in general (this is called *Newton's Law of Gravitation*)?
  
  
  
  
  
  
  
  
  
  
6. C: What are some differences between *mass* and *weight*?

7. C: What is the *normal force*? In which direction does it point? Draw an image.
8. C: What is the *force of friction*? In which direction does it point? Draw an image.
9. C: What is the equation for *surface friction*? Define each variable.
- 10.C: What is the meaning of *dynamic/kinetic*? *Static*? Which is greater: *kinetic friction* or *static friction*?
- 11.C: What is the meaning of a *rough surface*? A *smooth surface*?
- 12.C: For which object do we observe the *force of tension*? Draw an image.
- 13.C: What is the equation for the *spring force*? Define each variable. What is the name and what are the units of  $k$  in the spring force equation?



14.C: **Use a pencil and ruler!** Draw a *force vs. displacement* graph for a mass on a spring. What does the slope of a *force vs. displacement* graph tell us? What does the area under a *force vs. displacement* graph tell us?

15.C: What is the *buoyant force*? State its equation and define each variable.

16.C: State the equation for the viscous drag force acting on a small sphere opposing its motion through a fluid. Define each variable.

17.E: The coefficient of viscosity of blood at 37.0°C is approximately  $4.00 \times 10^{-3} \frac{\text{kg}}{\text{m}\cdot\text{s}}$ . What will be the acceleration of a metal sphere with a mass of 0.500 g and a diameter of 1.00 cm when it is falling vertically down a tube of blood with a speed of 0.200 m/s? The density of blood is approximately  $1,025 \frac{\text{kg}}{\text{m}^3}$ .

18.C: How do we draw a *free body diagram*? Here are the steps:

- Circle the object (or objects) in question
- Label all the external/outside forces on the object (or objects) with an arrow to show the direction and magnitude of each force
- Draw a convenient axis to minimize vector components
- For each object apply Newton's second law of motion for each axis

19.C: Label the forces on the following diagrams.

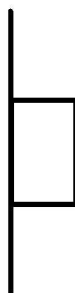
- A block is at rest on a horizontal surface.



- A man is pushing a block to the left with a horizontal force on a rough horizontal surface. The block does not move.

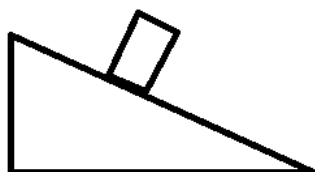


- An object is being pushed to the left on a wall.

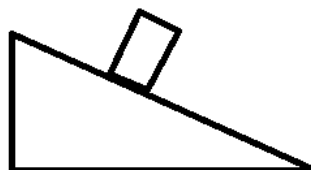


- d. A man is pushing a block on a slope which is  $20^\circ$  from the horizontal on a rough horizontal surface. The block does not move.

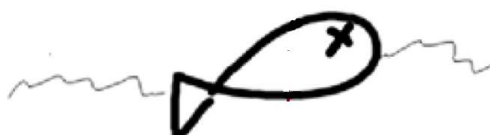
The man is pushing the block downwards. The push is parallel to the slope.



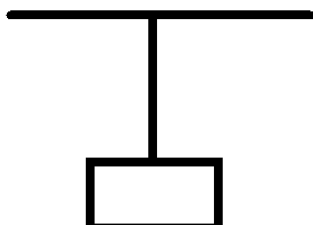
The man is pushing the block upwards. The push is parallel to the slope.



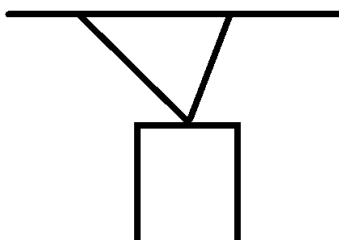
- e. A dead fish is floating on top of the plastic radioactive ocean water.



- f. A block is at rest and is hanging from the ceiling by one massless string.

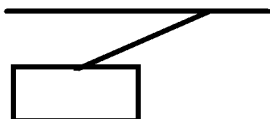


- g. A block is at rest and is hanging from the ceiling by two massless strings.

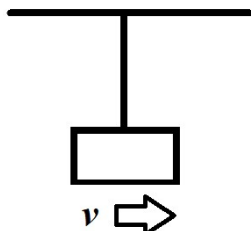


- h. A block is tied to a massless string and is raised up at an angle  $\theta$  from the vertical.

The block is released  
from rest.



The block is now at the  
bottom of its motion.



The block is now at the  
top of its motion.



- i. A car is moving in a straight line to the right with a constant speed



on a smooth horizontal surface.



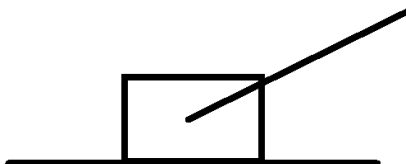
on a rough horizontal surface.



- j. A car is moving in a straight line to the right on a rough horizontal surface.

The car is slowing down (decelerating).	The car is speeding up (accelerating).
	
<p>You can also add force of brakes <math>F_B</math> pointing to the left.</p>	<p>You can also call the force of push <math>F_P</math> the driving force <math>F_D</math></p>

- k. A man pulls a massless string which is attached to a block with a constant speed at an angle  $\theta$  above the horizontal on a rough surface. Label the forces on the block, not the man.



- l. A ball is thrown vertically up and is moving upwards.

There is **no** force of air friction.

There **is** a force of air friction.



- m. A ball is thrown vertically up and is at its maximum height.

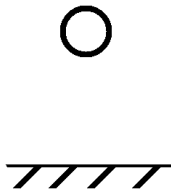
There is **no** force of air friction.

There **is** a force of air friction.

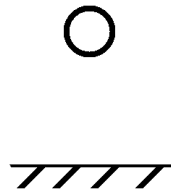


- n. A ball is **dropped** from rest from the top of a very tall building. There is no force of air friction. Draw a free body diagram of the ball ....

the moment the ball is dropped.



when the ball is halfway down.

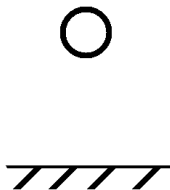


just before ball strikes the ground.

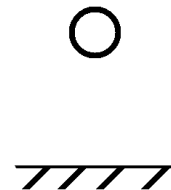


- o. A ball is **thrown** downwards from the top of a tall building. Draw a free body diagram of the ball the moment after the ball is thrown when ....

there is **no** force of air friction.

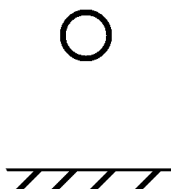


there **is** a force of air friction.

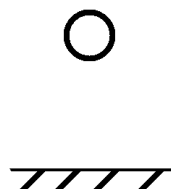


- p. A ball is released from rest from the top of a very tall building. There is air friction. Draw a free body diagram of the ball....

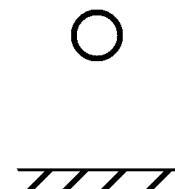
a few seconds before the ball reaches its terminal velocity.



the exact moment the ball reaches its terminal velocity.

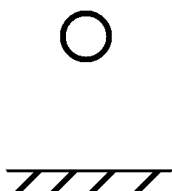


a few seconds after the ball reaches its terminal velocity.

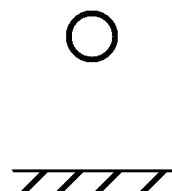


- q. A ball is thrown at an angle  $\theta = 45^\circ$  north of east from a horizontal surface. Draw a free body diagram of the ball the moment the ball is thrown when ....

there is **no** force of air friction.

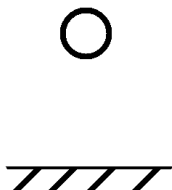


there **is** a force of air friction.

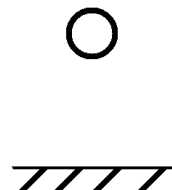


- r. A ball is thrown at an angle  $\theta = 45.0^\circ$  north of east from a horizontal surface. The ball is at its maximum vertical height.

There is **no** force of air friction.

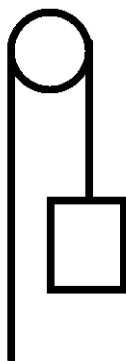


There **is** a force of air friction.

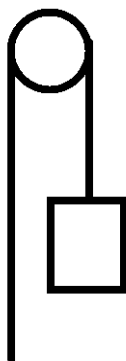


- s. A block is being pulled vertically upwards by a massless string pulley.

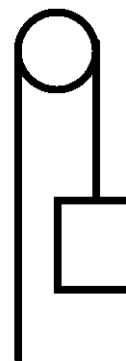
The speed of the block is constant.



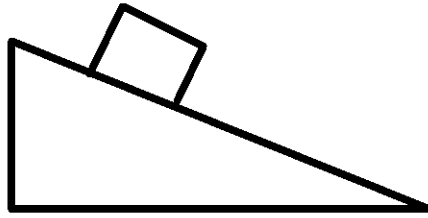
The block is accelerating.



The block is slowing down (decelerating).



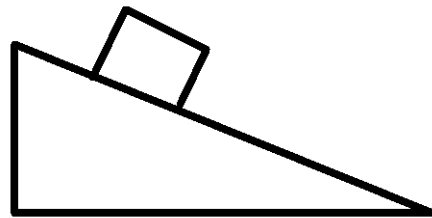
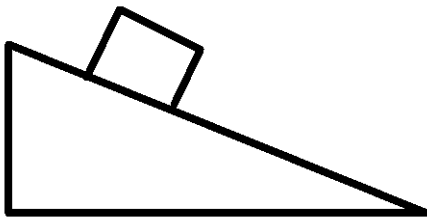
t. A block is at rest on an incline. There is surface friction.



u. A block moves down an incline. There is surface friction.

The speed of the block is constant.

The block accelerates.

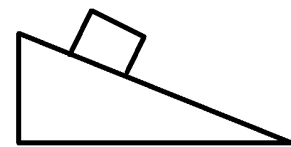
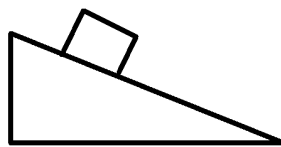
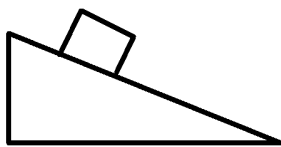


v. A block is pushed up an incline. There is surface friction.

The block slows down  
(decelerates).

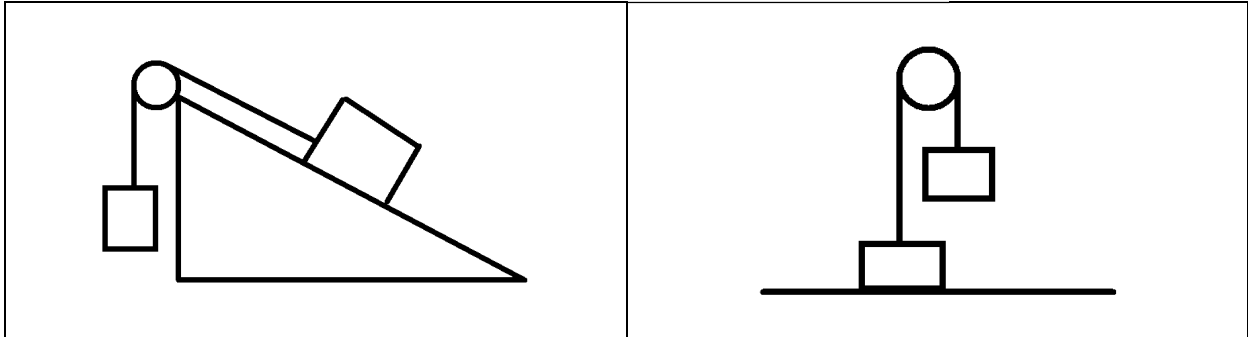
The speed of the block is  
constant.

The block speeds up  
(accelerates).

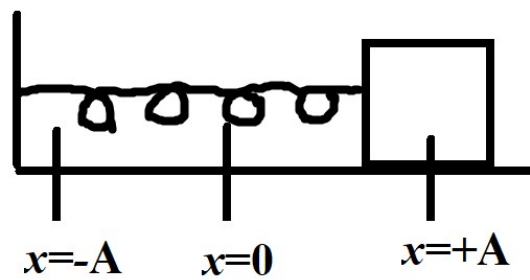




- w. Two blocks are attached to each other by a common string. There is surface friction.



- x. A mass lying on a rough horizontal surface is attached to a spring and is stretched from its equilibrium position. It is then released.



20.C: What is the meaning of *inertia*? What is *inertia* directly proportional to?

21.C: State the name of *Newton's first law of motion*. State the definition/meaning of *Newton's first law of motion*.

22.C: Why is it not safe to stand up when a bus, plane, or subway is moving?

23.C: State the name of *Newton's second law of motion*. Give the equation for *Newton's second law of motion*.

