

# Capacitor

**What is the current vs. time graph and potential difference vs. time graph when a capacitor is charging and discharging?**

Physics Internal Assessment

[gophysicsgo.com](http://gophysicsgo.com)

# Research Question

What is the current vs. time graph and potential difference vs. time graph when a capacitor is charging and discharging?

## 1. Introduction

When I played with my solar powered car (Figure 1) last year, I found that after being exposed in the sun for ten minutes, the car could travel quickly for a long distance with the bulb on the car becoming dimmer and dimmer gradually. If the car was exposed in the sun more than ten minutes, the car would not travel an obviously longer distance. However, if the car was exposed in the sun for much less than ten minutes, for example, 5 minutes, the car would travel a much shorter distance and the bulb turned off faster. I looked up the principle of solar powered car on the internet and learned that the solar powered car can convert solar energy into electric energy through the solar panels and store the electric energy in a capacitor when the car is exposed to the sun. When the capacitor is discharged, it can provide power to the electric motor to drive the wheels to rotate. When the capacitor is fully discharged, the motor will be out of power and the car will stop.

The solar powered car also reminded me of the knowledge of capacitors I had studied in physics classes. It is “any arrangement of two conductors separated from each other by insulating material (or a vacuum)” (Tsokos 457) and is a kind of electronic component that has the capacity to store electric charge. Capacitors are applied to a wide range of electronics products. They are indispensable for almost all electronic circuits in various electrical devices, apart from application in solar energy systems. Figure 2 illustrates a variety of capacitors. Recently, capacitors are considered as potential supplements to batteries of new energy vehicles and have attracted the public’s attention. Therefore, I would like to carry out a series of experiments to explore how capacitors are charged and discharged.



Figure 1. My solar car



Figure 2. Different types of capacitors  
(Five Years Out)

## 2. Rationale and Aim

### 2.1 The Background of Capacitors

A typical capacitor consists of two electrical conductors, such as metal foils, thin films or electrolytes “often in the form of metallic plates or surfaces separated by a dielectric medium,”

such as glass, ceramic, plastic, paper, etc. (Codepixer). Ideally, a capacitor never dissipates energy, which is different from a resistor (Figure 3). However, in real life, capacitors do dissipate a small amount of energy.

As studied in physics classes, the capacitor and the battery have different charging principles. Batteries make electrons flow through chemical reactions, while capacitors store electrical charge by accumulating electrons on conductive plates separated by insulating materials. According to the physics website Solectroshop.com, in a parallel-plate capacitor, “when an [electric potential] is applied across the terminals of a capacitor, an electric field [develops across the dielectric], and positive charges are accumulated on one of its plates and negative charges on the other”. If the voltage is withdrawn, the capacitor will discharge.

According to the book titled *The Story of Electrical and Magnetic Measurements: From 500 BC to the 1940s*, “the earliest forms of capacitors were created in the 1740s, when European scientists found that a glass bottle holding water could store electricity...such a bottle was called a ‘Leyden jars’” (23) (Figure 4). Later, in 1748, Benjamin Franklin “associated a progression of containers together to make what he called an ‘electrical battery’, from their visual similitude to a battery of gun” (Kumar et al. 12). The word has become a physics term since 1926. Today, capacitors are widely applied to different electronic circuits to block direct current while allow alternating current to pass.

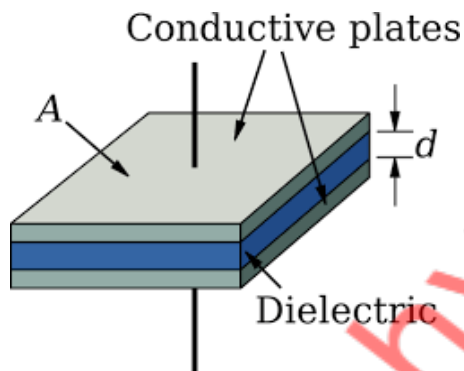


Figure 3. Capacitors (Solectroshop)

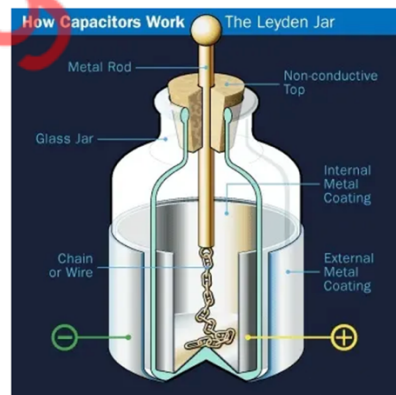


Figure 4. Leyden Jars (Keightley)

## 2.2 Theoretical Basis

### 2.2.1 Coulomb's Law

There is an electric force  $F_e$  between two charges  $q_1$  and  $q_2$ .

$$F_e = \frac{kq_1q_2}{r^2} \quad (\text{Eq-1})$$

where  $k$  is the coulomb constant ( $8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ ), and  $r$  is the distance between the two conductors. The charge on the first conductor of the capacitor will exert an electric force on the other conductor. The first conductor attracts the opposite polarity charge and repels the same polarity charge. The opposite polarity charge can always be induced to the other conductor. The

dielectric will be polarized. Therefore, both the conductors will have equal, but opposite charges and the dielectric creates an electric field.

