To what extent do the radius and mass of an object influence the centripetal force?

IB Physics IA

Introduction

The first time I learnt about the Roche Limit was in the library with my ex-boyfriend when I was learning astrophysics within the physics high-level elective content. The textbook said the Roche Limit is the distance between an object's own gravitational force and the tidal force formed by a second object. When the distance between objects is less than the Roche limit, small objects disintegrate and then become rings of the larger one. Listen to the sound of the pen tip rubbing against the paper and the steady breathing of my boyfriend on my side as he takes notes, I see the metaphor for boundary in human interpersonal relationships from the Roche Limit. In every healthy and stable relationship, people are attracted to each other but relatively independent. If they are too close, excess the boundary, the balance of relationship will be broken and the vulnerable tends to be the subordinate and sacrifice subjectivity. The first memorable encounter with the Roche Limit sparks my impression about circular motion of celestial bodies, incentivizing me to dig deeper about the principles of circular motion. The rotation of a satellite around a central object requires a "centripetal force", which is usually provided by the gravitational force between the two. When the angular velocity of the satellite's rotation is fixed, the "centripetal force ($F_{net} = ma_c = \frac{mv^2}{r}$) " is proportional to the radius of rotation, but the gravitation force is inversely proportional to the square of the radius of rotation.

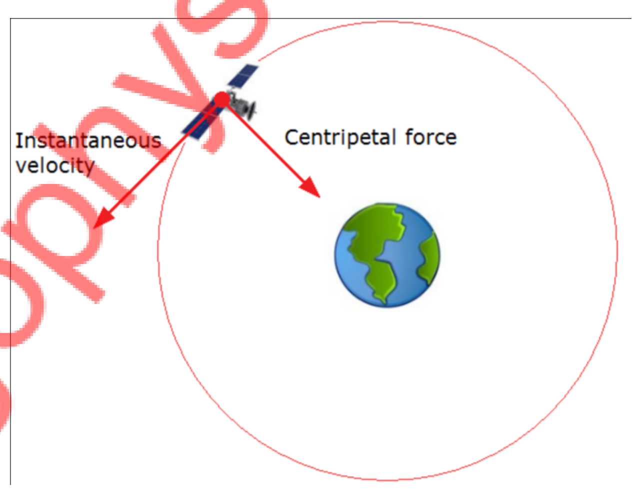

Figure 1.1 Newton's Law of Gravitation and Orbital Motion of Satellites

The two variables that are inversely related are opposite, resulting in uneven forces inside the satellite. On the side of the satellite near the central object, the centripetal force is less than the universal gravitational force; on the side away from the central object, the centripetal force is greater than the gravitational force. With the idea of "two iron balls falling to the ground at the same time", we consider the satellites as two iron balls chained together. For the iron ball near the central body, gravity provides a surplus of centripetal force; for the other iron ball, gravity cannot provide enough centripetal force. In this case, the chain in the middle is in balance by transferring the force to the "satellite" as a whole.

Gravitational force keeps the planets and satellites in their orbits. The same physical laws determine how an eraser on a string moves in a circle. In order to investigate the two factors, the radius of circular motion and the mass of the object, influence the centripetal force, I conduct the simulation experiment by taking erasers as the celestial bodies while keeping the controlling for mass or radius constant to obtain a single variable.

Research Question

To what extend the radius and mass influence the centripetal force?

Materials

The following materials are used for this experiment:

- ✓ 1.5 meter of light, strong string/thread
- ✓ 1 eraser：12.9g
- ✓ Electric balance
- ✓ Spring scale — 10 N
- ✓ Clamp to attach the spring balance to the table
- ✓ Hooked masses: 20g, 50g, 50g, 50g, 50g.
- ✓ Meter stick
- ✓ Cardboard tube
- ✓ Safety goggles — the experimenter could be swirling an object in a circle, so safety goggles prevent the possibility of eye injury

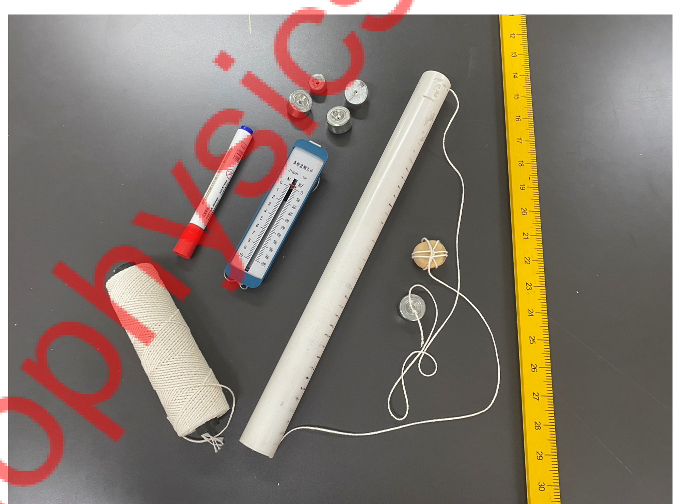

Figure 2.1 materials needed for conducting the experiment

Setup

1. Securely fasten the thread to the eraser.

2. Insert the string inside the cardboard tube.

3. Cut the string so that about 25 centimeters of it are below the tube, leaving about 1 meter of string between the tube and the eraser