24.C: True or false:

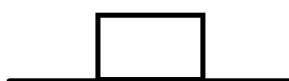
- a. According to Newton's second law of motion  $\sum \vec{F}$  and  $\vec{a}$  will always point in the same direction.
- b. According to Newton's second law of motion  $\sum \vec{F}$  and  $\vec{v}$  will always point in the same direction. In other words, there must be a net force in the same direction as the motion of the object.
- c. According to Newton's second law of motion  $\vec{v}$  and  $\vec{a}$  will always point in the same direction.

25.C: Give an example of an object when its net force (or acceleration) and velocity point in opposite directions.

26.C: What is the meaning of *static equilibrium*? What is the meaning of *translational/dynamic equilibrium*?

27.E: A 14.0 kg mass is at rest on a horizontal surface.

- a. Draw a free body diagram.



- b. What is the force of gravity acting on the object?
- c. What is the normal force acting on the object?

28.E: Ishmael pushes a 16.0 kg block to the left on a rough horizontal surface with a force of 70.0 N. The block does not move.

- a. Draw a free body diagram.

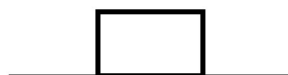


- b. What is the force of gravity acting on the object?
- c. What is the normal force acting on the object?
- d. What is the force of friction exerted on the block?

e. What is the coefficient of static friction?

29.E: Isaac pushes a 18.0 kg block to the left on a smooth horizontal surface with a force of 70.0 N.

a. Draw a free body diagram.



b. What is the force of gravity acting on the object?

c. What is the normal force acting on the object?

d. What is the horizontal acceleration of the block?

e. What is the vertical acceleration of the block?

30.E: Jacob pushes a 20.0 kg block to the left on a rough horizontal surface with a force of 70.0 N. The block moves at a constant speed of 2.00 m/s.

a. Draw a free body diagram.



b. What is the force of gravity acting on the object?

- c. What is the normal force acting on the object?
- d. What is the horizontal acceleration of the block?
- e. What is the vertical acceleration of the block?
- f. What is the force of friction exerted on the block?
- g. What is the coefficient of friction  $\mu$  between the block and the surface?

31.E: Adam pushes a block with a mass of 24.0 kg to the right on a rough horizontal surface with a coefficient of kinetic friction of 0.300. The block moves with a constant acceleration of  $2.00 \frac{\text{m}}{\text{s}^2}$ .

- a. Draw a free body diagram.



- b. What is the force of gravity acting on the object?
- c. What is the normal force acting on the object?
- d. What is the force of friction exerted on the block?
- e. What is the force of push given to the block?

32.E: Joseph is pulling a 65.0 kg block with a force of 800. N at an angle of  $45.0^\circ$  north of east above the horizontal of a rough horizontal surface. The coefficient of friction between the block and the surface is  $\mu = 0.300$ .

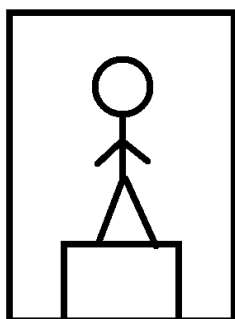
- a. Draw a free body diagram.



- b. What is the vertical acceleration of the block?
- c. What is the normal force acting on the block?
- d. What is the horizontal acceleration of the block?

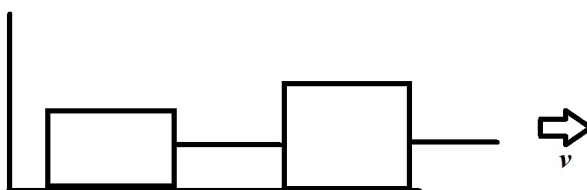
33.E: An 80.0 kg man is standing on a scale in an elevator. Determine the reading on the scale when

- a. the elevator is at rest.
- b. the elevator is moving up with a constant speed of  $2.00 \frac{\text{m}}{\text{s}}$ .
- c. the elevator is moving down with a constant speed of  $2.00 \frac{\text{m}}{\text{s}}$ .
- d. the elevator moves upwards with a constant acceleration of  $2.00 \frac{\text{m}}{\text{s}^2}$ .
- e. the elevator moves downwards with a constant acceleration of  $2.00 \frac{\text{m}}{\text{s}^2}$ .



34.E: A block with a mass  $m_2 = 20.0$  kg is on a rough horizontal surface with a coefficient of friction of  $\mu = 0.400$ . Attached to the right of  $m_2$  is a massless string which is pulling  $m_2$  to the right with a force of tension  $F_T$ . Attached to the right of the massless string is another block of mass  $m_1 = 30.0$  kg. Attached to the right of  $m_1$  is another massless string which pulls the whole system with a constant pulling force  $F_{\text{pull}} = 800$ . N and constant acceleration  $a$ .

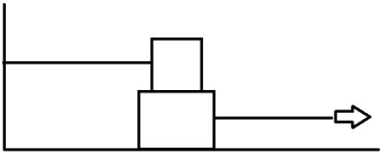
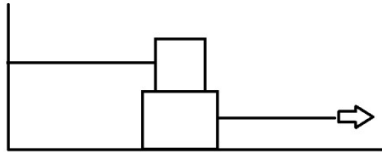
- a. Draw a free body diagram.



- b. Find the acceleration of the whole system  $a$ .  
c. Find the force of tension  $F_T$  of the massless string which attaches both masses.

35.E: A block with a mass  $m_2 = 15.0$  kg is on a rough horizontal surface. There is a string pulling it to the right with a force  $F_P$  at a constant speed. Above  $m_2$  there is a block with a mass  $m_1 = 12.0$  kg. There is a string attached to the left of  $m_1$  which is attached to a wall which has a force of tension  $F_T$ . The coefficient of friction between  $m_1$  and  $m_2$  is  $\mu_{1,2} = 0.250$  and the coefficient of friction between  $m_2$  and the surface is  $\mu_{2,\text{surface}} = 0.350$ .

a. Draw a free body diagram.

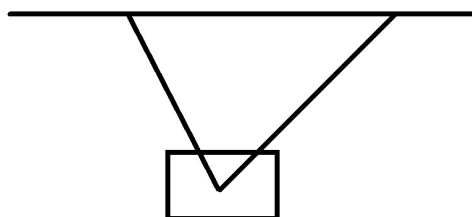
For $m_1$ :	For $m_2$ :
	

- b. Determine the force of tension  $F_T$ .  
 c. Determine the pulling force  $F_P$ .



36.E: A 12.0 kg block is held in the air by two strings attached to the wall. The first string makes an angle of  $\theta_1 = 60.0^\circ$  north of west. The second string makes an angle of  $\theta_2 = 45.0^\circ$  north of east.

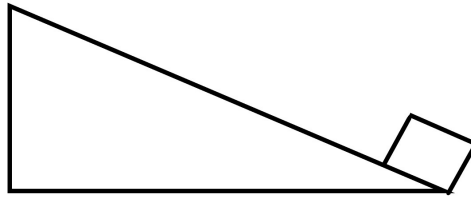
a. Draw a free body diagram.



b. Find the force of tension on each string.

37.E: A block with mass 15.0 kg is at rest on the bottom of an incline with  $\theta = 25.0^\circ$  which is 35.0 m long. The coefficient of friction between the block and the surface is  $\mu = 0.450$ . A man pushes the block up parallel to the incline with a force of 155 N.

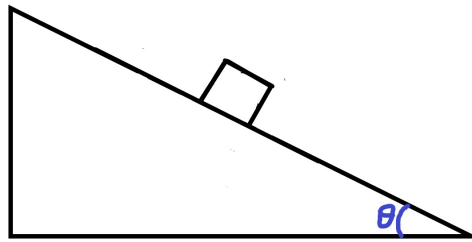
a. Draw a free body diagram.



- b. What is the acceleration of the block?
- c. What will be the final speed of the block when it reaches the top of the incline?
- d. How long will it take for the block to reach the top of the incline?

38.E: A block with mass 65.0 kg is initially at rest in the middle of an incline with  $\theta = 25.0^\circ$  which is 40.0 m long. The coefficient of friction between the block and the surface is  $\mu = 0.450$ . A man pushes the block down parallel to the incline with a force of 60.0 N. The block accelerates downwards at a constant rate. Let the acceleration from gravity be  $\vec{g} = 9.81 \frac{\text{m}}{\text{s}^2}$ .

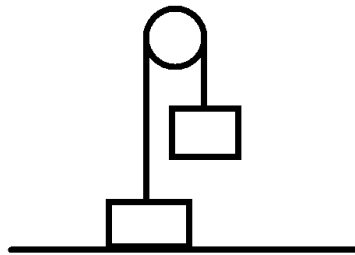
a. Draw a free body diagram.



- What is the magnitude of the acceleration of the block?
- What will be the final speed of the block when it reaches the bottom of the incline?
- How long will it take for the block to reach the bottom of the incline?

39.E: A massless frictionless pulley is attached to a ceiling. Mass  $m_1 = 16.0$  kg is at rest on the ground. It is attached to a massless string which goes over the massless frictionless pulley and is attached to another mass  $m_2 = 46.0$  kg which is also initially at rest in the air.  $m_2$  is released from rest and both masses accelerate at a constant rate.

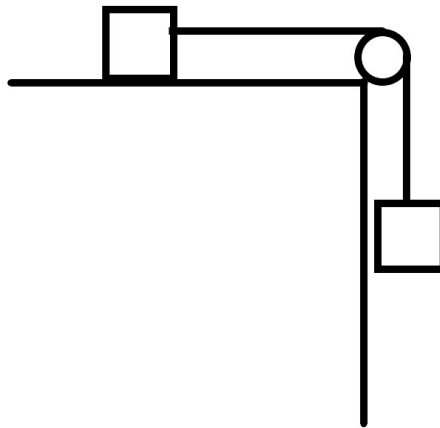
a. Draw a free body diagram.



- b. Find the common acceleration of the system.  
c. Find the force of tension  $F_{\text{tension}}$  of the massless string.

40.E: A block of mass  $m_1 = 12.0$  kg sits at rest on a horizontal surface with  $\mu = 0.240$ . Mass  $m_1$  is attached to a massless string which goes over a massless pulley which is attached to another block of mass  $m_2 = 36.0$  kg.

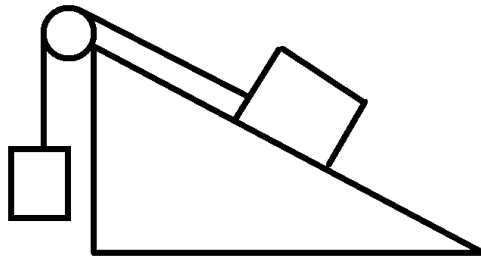
a. Draw a free body diagram.



- b. What is the common acceleration of the blocks?  
c. What is the force of tension on the string?

41.E: A 4.00 kg mass  $m_1$  is initially at rest on a  $\theta = 30.0^\circ$  incline. The surface has a coefficient of friction  $\mu = 0.400$ . The 4.00 kg mass has a massless string attached to it which goes over the top of the incline above a frictionless pulley to another mass  $m_2$  of 18.0 kg which is hanging in the air. Both objects are released from rest and move with a constant acceleration.  $m_2$  moves down while  $m_1$  moves up the incline.

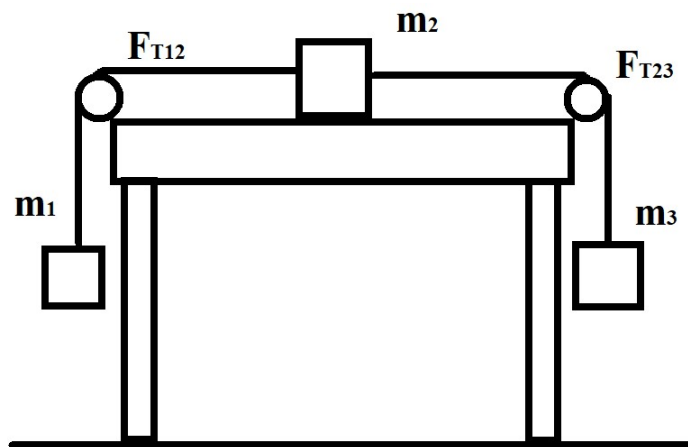
- a. Draw a free body diagram.



- b. What will be the common acceleration of each object?  
c. What will be the force of tension on the string?

42.E: A block with mass  $m_2 = 8.00$  kg is held at rest on a rough horizontal table which has a coefficient of friction of  $\mu = 0.200$ . It is attached by a string to a mass  $m_3 = 14.0$  kg which hangs to the right of the table and another string to a mass  $m_1 = 2.00$  kg which hangs to the left of table as shown below. Mass  $m_2$  is released from rest and the whole system accelerates with a constant rate.

- a. Draw a free body diagram.



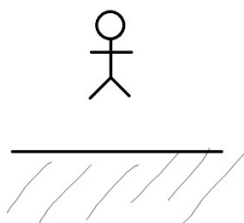
- b. Determine the acceleration of the system.  
c. Determine the force of tension of string  $F_{T12}$  and the force of tension of string  $F_{T23}$ .

43.C: State the name of *Newton's third law of motion*. State the equation for *Newton's third law of motion*.

44.C: Give three examples of *Newton's third law of motion* (For each example you need two sentences: one for the action and one for the reaction.). Three examples have been given to you:

- a. Man pushes wall forward. Wall pushes man backwards.
- b. Fish pushes water backwards. Water pushes fish forwards.
- c. Earth pulls man down. Man pulls Earth up.
- d.
- e.
- f.

45.E: An 80.0 kg man on Earth jumps vertically upwards. The acceleration due to gravity near the surface of the Earth is approximately  $9.81 \frac{\text{m}}{\text{s}^2}$ . The mass of the Earth is approximately  $5.97 \times 10^{24}$  kg. Use Newton's third law of motion to determine the acceleration of the Earth after the man jumps.



46.C: **Use a pencil and ruler!** Define *free fall*. Draw a *displacement vs. time* graph, a *distance vs. time* graph, a *velocity vs. time* graph, a *speed vs. time* graph, and an *acceleration vs. time* graph for an object dropped from rest in free fall.



47.C: **Use a pencil and ruler!** Define *terminal velocity*. What is the relationship between speed and the force of friction? Draw a *distance vs. time* graph, a *speed vs. time* graph, and an *acceleration vs. time* graph of an object being dropped from rest from a very high height above the surface of the Earth with both the force of friction and the force of gravity acting on it.

48.C: Use Newton's third law of motion to complete the next sentence: Man throws rock forward.

49.C: What is the meaning and equation for *impulse*  $\vec{J}$ ? Do not confuse impulse  $\vec{J}$  with current  $I$ !

50.C: What is the meaning, symbol, equation, and fundamental units for *momentum*  $\vec{p}$ ? Momentum is also called "*inertia in motion*." Why? Do not confuse momentum  $\vec{p}$  with pressure  $P$  or power  $P$  or density  $\rho$ !

51.C: Why are the front of cars built so weak? Why are cars so easy to damage during an accident?  
<https://www.youtube.com/watch?v=v9ML4GA47Rg>

52.C: Why do athletes have their elbows bent when catching a ball? Why do athletes have their knees bent when coming down after jumping?

53.C: What common mistake do people make when firing/shooting a gun?  
<https://www.youtube.com/watch?v=bYWzMDVgweg>

54.C: What does the law of *conservation of momentum* tell us? What is the equation for the law of conservation of momentum?

55.C: What is an *elastic collision*? Is momentum conserved? Is kinetic energy conserved? Is total energy conserved?

56.C: What is an *inelastic collision*? Is momentum conserved? Is kinetic energy conserved? Is total energy conserved?

57.C: What is a *perfectly inelastic collision*? Is momentum conserved? Is kinetic energy conserved? Is total energy conserved?

58.C: What does the area under a *force vs. time graph* tell us?

59.C: What does the slope of a line on a *force vs. time graph* tell us?

60.E: A 2.00 kg block is moving east with a speed of 5.00 m/s. It hits a wall and rebounds to the west at a speed of 4.00 m/s. What is the magnitude and direction of the change in momentum of the block?

61.E: A 2.00 kg block is moving east on a frictionless surface with a speed of 5.00 m/s. It then moves on a rough surface for three seconds. Finally it continues to move east on a frictionless surface with a new speed of 1.00 m/s. What is the force of friction of the rough surface?

62.E: A 3.00 kg block is moving west at 4.00 m/s on a frictionless horizontal surface. A 5.00 kg block is moving east at 6.00 m/s on the same surface. Both of them collide and stick together.

a. What is the final speed and direction of the block?

b. Is momentum conserved?

c. What is the original total kinetic energy?

d. What is the final total kinetic energy?

e. Is kinetic energy conserved?

f. Is this an elastic or inelastic collision?

g. Is total energy conserved?

63.E: A 7.00 kg block is moving north at 8 m/s on a frictionless horizontal surface. A 9.00 kg block is moving south at 10.0 m/s on the same surface. They collide. The 7.00 kg block is now moving south at 4.00 m/s.

a. What is the final speed and direction of the 9 kg block?

b. Is momentum conserved?

c. What is the original total kinetic energy?

d. What is the final total kinetic energy?

e. Is kinetic energy conserved?

f. Is this an elastic or inelastic collision?

g. Is total energy conserved?

64.E: A 12.0 kg block is initially at rest on a frictionless horizontal surface. It then explodes into three pieces. A 3.00 kg block moves west at 4.00 m/s. A 5.00 kg block moves east at 6.00 m/s.

a. What is the final speed and direction of the 4.00 kg block?

b. Is momentum conserved?

c. What is the original total kinetic energy?

d. What is the final total kinetic energy?

e. Is kinetic energy conserved?

f. Is total energy conserved?

65.E: A 12.0 kg block is moving east at 13.0 m/s on a frictionless horizontal surface. It then explodes into three pieces. A 4.00 kg block moves west at 5.00 m/s. A 6.00 kg block moves east at 7.00 m/s.

a. What is the final speed and direction of the 2.00 kg block?

b. Is momentum conserved?

- c. What is the original total kinetic energy?
- d. What is the final total kinetic energy?
- e. Is kinetic energy conserved?
- f. Is total energy conserved?

66.E: A 4.00 kg block is moving east at 5.00 m/s on a frictionless horizontal surface. It collides with a 6.00 kg block initially at rest. The 4.00 kg block then moves northeast at 3.00 m/s at an angle of  $30.0^\circ$  above the horizontal.

- a. **Use a pencil!** Draw an initial and final figure.

Initial	Final

- b. Use the law of conservation of momentum for each axis to determine the final speed (in m/s) and direction (in degrees) of the 6.00 kg block.

- c. Is momentum conserved?
- d. What is the original total kinetic energy?
- e. What is the final total kinetic energy?
- f. Is kinetic energy conserved?
- g. Is this an elastic or inelastic collision?
- h. Is total energy conserved?

67.C: Define *centripetal*.

68.C: Define *centrifugal*.

69.C: Are there *centripetal forces*?

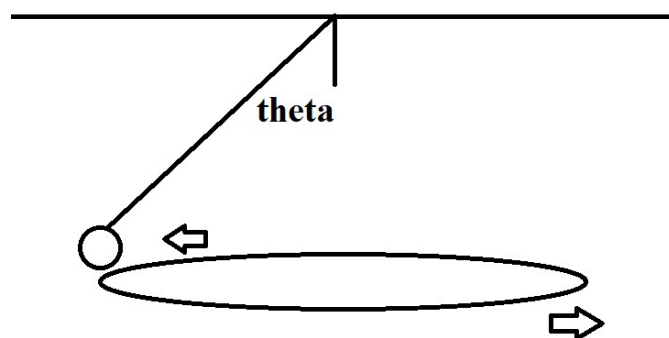
70.C: Are there *centrifugal forces*?

71.C: Imagine driving in a straight line with a constant speed of 60 km/h. You then quickly make a right turn. Do you feel a force? In which direction? Is it a centripetal force or a centrifugal force? Is it a real force? Why?

72.C: In circular motion how much work does the centripetal force do? Use the equation  $W = Fs \cos \theta$ .

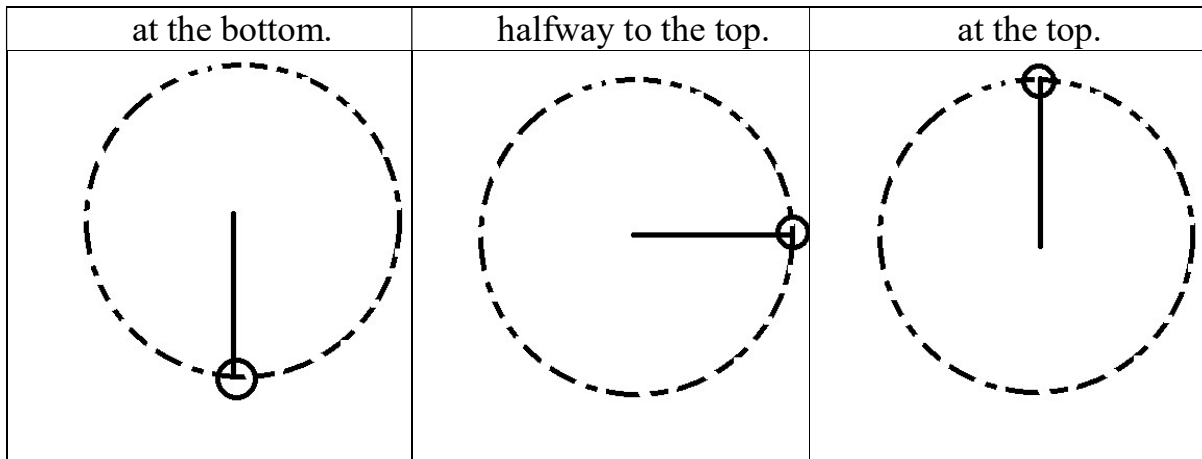
73.C: Label the forces on the following diagrams.

- a. An object is attached to a string. The object moves in a horizontal circle at an angle  $\theta$  from the vertical.

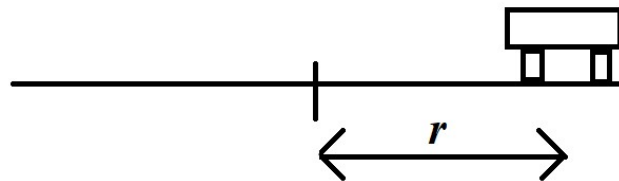




- b. An object is attached to a string. The object moves in a vertical loop. Draw a free body diagram when the object is



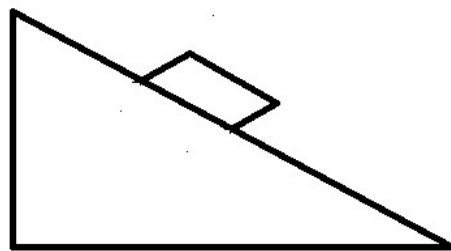
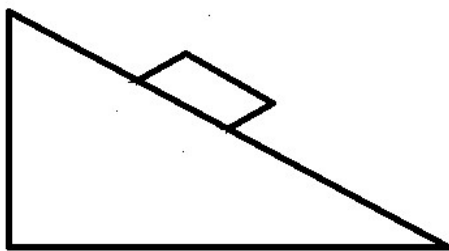
- c. A car moves in a horizontal circle at a constant speed with a radius  $r$ .



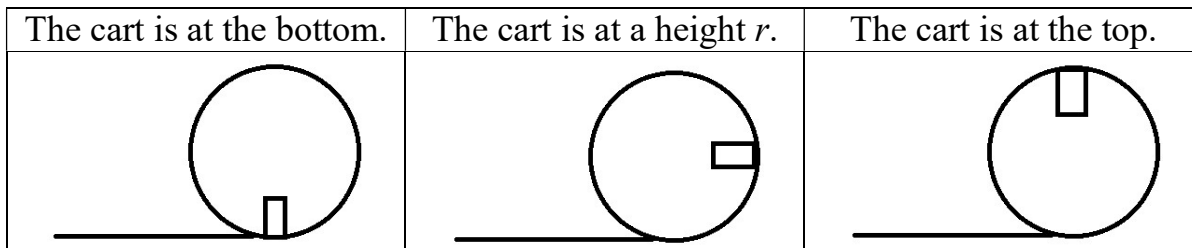
- d. A car moves in a circle on a banked road (cone) with a constant radius  $r$ . There is force of friction.

a. The car is moving slow.

b. The car is moving fast.



- e. A cart is moving up on a vertically circular roller coaster with a radius  $r$ . There is no force of friction.

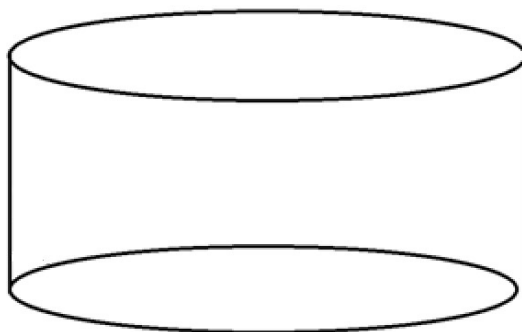


- f. A fast motorcycle moves around a nonmoving cylindrical wall.