Figure 1.2 Apparatus for exploring centripetal force by known-weight method

Procedure to Collecting Data

Investigation I: The centripetal force versus velocity

1. Don the protective eyewear. The experimenter should continue to move the eraser in a circle to measure the centripetal force.

2. Suspend a known weight from the thread, use the method depicted in Figure 1.2. The weight (in newtons), which is equal to the force, is calculated by multiplying the mass (in kg) by the gravitational acceleration (9.8 m/s^2).

3. Swing the eraser in a smooth, horizontal circle while holding the tube in one hand.

4. Calculate the time it takes to complete 10 revolutions, divide that number by ten, and to obtain the time it takes to complete one rotation.

5. Beginning at the loop, use the marker and tape to make a series of markings at 30 cm, 40 cm, 50 cm, and 60 cm.

6. Begin moving the eraser in a circle.

7. Calculate the duration, or the time it takes to complete 10 full rotations.

8. Using the formula $v = \frac{2\pi r}{T}$, where r is the radius (in meters) and T is the period, calculate the velocity (in meters per second).

9. As the eraser is spinning, gauge the force on the spring scale.

10. Because there is no change in the mass hanging from the thread, the force (in newtons) is equal to the mass's weight.

11. Alternately accelerate while keeping the fixed radius. Repeat for several velocity and force measurements at the same radius for each new speed.

Ethical and Safety Concerns

There were no issues with regards to ethics when conducting this lab. However, when rotating the eraser that conducting circular motion, the experimenter had better wear safety goggles in order to prevent the possibility from eye injury.

Raw Data

Table 1.1 below shows the time taken, period of circular motion, and speed respect to the constant force 0.50N

Radius for all trials = 0.5 meters

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Force (N)	0.50	0.50	0.50	0.50	0.50
Time (s) for 10 oscillations	6.75	5.95	5.97	5.83	6.31
Period (s)	0.675	0.595	0.597	0.583	0.631
V=2πr/T	4.65	5.28	5.26	5.39	4.98
Average V	5.11				

Table 1.2 below shows the time taken, period of circular motion, and speed respect to the constant force 1.00N

Radius for all trials = 0.5 meters

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Force (N)	1.00	1.00	1.00	1.00	1.00
Time (s) for 10 oscillations	5.52	5.37	5.17	5.42	5.15
Period (s)	0.552	0.537	0.517	0.542	0.525
$V=2\pi r/T$	5.69	5.85	6.08	5.80	5.98
Average V	5.88				

Table 1.3 below shows the time taken, period of circular motion, and speed respect to the constant force 1.50N

Radius for all trials = 0.5 meters

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Force (N)	1.50	1.50	1.50	1.50	1.50
Time (s) for 10 oscillations	4.39	4.71	4.30	4.43	4.50
Period (s)	0.439	0.471	0.430	0.443	0.450
$V=2\pi r/T$	7.16	6.67	7.31	7.09	6.98
Average V	7.04				

Table 1.4 below shows the time taken, period of circular motion, and speed respect to the constant force 2.00N

Radius for all trials = 0.5 meters

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Force (N)	2.00	2.00	2.00	2.00	2.00
Time (s) for 10 oscillations	3.97	4.16	3.88	3.98	4.12
Period (s)	0.397	0.416	0.388	0.398	0.412
$V=2\pi r/T$	7.91	7.55	8.10	7.89	7.63
Average V	7.82				

Data Processing

Table 1.5 below shows the force respect to the speed after averaging over multiple measurements

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Force(N)	0.50N	1.00N	1.50N	2.00N
Average Speed(m/s)	5.11	5.88	7.04	7.82

Figure 2.1.1 below shows the centripetal force versus average speed with constant radius and mass in the circular motion

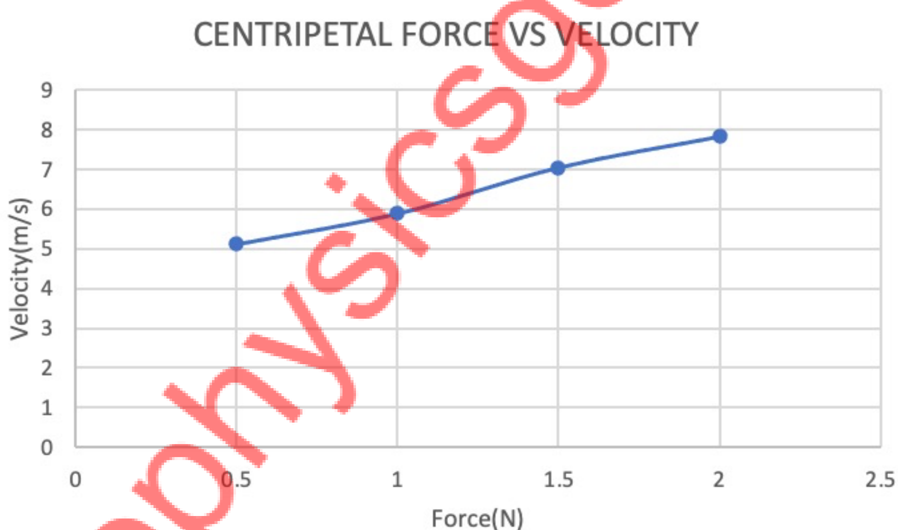

Figure 2.1.1 Centripetal force versus speed

According to Logger Pro, the proportional relationship between centripetal force and average speed is quadratic relationship, $y = Ax^2 + Bx + C$. As the force increase, the average speed of the object rises as well, under the condition of constant radius and constant mass. Plugging in data shown in Table 1.5, $A = 0.01 \pm 0.34, B = 1.83 \pm 0.87, C = 4.15 \pm 0.48$. The simplified equation of the best fit line is $y = 0.01x^2 + 1.83x + 4.15$. The correlation is 0.9966, which means strong positive relation between two variables.

Investigation II：The centripetal force versus radius

1. Set the spring balance to zero

2. Attach one end of the spring balance to the string coming from the tube and the other end to a clamp on the table. See Figure 1.2

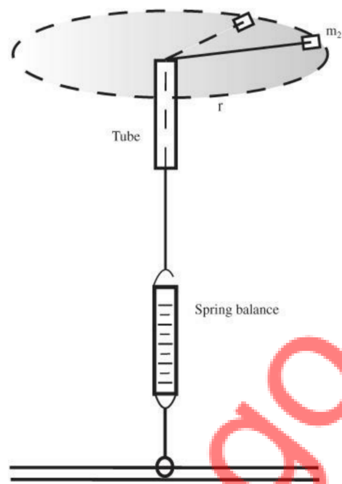

Figure 1.2 Using a spring scale to measure centripetal force

3. Begin spinning the eraser in a circular motion.

4. Measure the distance between the eraser and the circle's center (in meters) as radius. For all of these metrics, this ought to remain essentially unchanged.

5. Calculate the period, or the time it takes for 10 full rotations, in seconds, divide the time by 10 to obtain the period.

6. Use the formula $v = \frac{2\pi r}{T}$ to obtain the speed (in meters per second), where r is the radius (in meters) and T is the period (in seconds).

7. As the eraser is spinning, gauge the force on the spring scale. When utilizing a mass suspended from a string, the force (measured in newtons) is equal to the mass's weight.

8. Change the speed while keeping the radius constant. Measure the force on the spring scale for each new speed. Plot the findings after obtaining sufficient data of speed and force measurements at almost the same radius.

Table 2.1 below shows the time taken, period of circular motion, and speed respect to the constant force 1.00N

<u>Period for all trials = 0.4 sec</u>

	Trial 1	Trial 2	Trial 3	Trial 4
Radius (m)	0.3	0.4	0.5	0.6
Time (s)	4	4	4	4
Force (N)	0.954	1.696	2.650	3.816
V=2πr/T	4.71	6.28	7.85	9.42

Figure 2.2.1 shows the centripetal force versus radius with constant period and mass