**"Mauth Ka Kua" (The Well Of Death): Basic physics at its best!**

Swastik Ghosh

<https://www.youtube.com/watch?v=cFLNknvi7QE>

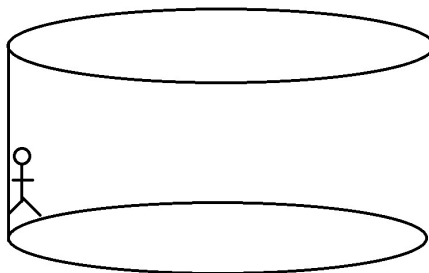


- g. A man is on the edge of a moving cylindrical wall.

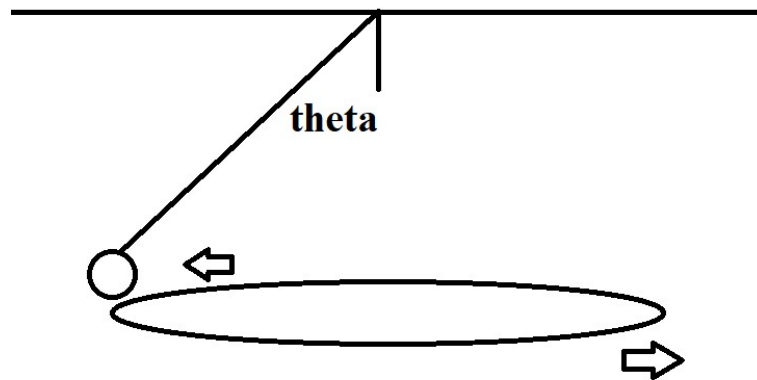
**CENTRIFUGEUSE - ROTOR @ FOIRE DU TRONE (GoPro)**

josselinz86

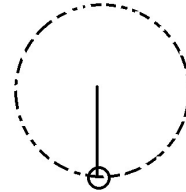
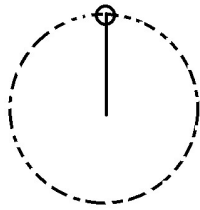
<https://www.youtube.com/watch?v=GspwbZSjABA>



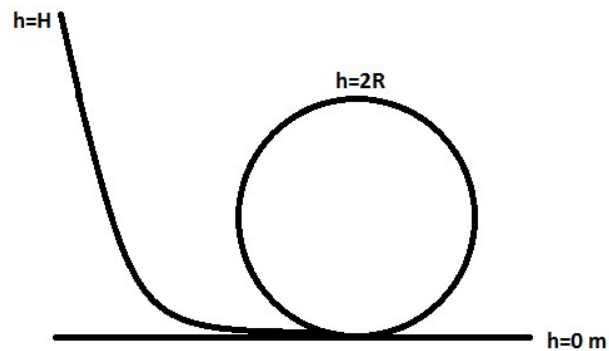
74.E: Draw a free body diagram and use Newton's second law of motion to obtain an equation for the force of tension and then the speed of a mass on a string in horizontal circular motion which makes an angle  $\theta$  from the vertical. Your answer should be in terms of the mass of the object  $m$ , the length of the string  $l$ , the angle from the vertical  $\theta$ , and the acceleration from gravity  $g$ .



75.E: A point mass is attached to a massless string with length  $r$ . The mass and string are moving in vertical circular motion with a constant speed  $v$ . Draw free body diagrams and use Newton's second law of motion to obtain equations of the force of tension at the top and bottom of the string. Where is the force of tension greater? Your answers should be in terms of the mass of the object  $m$ , the radius of the string  $r$ , the speed of the point mass  $v$ , and the acceleration from gravity  $g$ .



76.E: An object is released from rest from a height  $H$ . First use the law of conservation of energy to obtain an equation for the speed of the object when it has reached the top of the loop of the roller coaster. Then use Newton's second law of motion to obtain an equation for the normal force on the object when it has reached the top of the loop of the roller coaster. Your answer for the normal force should be in terms of the mass of the object  $m$ , the initial height of the object  $H$ , the radius of the loop  $r$ , and the acceleration from gravity  $g$ .

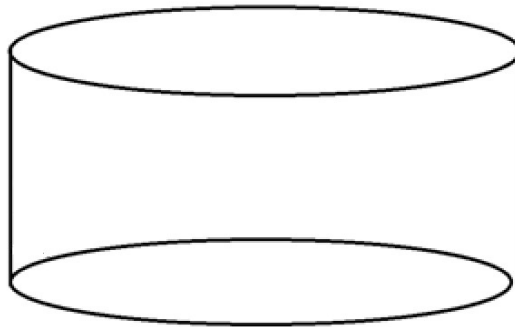


77.E: Draw a free body diagram and use Newton's second law of motion to obtain an equation for the speed of an object in the amusement park ride "The Well of Death." Your answer should be in terms of the radius of the cylinder/well  $R$ , the coefficient of friction  $\mu$ , and the acceleration from gravity  $g$ .

**"Mauth Ka Kua" (The Well Of Death): Basic physics at its best!**

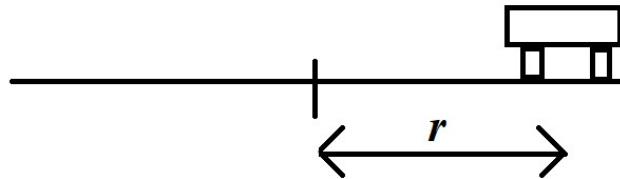
Swastik Ghosh

<https://www.youtube.com/watch?v=cFLNknvi7QE>

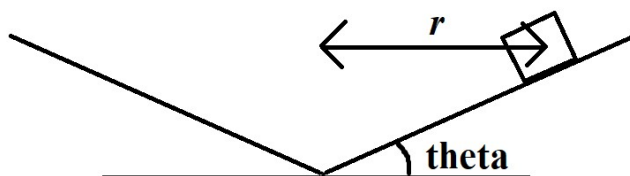


## 78.E: Cars

- a. Use Newton's second law of motion to find an equation for the speed of a car moving in circular motion on a horizontal road with surface friction. Your answer should be in terms of the radius of the track  $r$ , the coefficient of friction  $\mu$ , and the acceleration from gravity  $g$ .

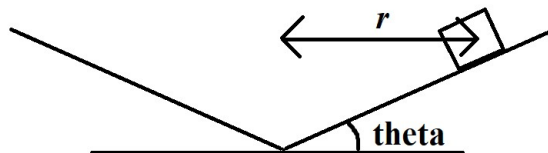


- b. Use Newton's second law of motion to find an equation for the speed of a car moving at an angle  $\theta$  to the horizontal in circular motion on a banked/angled road with no friction. Your answer should be in terms of the radius of the track  $r$ , the angle of the banked road  $\theta$ , and the acceleration from gravity  $g$ .

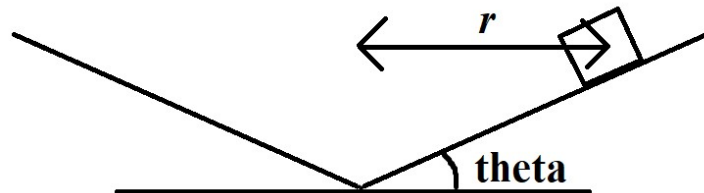




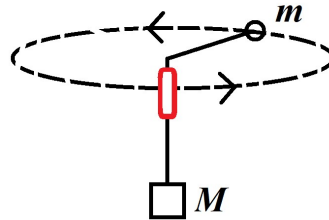
- c. Use Newton's second law of motion to find an equation for the speed of a **slow moving car** moving at an angle  $\theta$  to the horizontal in circular motion on a banked/angled road with surface friction. Your answer should be in terms of the radius of the track  $r$ , the angle of the banked road  $\theta$ , the coefficient of friction  $\mu$ , and the acceleration from gravity  $g$ .



- d. Use Newton's second law of motion to find an equation for the speed of a **fast moving car** moving at an angle  $\theta$  to the horizontal in circular motion on a banked/angled road with surface friction. Your answer should be in terms of the radius of the track  $r$ , the angle of the banked road  $\theta$ , the coefficient of friction  $\mu$ , and the acceleration from gravity  $g$ .



79. You are holding a red hollow cylinder and spinning a rubber stopper over your head. The rubber stopper has a mass of 18.5 g and is moving in a circle which has a radius of 110 cm. You measure that the rubber stopper moves 10 times around your head every 9.00 s.

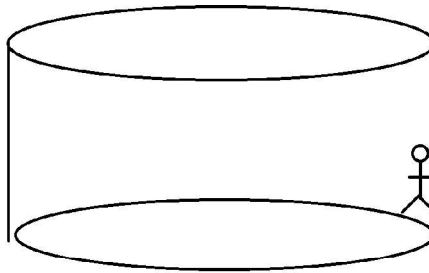


- What is the distance once around this circular path?
- What is the distance ten times around this circular path?
- What is the average speed of this rubber stopper as it circles above your head?
- What is the direction of the centripetal acceleration of the stopper as it circles above your head?
- What is the magnitude of the centripetal acceleration of the stopper as it circles over your head?
- What is the direction of the velocity of this stopper when in the position shown in the diagram?
- How much force would be required to keep this stopper moving in the given circular path?

- h. How much mass  $M$  must be hung on the lower end of the string to keep the stopper moving in the given circular path?

80. There is an amusement ride called the “ROTOR” where you enter a cylindrical room. The room begins to spin very fast until at some point the floor beneath you “falls out.” Suppose this that room has a radius of 4.20 m and that the room rotates such that you make one complete revolution in 3.65 s.

- a. What will be your linear speed as the room spins at this speed?
- b. What is the magnitude of your centripetal acceleration? How many “g’s” is this?
- c. Label all of the forces acting on the rider.



- d. What will be the magnitude of the centripetal force acting on a 50.0 kg person on this ride?
- e. What will be the magnitude of the normal force acting on this person?
- f. What will be the minimum frictional force acting on this person?
- g. What is the minimum coefficient of friction between the rider and the wall?

81. Write down the common terms and equations for circular motion.

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.3 Work, Energy, and Power

### Understandings

- The principle of the conservation of energy.
- Work done by a force is equivalent to a transfer of energy.
- Energy transfers can be represented on a Sankey diagram.
- Work  $W$  done on a body by a constant force depends on the component of the force along the line of displacement as given by  $W = Fs \cos \theta$ .
- Work done by the resultant force on a system is equal to the change in the energy of the system.
- Mechanical energy is the sum of kinetic energy, gravitational potential energy, and elastic potential energy.
- In the absence of frictional, resistive forces, the total mechanical energy of a system is conserved.
- If mechanical energy is conserved, work is the amount of energy transformed between different forms of mechanical energy in a system such as
  - the kinetic energy of translational motion as given by  $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
  - the gravitational potential energy, when close to the surface of the Earth as given by  $\Delta E_p = mg\Delta h$
  - the elastic potential energy as given by  $E_H = \frac{1}{2}k\Delta x^2$
- Power developed  $P$  is the rate of work done, or the rate of energy transfer, as given by  $P = \frac{\Delta W}{\Delta t} = Fv$
- Efficiency  $\eta$  in terms of energy transfer or power as given by  $\eta = \frac{E_{\text{output}}}{E_{\text{input}}} = \frac{P_{\text{output}}}{P_{\text{input}}}$
- Energy density of the fuel sources.

**Equations**

$$W = Fs \cos \theta$$

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta E_p = mg\Delta h$$

$$E_H = \frac{1}{2}k\Delta x^2$$

$$P = \frac{\Delta W}{\Delta t} = Fv$$

$$\eta = \frac{E_{\text{output}}}{E_{\text{input}}} = \frac{P_{\text{output}}}{P_{\text{input}}}$$

**Equation not given in IB Physics Data Booklet:**

$$W = \Delta E_k = E_{k,f} - E_{k,i}$$

The solutions can be found on the YouTube channels Go Physics Go

<https://www.youtube.com/@gophysicsgo/playlists>

**Part 1: Answer the following questions**

1. C: What is the meaning of *work*? Equation? Units? Is it a scalar or vector?
  
2. C: What is the meaning of *energy*? Units? Is it a scalar or vector?
  
3. C: What is the meaning of *kinetic energy*? Equation? Define each variable.
  
4. C: What will happen to the kinetic energy of a moving object if its
  - a. Mass halves and speed halves?
  
  - b. Mass doubles and speed doubles?
  
  - c. Mass decreases by three (one third) and speed increases by four (quadruples)?
  
5. C: What is the meaning of *potential energy*?



6. C: What is the meaning of *gravitational potential energy*? What is the equation for *gravitational potential energy* of an object near the surface of a planet? Define each variable. What is the general equation for the *gravitational potential energy* between two objects? Define each variable.
7. C: What is the equation for the *elastic potential energy* of a compressed or stretched spring? Define each variable. What is the meaning and units for the *spring constant k*?
8. C: What is the equation for the *total mechanical energy* of an object?
9. C: True or false: Work is done on an object if the object moves.
- 10.C: What is the work done on a 3.00 kg rock if it travels 60.0 m with a constant speed of 4.00 m/s in outer space?
- 11.C: What does the slope of a *force vs. displacement* graph tell us?
- 12.C: What does the area under a *force vs. displacement* graph tell us?

13.C: What is the meaning of *power*  $P$ ? Equation? Units? Is it a scalar or vector? Do not confuse power  $P$  with pressure  $P$  or momentum  $\vec{p}$  or density  $\rho$ !

14.C: What is the meaning of and the equation for the *law of conservation of energy*?

15.C: State the equation for the work-energy theorem.

16.C: What are some characteristics of a *Sankey diagram*? Sketch a simple *Sankey diagram*.

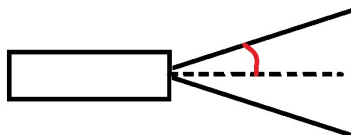
<http://sankeymatic.com/>

17.C: What is *efficiency*? Equation? Units? Is it a scalar or vector?

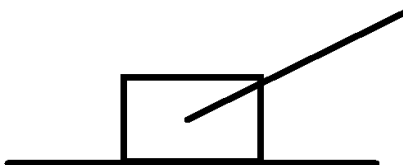
**Part 2: Answer the following free response questions**

1. E: A force of 120. N is applied to the front of a sled at an angle of  $28.0^\circ$  above the horizontal so as to pull the sled a distance of 165 m. How much work was done by the applied force?
2. E: How much work would be required to lift a 12.0 kg mass up onto a table 1.15 m high?

3. E: A barge is being pulled along a canal by two cables being pulled as shown below. The tension in each cable is  $1.40 \times 10^4$  N and each cable is being pulled at an angle of  $18.0^\circ$  relative to the direction of motion. How much work will be done in pulling this barge a distance of 3.00 km?

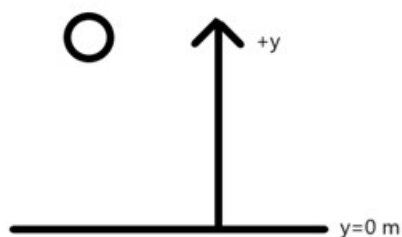


4. E: Job pulls a massless rope at an angle of  $40.0^\circ$  from the horizontal which is attached to a block of mass  $m = 60.0$  kg on a rough horizontal surface with a coefficient of friction of  $\mu = 0.200$  with a constant speed of 2.00 m/s for 300. m.
- a. Draw a free body diagram.



- b. Use Newton's second law of motion to find  $F_{\text{pull}}$ .

- c. How much work was done by Job?
- d. What is the average power performed by Job?
5. E: A 10.0 kg object initially at rest is 12.0 m above the surface of the Earth. It is released. There is no air friction.
- a. Draw a figure.

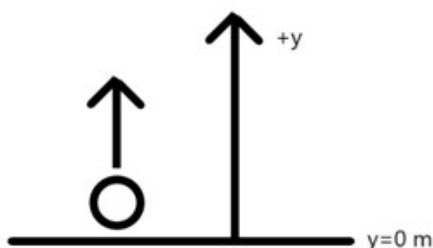


- b. What is the initial kinetic energy of the object?
- c. What is the initial gravitational potential energy of the object?
- d. What is the initial total energy of the object?
- e. What is the kinetic energy of the object when it is halfway to the surface?

- f. What is the gravitational potential energy of the object when it is halfway to the surface?
- g. What is the total energy of the object when it is halfway to the surface?
- h. What is the final gravitational potential energy of the object just before it reaches the surface?
- i. What is the final kinetic energy of the object just before it reaches the surface?
- j. What is the total energy of the object just before it reaches the surface?
- k. What is the final speed of the object just before it reaches the ground?  
From part d the total energy of the object is 1,177.2 J.
- l. Draw a *gravitational potential energy vs. height* graph, a *kinetic energy vs. height* graph, and a *total energy vs. height* graph.

6. E: Jethro throws a 5.00 kg object from the surface of the Earth vertically upwards with an initial speed of 8.00 m/s. There is no air friction.

a. Draw a figure.



b. What is the initial gravitational potential energy of the object?

c. What is the initial kinetic energy of the object?

d. What is the initial total energy of the object?

e. What is the maximum height the object will travel?

f. How long will it take for the object to reach its maximum height?

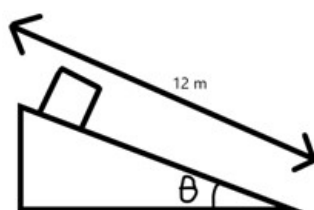
g. What is the gravitational potential energy of the object when it is halfway to its maximum height?

h. What is the kinetic energy of the object when it is halfway to its maximum height?

- i. What is the total energy of the object when it is halfway to its maximum height?
  - j. What is the gravitational potential energy of the object when it reaches its maximum height?
  - k. What is the kinetic energy of the object when it reaches its maximum height?
  - l. What is the total energy of the object when it reaches its maximum height?
  - m. What is the speed of the object when it reaches its maximum height?
  - n. Draw a *gravitational potential energy vs. height* graph, a *kinetic energy vs. height* graph, and a *total energy vs. height* graph.
7. E: A 8.00 kg object is falling vertically freely with a speed of 40.0 m/s at an elevation of  $h_1$ . What will be the speed of the object after it has fallen a distance of 70.0 m? Round your answer to two decimal places.

8. E: A 8.00 kg object is falling down with a speed of 40.0 m/s at an elevation of 300. m. After the object has fallen a distance of 90.0 m its speed is now 45.0 m/s.
- a. What is the magnitude of energy lost from air friction? Round your answer to two decimal places.
- b. What is the magnitude of the force of air friction during this 90.0 m? Round your answer to two decimal places.

9. E: A 7.00 kg object is placed on top of a 12.0 m long smooth incline which is  $30.0^\circ$  above the horizontal. It is released and slides down.
- a. Draw a figure.



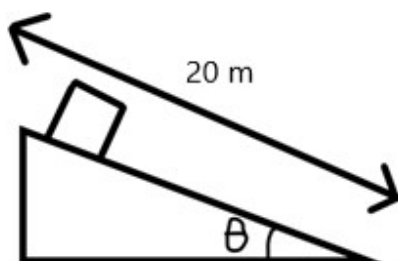
- b. What is the initial height of the object?
- c. What is the initial gravitational potential energy of the object?



- d. What is the initial kinetic energy of the object?
- e. What is the initial total energy of the object?
- f. What is the final speed of the object when it reaches the bottom of the incline?
- g. What is the final kinetic energy of the object?
- h. What is the final gravitational potential energy of the object?
- i. What is the acceleration of the object?
- j. How long does it take for the object to reach the bottom of the incline?

10.E: A 4.00 kg block is placed on top of a 20.0 m long rough incline which is  $30.0^\circ$  above the horizontal. The coefficient of friction between the block and the incline is  $\mu = 0.300$ . The block is released and slides down.

- a. Draw a figure.



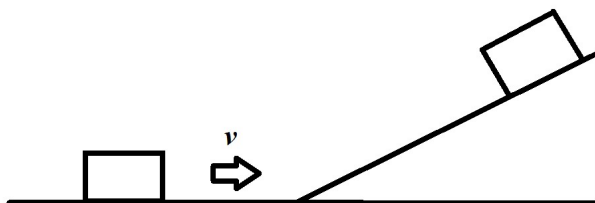
- b. What is the initial height of the object?



- j. What is the final kinetic energy of the object?
- k. What is the final gravitational potential energy of the object?
- l. What is the final total energy of the object?
- m. How long will it take for the object to reach the bottom of the incline?
- n. How much energy was lost by the block?

11.E: A 6.00 kg block is moving to the right with a constant speed of 22.0 m/s on a horizontal frictionless surface. The block then goes up a  $30.0^\circ$  incline which has a coefficient of friction of 0.800.

- a. Draw a figure.



- b. How many meters up the incline and how high does the block move?

12.E: A force of 35.0 N is applied to a spring and as a result the spring stretches a distance of 12.0 cm.

- a. What is the spring constant  $k$  for this spring?
- b. How much energy will be stored in this spring?

13.E: A spring, which has a spring constant  $k$ , is hung from the ceiling. A mass of 3.00 kg is added to the end of the spring and is then slowly lowered until equilibrium is reached. At this point the bottom of the mass has been lowered a distance of 52.0 cm.

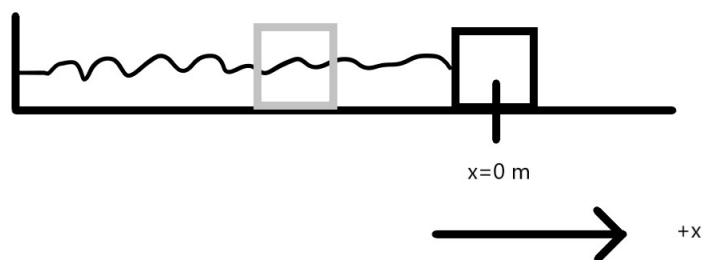
- a. What is the magnitude of the force being exerted by the spring when the system reaches equilibrium?
- b. What is the spring constant of this spring?
- c. How much energy is stored in the spring when equilibrium is reached?

14.E: A mass of 5.00 kg is dropped from a height of 2.20 m above a vertical spring sitting on a horizontal surface. Upon colliding with the spring the mass compresses the spring 30.0 cm before it momentarily comes to a halt. Assume  $h = 0.00$  m at the lowest point.

- a. How much gravitational potential energy was contained in the 5.0 kg mass before it was dropped?
- b. How much energy will be stored in the spring when the mass comes briefly to a halt?
- c. What is the spring constant of this spring?

15.E: A horizontal spring with a spring constant  $k = 3.00 \times 10^4 \frac{\text{N}}{\text{m}}$  is compressed 6.00 cm by an 800. gram block which is resting on a frictionless surface. The block is then released from rest.

a. Draw a figure.



b. What is the initial potential energy of the spring?

c. What is the kinetic energy of the block after it leaves the spring?

d. What is the final speed of the block after it leaves the spring?

e. After some distance the block moves through a rough surface with a coefficient of friction  $\mu = 0.0500$ . What is the total distance the block travels along the rough surface?

16.E: A mass of 2.20 kg is placed on a stiff vertical spring, which has a spring constant of 950. N/m. The object is then pressed against the spring until it has been compressed a distance of 77.0 cm. The mass is then released and is allowed to be thrown up into the air.

- a. What will be the elastic potential energy stored in the spring just before the mass is released?
- b. What will be the gravitational energy of this mass when it reaches the highest point?
- c. How high in the air will the mass be thrown?
- d. What will be the velocity of the mass just as it leaves the end of the spring?

17.E: A vertical spring is hung from one end. A mass of 5.00 kg is hung from the end of the spring. As a result of the addition of this mass the spring is stretched a distance of 125 cm.

- a. What is the spring constant for this spring?
- b. How much would this spring be stretched if the mass is 15.0 kg?

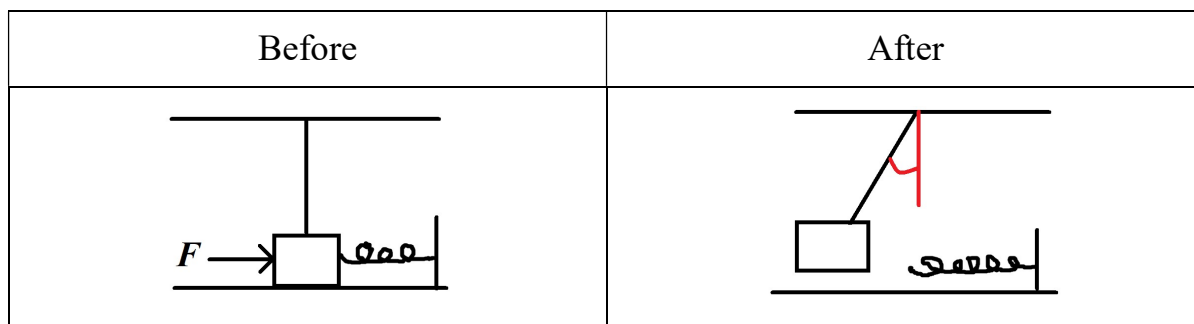
**The 5.00 kg mass is then lifted up until the spring is unstretched. The mass is then released and is allowed to fall until at some lower point it stops. Assume that at this point  $h = 0$  m.**

- c. How far will the mass have fallen when it stops at the lowest point?