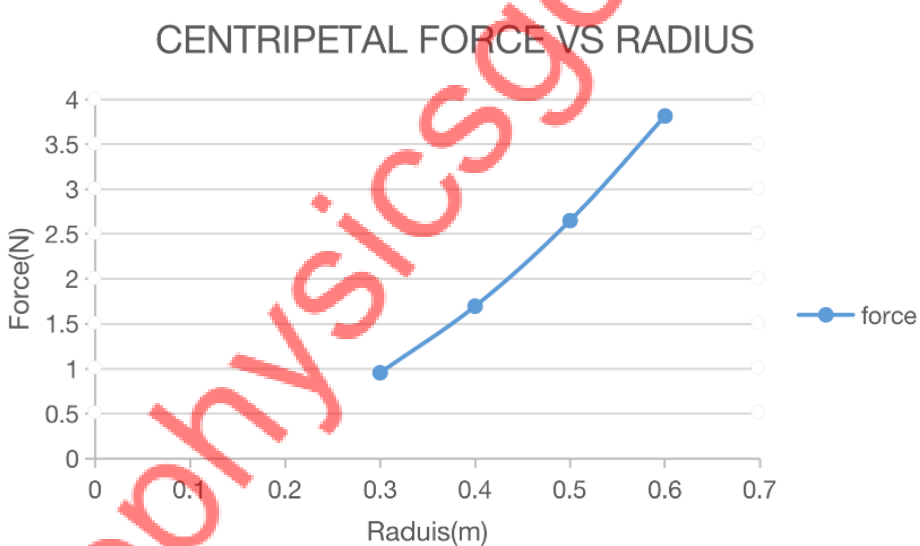

Figure 2.2.1 Centripetal force versus radius

According to Logger Pro, the proportional relationship between centripetal force and average speed is linear relationship, $y = mx + b$. As the radius of the circular motion increases, the centripetal force rises as well, under the condition of constant period and constant mass. Plugging in data shown in Table 2.1, $m = 9.54 \pm 0.67, b = -2.01 \pm 0.31$.

The simplified equation of the best fit line is $y = 9.54x - 2.01$. The correlation is 0.9951, which means strong positive relation between two variables.

Conclusion

1. The faster the rotation (the shorter the period of rotation), the greater the centripetal force needed to maintain circular motion.

For a 12-gram eraser, the results are shown in Figure 2.1.1. This shows the relationship is not linear, but that it increases more rapidly as the velocity increases.

2. The greater the mass, the greater the centripetal force needed to keep the eraser going at a given speed at a particular radius. For a given rotational speed, the shorter the string, the greater the centripetal force needed.

For a 12-gram eraser, as shown in Figure 2.2.1, there is an inverse relationship between force and string length.

In conclusion, *centripetal force* keeps an object rotating in a circle. The centripetal force equals the mass of the object times the velocity squared divided by the radius.

Errors and Limitations of the Investigation

With the help of fairly basic tools, this project allows you to find the centripetal force model and works reasonably well. The following are some systematic errors and random error caused by human error that could affect the outcomes:

1. The force needed is overestimated due to friction between the sting and the tube.

2. The speed is a little bit slower as a result of air resistance.

3. The circle may not be completely moving horizontally at slower speeds, and gravity may complicate matters.

4. The radius might not remain at a constant 30cm during the process of rotating, even though the marks of length is remarkable.

5. The reaction time for time recorder is inevitable, leading the periods of rotation is not accurate, which is a random error due to human error.

6. While rotating, the tube will lead the indicator on spring scale to move up and down, which makes hard to read the displayed number.

Exploration

As an alternative method, an experimenter can use the conic pendulum to roughly verify the determinants of centripetal force, The experimental setup is shown in the figure below. First hang a steel ball with a line, the upper end fixed on the iron stand, the flower oil concentric circles of white paper placed on the horizontal table, so that the steel ball at rest just at the center of the circle, and white paper contact but no extrusion, with the hand driving the steel ball, so that it is along the paper surface for circular motion.

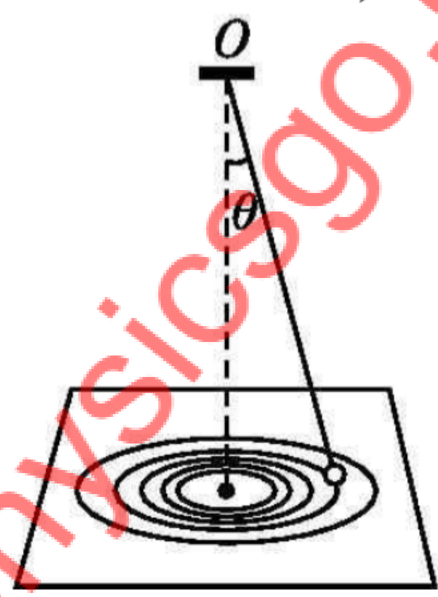

Figure 3.1 conic pendulum method

The concept of centripetal force is introduced through the analysis of the phenomenon of uniform circular motion on a horizontal table by pulling a ball with a rope in a demonstration experiment. Through the analysis of the special uniform circular motion, the direction of the centripetal force points to the center of the circle, and then the expression of the magnitude of the centripetal force is inductively derived through experimental investigation. The method used in this experiment is similar to the previous experiments to investigate the relationship between acceleration and external forces and masses, both using the method of controlling variables. In the teaching, students should be bold in their assumptions and careful in their proofs, and finally arrive at the quantitative relationship between the magnitude of centripetal force and mass, velocity (linear velocity, period, frequency) and radius.

The “string” that keeps an object going around in a circle is provided by a centripetal force. In the case of a satellite or planet, the “string” is the gravitational force. The faster the object goes (for a given radius), the greater the force, according to the equation: $F = \frac{\gamma m v^2}{r}$

where F_c is the centripetal force, m is the mass of the spinning object (the eraser in our case), v is the velocity, and r is the radius of the circle.

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