- d. What will be the gravitational potential energy stored in this system when the mass is at the lowest point?
- e. What will be the kinetic energy of this system when the mass reaches the lowest point?
- f. What will be the elastic potential energy stored in the spring when the mass is at the lowest point?
- g. What will be the elastic potential energy stored in the system when the mass is at the highest point?
- h. What will be the kinetic energy of this system when the mass is at the highest point?
- i. What will be the gravitational potential energy of this system when the mass is at the highest point?
- j. What will be the total energy of this system at the highest point?
- k. What will be the total energy of this system at the lowest point?
- l. What will be the total energy of this system when the mass is 65.0 cm below the highest point?
- m. What will be the gravitational energy of this system when the mass is 65.0 cm below the highest point?

- n. What will be the elastic potential energy of this system when the mass is 65.0 cm below the highest point?
- o. What will be the kinetic energy of this system when the mass is 65.0 cm below the highest point?
- p. What will be the velocity of the mass when it is 65.0 cm below the highest point?

18.E: A mass of 4.40 kg is attached at the end of a string, which is 2.60 m long, is pressed against a horizontal spring with  $k = 5.60 \times 10^2 \frac{\text{N}}{\text{m}}$  as shown below. The other end of the string is attached to the ceiling. The spring is compressed by 12.0 cm by the applied force. The mass is then released and is allowed to swing outward until at some point it stops.

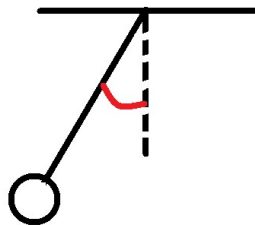


- a. What will be the total energy of this system just before the mass is released?
- b. How much force is needed to press this mass against the spring?
- c. What will be the velocity of the mass just as it leaves the spring?



- d. What will be the total energy of the mass when it reaches the highest point?
- e. How high, in cm, will the mass be when it stops at the highest point?
- f. What will be the angle  $\alpha$  between the string and the vertical line as shown in the diagram when the mass reaches the highest point?

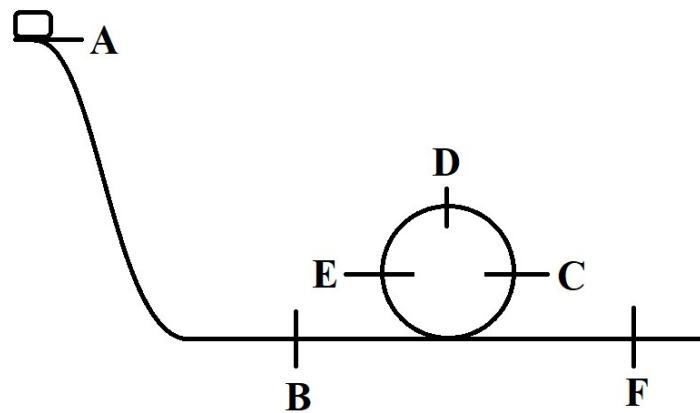
19.E: A child, which has a mass of 23.0 kg, is sitting on a swing. The ropes of the swing are 5.20 m long and the child is pulled back until the angle between the ropes of the swing and the vertical is  $35.0^\circ$  as shown below. The child is released and is allowed to swing back and forth.



- a. What is the gravitational potential energy, relative to the lowest point reached by the swing as it swings back and forth, of the child at the moment the child is released?
- b. What will be the total kinetic energy of the child at the lowest point of the swing?

- c. What will be the child's velocity at the bottom of the swing?
- d. What will be the tension in the ropes of the swing when the child swings through the lowest point?

20.E: A 425. kg roller coaster begins from rest at a height  $h_1 = 140.$  m above the surface of the Earth. The roller coaster makes a circular loop with a radius of  $r = 24.0$  m.



- a. Determine the total energy of the roller coaster at points A, B, C, D, E, and F. Write neatly, show all your work, and place a box around all six of your answers.
- b. Determine the gravitational potential energy of the roller coaster at points A, B, C, D, E, and F. Write neatly, show all your work, and place a box around all six of your answers.

c. Determine the kinetic energy of the roller coaster at points A, B, C, D, E, and F. Write neatly, show all your work, and place a box around all six of your answers.

d. Determine the speed of the roller coaster at points A, B, C, D, E, and F. Write neatly, show all your work, and place a box around all six of your answers.

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.4 Rigid Body Mechanics

### Additional HL Understandings

- The torque  $\tau$  of a force about an axis as given by  $\tau = Fr \sin \theta$ .
- Bodies in rotational equilibrium have a resultant torque of zero.
- An unbalanced torque applied to an extended, rigid body will cause angular acceleration.
- The rotation of a body can be described in terms of angular displacement, angular velocity, and angular acceleration.
- Equations of motion for uniform angular acceleration can be used to predict the body's angular position  $\theta$ , angular displacement  $\Delta\theta$ , angular speed  $\omega$ , and angular acceleration  $\alpha$  as given by
  - $\Delta\theta = \frac{\omega_f + \omega_i}{2} t$
  - $\omega_f = \omega_i + \alpha t$
  - $\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$
  - $\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$
- The moment of inertia  $I$  depends on the distribution of mass of an extended body about an axis of rotation.
- The moment of inertia for a system of point masses as given by  $I = \sum mr^2$ .
- Newton's second law for rotation as given by  $\tau = I\alpha$  where  $\tau$  is the average torque.
- An extended body rotating with an angular speed has an angular momentum  $L$  as given by  $L = I\omega$ .
- Angular momentum remains constant unless the body is acted upon by a resultant torque.
- The action of a resultant torque constitutes an angular impulse  $\Delta L$  as given by  $\Delta L = \tau \Delta t = \Delta(I\omega)$
- The kinetic energy of rotational motion as given by  $E_k = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$ .

**Additional HL Equations**

$$\tau = Fr \sin \theta$$

$$\Delta\theta = \frac{\omega_f + \omega_i}{2} t$$

$$\omega_f = \omega_i + \alpha t$$

$$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$I = \sum mr^2$$

$$\tau = I\alpha$$

$$L = I\omega$$

$$\Delta L = \tau\Delta t$$

$$\Delta L = \Delta(I\omega)$$

$$E_k = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$$

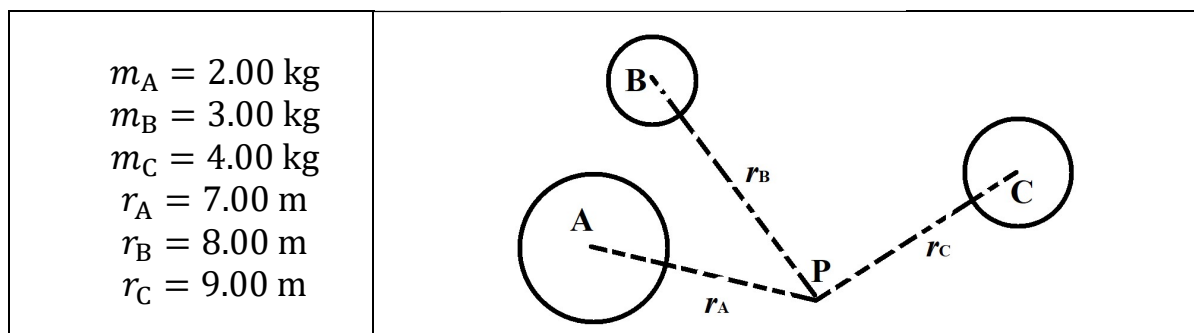
The solutions can be found on the YouTube channel Go Physics Go:

<https://www.youtube.com/@gophysicsgo/playlists>

1. C: Define, state the equation, and give the units of *angular position*  $\theta$ .
2. C: Define, state the equation, and give the units of *angular speed*  $\omega$ .
3. C: Define, state the equation, and give the units of *angular acceleration*  $\alpha$ .
4. C: Convert the *suvat* equations from linear motion to circular motion.

5. C: Define, state the equation, define each variable, and give the units for the *moment of inertia*  $I$ . What is the *moment of inertia*  $I$  equivalent to in translational motion?

6. E: Determine the moment of inertia of three objects rotating around a point P. Treat the three objects as point masses.



7. C: Define, state the equation, define each variable, and give the units for *torque*.



8. C: State the equations for *Newton's second law of motion for linear motion* and *Newton's second law of motion for rotational motion*.
9. C: Define and state the conditions for *translational equilibrium* and *rotational equilibrium*.
- 10.C: State the equations for *power for linear motion* and *power for rotational motion*.
- 11.C: State the equations for *linear momentum* and *angular momentum*. Also state the equations for *linear impulse* and *angular impulse*.
- 12.C: State the equations for *translational kinetic energy* and *rotational kinetic energy*.

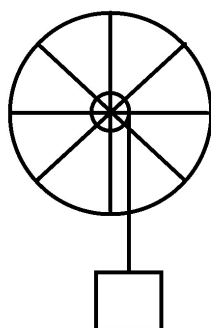
13.C: State the *law of conservation of linear momentum* and the *law of conservation of angular momentum*. Also state their equations.

14.E: A uniform disc, which has a mass of 3.50 kg and a radius of 28.0 cm, is accelerated from rest by a tangential force of 28.0 N applied to the outer edge of the disc. The moment of inertia of a disc is  $I = \frac{1}{2}mr^2$ .

- a. What is the moment of inertia of this disc?
- b. What is the magnitude of the torque being applied to this disc?
- c. What will be the angular acceleration of this disc?
- d. What will be the angular velocity of this disc after 8.00 s?
- e. What will be the linear velocity of a point on the outer edge of the disc at the end of 8.00 s?
- f. What will be the angular displacement of this disc during the 8.00 s period?

- g. What will be the linear displacement of a point on the outer edge of this disc during the 8.00 s period?
- h. How much work was done on this disc during the 8.00 s time period?
- i. What will be the final angular momentum of this disc?
- j. What is the final angular kinetic energy of this disc?

15.E: A bicycle wheel is mounted as in the lab and as shown below. This wheel has a mass of  $m_w = 6.55$  kg, a radius of  $r_w = 38.0$  cm, and is in the shape of a ring. A mass  $M = 1.85$  kg is attached to the end of a string which is wrapped around an inner hub which has a radius  $r_i = 5.40$  cm. Initially the mass  $M$  is a distance  $h = 72.0$  cm above the floor. Assume friction is negligible. The moment of inertia of a wheel is  $I = mr^2$ .



- a. What will be the resulting angular acceleration of this wheel?

- b. How long will it take for the mass  $M$  to reach the floor?
- c. What will be the total angular displacement of the wheel during the time in which the mass  $M$  is falling to the floor?
- d. How much work was done on the wheel by the external torque as the mass  $M$  falls to the floor?
- e. What will be the angular kinetic energy of this wheel just as the mass  $M$  reaches the floor?

16.E: A sphere, which has a mass of 2.10 kg and a radius of 35.0 cm, is rolling along a horizontal surface with a velocity of 12.2 m/s when the sphere encounters an inclined plane which meets the horizontal at an angle of  $22.0^\circ$ . The moment of inertia of a sphere is  $I = \frac{1}{2}mr^2$ .



- a. What is the angular velocity of this sphere?

- b. What is the linear kinetic energy of this sphere?
- c. What is the angular kinetic energy of this sphere?
- d. What is the total kinetic energy of this sphere?
- e. How far up this incline will the sphere roll before it comes to a halt?
- f. How long will it take for this sphere to come to a halt?
- g. What will be the linear acceleration of this sphere as it rolls up the incline?
- h. What will be the angular acceleration of this sphere as it rolls up the incline?

- i. What is the moment of inertia of this sphere?
- j. What will be the net torque acting on this sphere as it rolls up the incline?
- k. What will be the gravitational potential energy of this sphere just as it comes to a halt on the incline?

17.E: A disc, which has a mass of 12.0 kg and a radius of 65.0 cm, sits at the top of an inclined plane, which is 8.40 m long and 1.50 m high. At  $t = 0.00$  s the disc is released and is allowed to roll to the bottom of the incline without slipping.

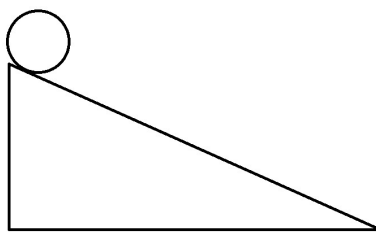
The moment of inertia of a disc is  $I = \frac{1}{2}mr^2$ .

- a. What is the gravitational potential energy of this disc as it sits at the top of this incline?
- b. What will be the total kinetic energy of this disc as it reaches the bottom of the incline?
- c. What will be the linear velocity of this disc when it reaches the bottom of the inclined plane?
- d. What would the linear velocity be if the disc is replaced by a sphere?

- e. What would the velocity be if the object was a ring?

18.E: A solid chocolate sphere with a mass of 9.00 kg and a diameter of 80.0 cm is placed on top of a rough incline ( $\mu = 0.700$ ) with a length of 6.00 m at an angle of  $50.0^\circ$ . The solid chocolate sphere begins from rest and rolls down the incline. The moment of inertia of a sphere is  $I = \frac{2}{5}mr^2$ .

- a. Draw a figure.



- b. What is the initial height of the solid chocolate sphere?
- c. How many revolutions will it take for the solid chocolate sphere to reach the bottom of the incline?
- d. What will be the final linear speed of the solid chocolate sphere at the bottom of the incline?

- e. What will be the final angular speed of the solid chocolate sphere at the bottom of the incline?
- f. What will be the angular acceleration of the solid chocolate sphere?
- g. What will be the linear acceleration of the solid chocolate sphere?
- h. How long will it take for the solid chocolate sphere to reach the bottom of the incline?

19.E: A solid chocolate sphere with a mass of 8.00 kg and a diameter of 70.0 cm is rolling to the right on a frictionless horizontal surface with a linear speed of 6.00 m/s. The surface then becomes rough with a coefficient of dynamic friction of 0.150. The moment of inertia of a sphere is  $I = \frac{2}{5}mr^2$ .

- a. Draw a figure.

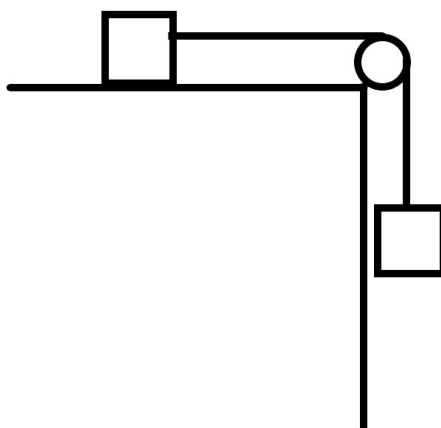






20.E: A block of mass  $m_1 = 7.00$  kg sits at rest on a horizontal surface with  $\mu = 0.200$ . Mass  $m_1$  is attached to a massless string which is wrapped around a pulley. Another massless string is wrapped around the same pulley and is holding another block of mass  $m_2 = 47.0$  kg in the air. The pulley is a cylinder which has a mass of  $m_C = 12.0$  kg and diameter of 10.0 cm. The moment of inertia of the rotating pulley is  $I = \frac{1}{2}mr^2$ .

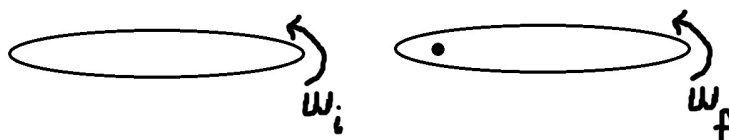
a. Draw a free body diagram.



b. What is the common linear acceleration of the system?

c. What is the force of tension on the two massless strings?

21.E: A thin disk with a mass of 250. g and diameter of 40.0 cm is spinning with an angular speed of 3.00 rad/s. A point mass with a mass of 350. g strikes and sticks to the top of the thin disk 4.00 cm from the edge. What is the final angular speed of the system? The moment of inertia of a disc is  $I = \frac{1}{2}mr^2$ .



22.E: A star, which has a mass of  $4.60 \times 10^{30}$  kg and a radius of  $9.30 \times 10^8$  m, rotates on its axis once every 16.0 days. The moment of inertia of a sphere is  $I = \frac{2}{5}mr^2$ .

- What is the moment of inertia of this star?
- What is the angular velocity of this star?
- What will be the linear velocity of a sunspot that is located on the equator of this star?
- What is the angular momentum of this star?

**As this star finally consumes most of its hydrogen fuel it starts to contract and heat up until it gets hot enough to start burning helium as a fuel. The burning of the helium fuel is very intense and the star quickly becomes a red giant as it expands to a new radius of  $4.65 \times 10^9$  m.**

- e. What will be the angular momentum of this star after it has expanded into a red giant?
  
- f. What will be the angular velocity of the star after it has expanded into a red giant?
  
  
  
  
  
  
  
  
  
  
- g. What will be the new rotational period of this star (in days) after it has become a red giant?

**After a few million years the helium fuel is all used up and the star again begins to contract, becomes extremely hot, and then undergoes a massive supernova explosion. After the explosion the remaining neutron star has a radius of only 1,200 km. Assume that the mass of the star remains unchanged.**

- h. What will be the angular momentum of this star after it has collapsed into a neutron star?
  
  
  
  
  
  
  
  
  
  
- i. What will be the moment of inertia of this star after it has collapsed into a neutron star?

j. What will be the angular velocity of this star after it has collapsed?

k. What will be the rotational period of this star (in seconds) after it has collapsed?

23.E: You are holding a sphere, which has a mass of 3.55 kg and a radius of 0.380 m, by an axle inserted vertical through its center as shown below. Initially, the sphere is rotating about its vertical axis 12.0 times every 5.00 seconds. The moment of inertia of a sphere is  $I = \frac{2}{5}mr^2$ .



a. What is the moment of inertia of this sphere?

b. What is the magnitude of the angular velocity of this sphere?

c. What is the magnitude of the angular momentum of this sphere?

d. What is the direction of this spheres angular momentum?

**Assume that you are sitting on a freely rotating chair, that your mass is 65.5 kg, and that your radius of gyration is 14.0 cm. Suddenly, you invert the sphere. The sphere continues to rotate with the same linear speed.**

e. What will be the spheres final angular momentum?

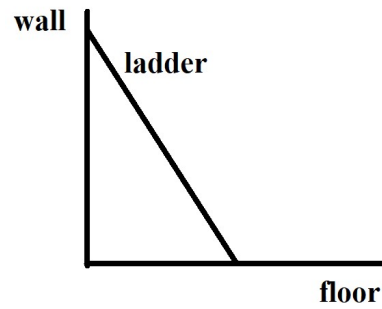
f. What will be the direction of the spheres final angular momentum?

g. What will be the magnitude of your final angular momentum?

h. What is the magnitude and direction of the impulse delivered to the sphere?

i. How much work was done in inverting the sphere?

24.E: A ladder has a mass of 20.0 kg and is 6.00 m long is leaning against a frictionless wall. The ladder is at rest and makes an angle of  $30.0^\circ$  from the wall. Draw a figure and write down the equations for static equilibrium.



25.C: Write down the common terms and equations for rigid body mechanics.



Name: \_\_\_\_\_

Class: \_\_\_\_\_

Due Date: \_\_\_\_\_

## A.5 Galilean and Special Relativity

### Additional HL Understandings

- Reference frames.
- Newton's laws of motion are the same in all inertial reference frames and this is known as Galilean relativity.
- In Galilean relativity the position  $x'$  and time  $t'$  of an event are given by  $x' = x - vt$  and  $t' = t$ .
- Galilean transformation equations lead to the velocity addition equation as given by  $u' = u - v$ .
- Two postulates of special relativity.
- The postulates of special relativity lead to the Lorentz transformation equations for the coordinates of an event in two inertial reference frames as given by
 
$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
 where  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ .
- Lorentz transformation equations lead to the relativistic velocity addition equation as given by  $u' = \frac{u-v}{1 - \frac{uv}{c^2}}$ .
- The space-time interval  $\Delta s$  between two events is an invariant quantity as given by  $(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$ .
- Proper time interval and proper length.
- Time dilation as given by  $\Delta t = \gamma\Delta t_0$ .
- Length contraction as given by  $L = \frac{L_0}{\gamma}$ .
- The relativity of simultaneity.
- Space-time diagrams.
- The angle between the world line of a moving particle and the time axis on a space-time diagram is related to the particle's speed as given by  $\tan \theta = \frac{v}{c}$ .
- Muon decay experiments provide experimental evidence for time dilation and length contraction.

**Additional HL Equations**

$$x' = x - vt$$

$$t' = t$$

$$u' = u - v$$

$$x' = \gamma(x - vt) \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \gamma \left( t - \frac{vx}{c^2} \right)$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$\tan \theta = \frac{v}{c}$$

**The solutions can be found on the YouTube channel Go Physics Go:**

<https://www.youtube.com/@gophysicsgo/playlists>

1. C: Define *observer*.
2. C: Define *reference frame*.
3. C: Define *inertial reference frame*.
4. C: Define *event*.
5. C: Define *simultaneous events*.
6. C: Define and state the equations for *Galilean transformations*.
7. C: Define *Galilean relativity*.
8. C: State *the two postulates of relativity*.

9. E: A train is traveling horizontally to the right with a speed of  $18.0 \frac{\text{m}}{\text{s}}$ . A man on the train is walking to the right with a speed of  $4.00 \frac{\text{m}}{\text{s}}$ . Determine the velocity of the man by an observer at rest outside the train.
- 10.E: A train is traveling horizontally to the left with a speed of  $32.0 \frac{\text{m}}{\text{s}}$ . A man on the train is walking to the right with a speed of  $4.00 \frac{\text{m}}{\text{s}}$ . Determine the velocity of the man by an observer at rest outside the train.
- 11.E: Usain Bolt and Florence Griffith-Joyner race each other. They both begin at the same time. Usain Bolt runs with a constant speed of  $10.44 \frac{\text{m}}{\text{s}}$  while Florence Griffith-Joyner runs with a constant speed of  $9.53 \frac{\text{m}}{\text{s}}$ .
- What is the distance between Usain Bolt and the starting line after two minutes?
  - According to Florence Griffith-Joyner, what is the distance to Usain Bolt after two minutes?
  - Determine the speed of Usain Bolt as observed by Florence Griffith-Joyner.
- 12.E: Wayde van Niekerk and Marita Koch race each other. They both begin at the same time. Wayde van Niekerk runs with a constant speed of  $9.30 \frac{\text{m}}{\text{s}}$  while Marita Koch runs with a constant speed of  $8.40 \frac{\text{m}}{\text{s}}$ .
- What is the distance between Wayde van Niekerk and the starting line after three minutes?

- b. According to Marita Koch, what is the distance to Wayde van Niekerk after three minutes?
- c. Determine the speed of Wayde van Niekerk as observed by Marita Koch.

13.C: What is the conclusion of the Michelson-Morley experiment?

14.C: State the *Lorentz transformation equations*. Define each variable.

15.C: **Use a pencil and ruler!** State and draw a *Lorentz factor vs. speed* graph.

16.E: An event occurs at position  $x = 4.00 \times 10^3$  m and time  $t = 6.00$  s. Where and when did the event occur according to a rocket traveling horizontally in the positive direction at a speed of  $v = 0.400c$ ? The origins of both reference frames are zero when both clocks read zero.

17.E: An event occurs at position  $x = -6.00 \times 10^3$  m and time  $t = 4.00$  s. Where and when did the event occur according to a rocket traveling horizontally in the positive direction at a speed of  $v = 0.100c$ ? The origins of both reference frames are zero when both clocks read zero.

18.E: A rocket moves to the right past an IB school with a speed of  $0.600c$ . The IB school is located at the origin. Determine the time reading of the rocket if  $x = 300$  m. The origins of both reference frames are zero when both clocks read zero.

19.E: A rocket moves to the right past an IB school with a speed of  $0.255c$ . The IB school is located at the origin. Determine the time reading of the rocket if  $x = 425$  m. The origins of both reference frames are zero when both clocks read zero.

20.C: Describe *time dilation* and *proper time interval*.

21.C: Describe *length contraction* and *proper length*.

22.C: Describe the *twin paradox*.

23.E: A man at rest sees two objects traveling towards each other each with a speed of  $0.500c$ . What is the speed of one object as seen by the other object? What would this value be using classical physics?

24.E: A man at rest sees two objects traveling towards each other each with a speed of  $0.900c$ . What is the speed of one object as seen by the other object? What would this value be using classical physics?



- 25.E: Romeo is behind Juliet and is running towards her with a speed of  $0.600c$  as seen by a man at rest. Juliet is running away from Romeo with a speed of  $0.400c$  as seen by the same man at rest. What is the speed of Romeo as observed by Juliet? What would this value be using classical physics?
- 26.E: Clyde is behind Terry and is running towards him with a speed of  $0.355c$  as seen by a man at rest. Terry is running away from Clyde with a speed of  $0.255c$  as seen by the same man at rest. What is the speed of Clyde as observed by Terry? What would this value be using classical physics?
- 27.C: A rocket is moving in a straight line with a constant speed  $v$  in space. It sends out a pulse of light ahead of it. How fast does the pulse of light move relative to an observer at rest on Earth?
- 28.E: A rocket at rest on Earth is measured to have a length of 45.0 m and a diameter of 5.60 m. The rocket is then launched toward Alpha Centauri, which is 4.30 light years from the Earth, at  $2.20 \times 10^8 \frac{\text{m}}{\text{s}}$ .
- What will be the length of this rocket as measured by an observer on the Earth?
  - What will be the length of the rocket as measured by an astronaut on board the rocket?
  - What will be the diameter of the rocket as measured by an observer on the Earth?

- d. What will be the distance to Alpha Centauri as measured by the astronaut on board the rocket?

29.E: A rocket at rest on Earth is measured to have a length of 65.5 meters and a diameter of 7.65 meters. The rocket is then launched toward Betelgeuse, which is 427 light years from the Earth, at  $1.25 \times 10^8 \frac{\text{m}}{\text{s}}$ .

- a. What will be the length of this rocket as measured by an observer on the Earth?
- b. What will be the length of the rocket as measured by an astronaut on board the rocket?
- c. What will be the diameter of the rocket as measured by an observer on the Earth?
- d. What will be the distance to Alpha Centauri as measured by the astronaut on board the rocket?

30.E: The distance to the star Epsilon Indi, as measured from the Earth frame of reference, is  $1.07 \times 10^{17}$  m.

- a. What is this distance in light years?
- b. What will be the distance in light years to Epsilon Indi as measured by an observer on a rocket heading toward this star at 85.0% of the speed of light?
- c. How long in years will it take for this rocket to reach Epsilon Indi according to an observer on the Earth?

31.E: The maximum distance to the planet Pluto, as measured from the Earth frame of reference, is approximately  $7.50 \times 10^9$  m.

- a. What is this distance in light years?
- b. What will be the maximum distance in light years to Pluto as measured by an observer on a rocket heading toward this star at 80.0% of the speed of light?
- c. How long in seconds will it take for this rocket to reach Pluto according to an observer on the Earth?

32.E: A rocket is moving toward the star Delta Pavonis with a velocity of  $2.40 \times 10^8 \frac{\text{m}}{\text{s}}$ . The distance to Delta Pavonis as measured from the rest frame on Earth is measured to be  $1.88 \times 10^{17}$  m.

- a. What will be the velocity of this rocket as a decimal fraction of the speed of light?
- b. What is the distance to Delta Pavonis in light years?
- c. What will be the distance to Delta Pavonis as measured by an astronaut on board the rocket?

33.E: A rocket is moving toward the star Eta Cassiopeiae with a velocity of  $2.00 \times 10^7 \frac{\text{m}}{\text{s}}$ . The distance to Eta Cassiopeiae as measured from the rest frame on Earth is measured to be  $1.82875 \times 10^{17}$  m.

- a. What will be the velocity of this rocket as a decimal fraction of the speed of light?

- b. What is the distance to Eta Cassiopeiae in light years?
- c. What will be the distance to Eta Cassiopeiae as measured by an astronaut on board the rocket?

34.E: As measured from the Earth the distance to Tau Ceti is determined to be 11.8 light years. How fast in  $\frac{\text{m}}{\text{s}}$  must a rocket be moving toward Tau Ceti so that the distance to Tau Ceti is reduced to 1.18 light years?

35.E: As measured from the Earth the distance to AD Leonis is determined to be 16.1939 light years. How fast in  $\frac{\text{m}}{\text{s}}$  must a rocket be moving toward AD Leonis so that the distance to AD Leonis is reduced to 12.0 light years?

36.E: How fast must a rocket be moving in order for its length to be reduced to 50.0% of its rest length?

37.E: How fast must a rocket be moving in order for its length to be reduced to 1.00% of its rest length?

- 38.E: According to an observer on the Earth the time it should take to reach a certain star is determined to be 7.50 years. How long will it take to reach this star according to an observer on board a rocket moving toward this star with a velocity of  $0.985c$ ?
- 39.E: According to an observer on the Earth the time it should take to reach a certain star is determined to be 22.0 years. How long will it take to reach this star according to an observer on board a rocket moving toward this star with a velocity of  $0.800c$ ?
- 40.E: The distance to Barnard's star is measured to be 6.00 ly by an observer at rest on the Earth.
- Assuming that a rocket is moving toward this star at  $0.980c$ , how long in years will it take for this rocket to reach Barnard's star according to an observer on the Earth?
  - How long in years will it take to reach this star according to an observer on board the rocket?
  - What will be the distance in light years to Barnard's star according to an observer on board the rocket?
  - Assuming that the astronaut was 21.0 years old when she left the Earth in 1988, in what year will the astronaut arrive at Barnard's star according to an observer on Earth?
  - How old will the astronaut be when she arrives at Barnard's star according to an observer on board the rocket?

- f. Suppose that the astronaut has a normal heart rate of 65.0 beats per minute when measured while at rest on the Earth. What will be the astronaut's heart rate as monitored by an observer on board the rocket with the astronaut?
- g. What will the astronaut's heart rate be while on the rocket moving toward Barnard's star as monitored by an observer on the Earth?

41.E: The distance to Groombridge 34 A is measured to be 11.6191 ly by an observer at rest on the Earth.

- a. Assuming that a rocket is moving toward this star at  $0.822c$ , how long in years will it take for this rocket to reach Groombridge 34 A according to an observer on the Earth?
- b. How long in years will it take to reach this star according to an observer on board the rocket?
- c. What will be the distance in light years to Groombridge 34 A according to an observer on board the rocket?
- d. Assuming that the astronaut was 32 years old when she left the Earth in 1988, in what year will the astronaut arrive at Groombridge 34 A according to an observer on Earth?
- e. How old will the astronaut be when she arrives at Groombridge 34 A according to an observer on board the rocket?
- f. Suppose that the astronaut has a heart rate of 62 beats per minute when measured while at rest on the Earth. What will be the astronaut's heart rate as monitored by an observer on board the rocket with the astronaut?

g. What will the astronaut's heart rate be while on the rocket moving toward Groombridge 34 A as monitored by an observer on the Earth?

42.E: Two astronauts play a game of chess on a rocket moving with a velocity of  $0.999c$  away from the Earth. According to the astronauts the game takes 2.50 hours. How long, in hours, does the game take according to an observer at rest on Earth?

43.E: Two astronauts are taking a very boring online IB HL physics class on a rocket moving with a velocity of  $0.900c$  away from the Earth. According to the astronauts the boring online IB HL physics class takes 90.0 min. How long, in minutes, does the boring online IB HL physics class take according to an observer at rest on Earth?

44.E: A rocket is moving toward Epsilon Eridani, which is 11.3 ly away as measured by an observer at rest on the Earth, at  $0.998c$ . When the astronaut leaves the Earth in 1991 he has just had his 22<sup>nd</sup> birthday and his young daughter has just turned 1.0 years old. The rocket travels to the star, remains there for 6.00 months, and finally returns to the Earth at the same speed.

a. In what year will the rocket return to the Earth?

b. How many years will the journey take according to the astronaut on board the rocket?

c. How old will the astronaut be when he returns to the Earth?

d. How old will his daughter be when he returns to the Earth?

45.E: A rocket is moving toward Luyten's Star, which is 12.3485 ly away as measured by an observer at rest on the Earth, at  $0.855c$ . When the astronaut leaves the Earth in 1992 he has just had his 32<sup>nd</sup> birthday and his young son has just turned 2.00 years old. The rocket travels to the star, remains there for 3.00 years, and finally returns to the Earth at the same speed.

a. In what year will the rocket return to the Earth?

b. How many years will the journey take according to the astronaut on board the rocket?

c. How old will the astronaut be when he returns to the Earth?

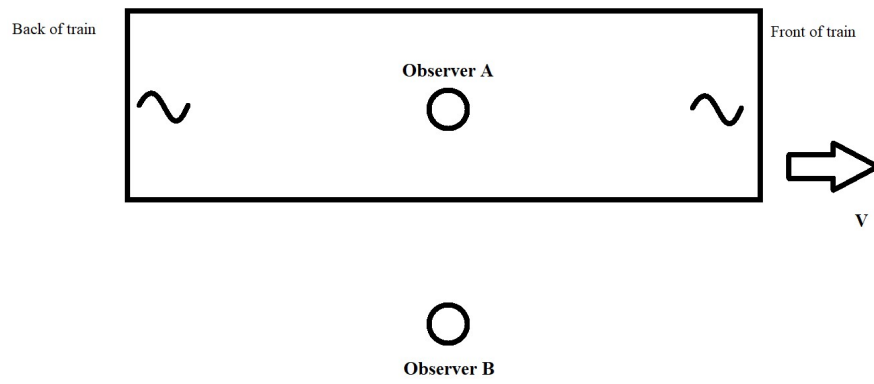
d. How old will his son be when he returns to the Earth?



- 46.E: Two twins are 22.0 years old when one of them sets out on a journey through space with a constant speed. The twin in the spaceship measures time with an accurate watch. When he returns to Earth, he claims to be 28.0 years old while the twin left on Earth is 40.0 years old. What was the speed of the spaceship?
- 47.E: Two twins are 34.0 years old when one of them sets out on a journey through space with a constant speed. The twin in the spaceship measures time with an accurate watch. When he returns to Earth, he claims to be 38.0 years old while the twin left on Earth is 44.0 years old. What was the speed of the spaceship?
- 48.E: A neutron outside the confines of the nucleus of an atom is unstable and has a life expectancy (half-life) of 6.00 min. Suppose that a fast moving alpha particle collides with a block of beryllium and knocks a neutron out of the nucleus with a speed of  $8.15 \times 10^7 \frac{\text{m}}{\text{s}}$ . What will be the expected lifetime of this neutron as measured by an observer in the rest frame?
- 49.E: A neutron outside the confines of the nucleus of an atom is unstable and has a life expectancy (half-life) of 6.00 min. Suppose that a fast moving alpha particle collides with a block of beryllium and knocks a neutron out of the nucleus with a speed of  $7.25 \times 10^7 \frac{\text{m}}{\text{s}}$ . What will be the expected lifetime of this neutron as measured by an observer in the rest frame?

50.C: Describe the equation of the *space-time interval*  $\Delta s$ . Describe *proper time interval* and *proper length* in terms of the *space-time interval*  $\Delta s$ .

51.C: Describe the *relativity of simultaneity*.



52.C: **Use a pencil and ruler!** Draw and label the axes of a space-time diagram.

53.C: **Use a pencil and ruler!** Draw a space-time graph where two events occur at the same time and another space-time graph where two events which occur at the same location.

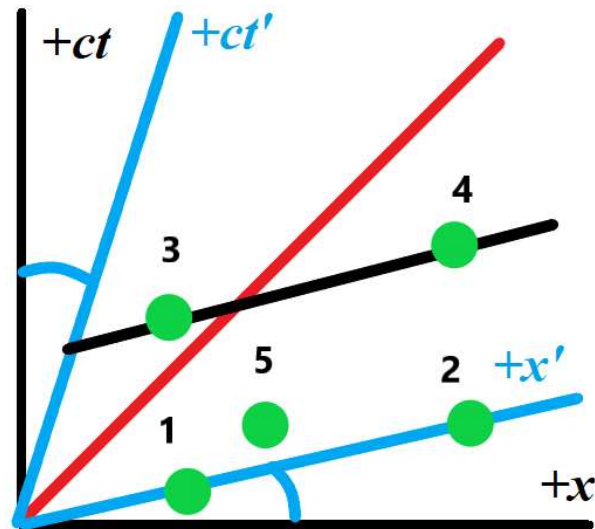
54.C: **Use a pencil and ruler!** Define the *world line*. Draw a *world line* in a space-time graph of an object at rest, a second object moving with a velocity of  $0.4c$ , and a third object moving with a velocity of  $0.8c$ .

55.C: Describe the relationship between the angle between the world line of a moving particle and the time axis on a space-time diagram.

56.C: **Use a pencil and ruler!** Add another reference frame to a space-time diagram. On the space time diagram label an event and describe how that event takes place at two different times in both reference frames.

57.C: **Use a pencil and ruler!** Draw a diagram with two reference frames. Label four stationary events in an  $(x', ct')$  graph. Draw a second diagram with two reference frames. Label four events which occur at the same time in an  $(x', ct')$  graph.

58.C: Below is a space-time diagram with two reference frames and five events. Determine the order of events according to the  $(x, ct)$  graph and the  $(x', ct')$  graph.



59.C: Use a pencil and ruler! Draw and describe a graph of  $\Delta s^2 = -1$ .

60.C: Use a pencil and ruler! Draw and describe a graph of  $\Delta s^2 = +1$ .



61.C: Three observers  $L$ ,  $M$ , and  $R$  are separated a distance  $A$  from each other on a parallel line. They all travel from rest to a speed of  $0.4c$  to the right and remain a distance  $A$  from each other. The observer in the middle  $M$  sends out a light pulse in all directions. The light gets reflected from observer  $L$  and observer  $R$  and returns to  $M$ . Does observer  $M$  receive the light pulse from observer  $L$  or observer  $R$  first?

62.C: Describe *muon decay*.