### 2.2.2 Capacitance

An ideal capacitor has a constant capacitance  $C$ . Capacitance is defined as “the charge  $Q$  per unit voltage  $V$  that can be stored on the capacitor” (Tsokos 457).

$$C = \frac{Q}{V} \quad (\text{Eq-2})$$

The unit of capacitance is Farads (F), charge is Coulomb (C) and voltage is Volt (V) in the SI system of units.

$$Q = It \quad (\text{Eq-3})$$

$I$  represents the current and  $t$  represents time. One current is defined as the flow of one charge per second.

From Eq-3 and Eq-2, one can obtain

$$C = \frac{It}{V} = \frac{t}{R} \quad (\text{Eq-4})$$

Two thin parallel conductive plates are separated by a gap  $d$  and each has a height  $h$  and area of  $A$ . They are separated by a uniform gap with a distance. The gap is filled with a dielectric with permittivity  $\epsilon$ . Assuming  $d$  is smaller than  $A$ , the capacitance of a capacitor can be calculated by using the following equation

$$C = \frac{\epsilon A}{d} \quad (\text{Eq-5})$$

Eq-5 implies that as the conductors move closer, the opposite charges on both conductors will attract each other, which enables the capacitor store more charge for a given voltage. Correspondingly, the capacitance will be larger.

### 2.2.3 Kirchhoff's Law

A typical series circuit containing a capacitor is shown in Figure 5.

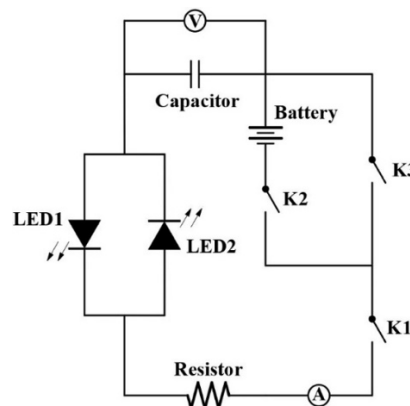


Figure 5. Circuit drawing

Based on Kirchhoff's Law, the current and the voltage for the capacitor can be calculated. In the charging process (when switches K1, K2 are open and K3 is closed), the current and voltage changes based on Eq-6 and Eq-7.

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad (\text{Eq-6})$$

$$V(t) = V_0 (1 - e^{-\frac{t}{RC}}) \quad (\text{Eq-7})$$

In the discharging process (when the switches K1, K3 are open and K2 is closed), the current and voltage changes based on Eq-8 and Eq-9.

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad (\text{Eq-8})$$

$$V(t) = V_0 e^{-\frac{t}{RC}} \quad (\text{Eq-9})$$

where  $V_0$  is the voltage of the battery.

## 2.3 Aim of the Investigation

This essay will investigate how the current and voltage changes during the time period of charging and discharging of a capacitor, and compare the results with the established theories to discuss how capacitor can be charged and discharged.

## 2.4 Hypothesis

According to Eq-4, I can assume that the capacitor's capacitance is positively proportional to current  $I$  and inversely proportional to voltage  $V$ . When the capacitor is being charged, as the time constant increases, current  $I$  will decrease and voltage  $V$  will increase. When the capacitor is being discharged, current  $I$  will increase and voltage  $V$  will decrease.

## 3. Variables

Table 1 below shows the various variables in the experiment.

Variables	Items	How	Why
Controlled Variables	Electric resistance of resistor $R$ Voltage of the battery $V_0$	I will use the same battery and capacitor in the experiment.	The investigation aims to explore the relationship between time $t$ and current $I$ and the relationship between time $t$ and voltage $V$ . Resistor $R$ , voltage of the battery $V_0$ and the capacitor are the determinant of current and voltage. For

	Capacitance of the capacitor $C$	I will use four different resistors in the experiment.	each resistor $R$ , $V_0$ and capacitance are kept unchanged.  I use four different resistors settings to further study the influence of resistance on charge/discharge of capacitors.
Independent Variables	Time $t$ of charging  Time $t$ of discharging	Use my mobile phone to record videos of the charging and discharging processes, and record the measured voltage and current from the videos.	/
Dependent Variables	Current $I$  Voltage $V$	Measured by the DC ammeter  Measured by the DC voltmeter	/

#### 4. Electrical Components

I used the following electric components to conduct this experiment and set up the actual circuit as shown in Figure 6, according to Figure 5:

- 1) 320  $\mu\text{F}$  (micro-Farad) capacitor, 100  $\mu\text{F}$  capacitor and 220  $\mu\text{F}$  capacitor
- 2) 4 resistors with different resistances: 14.21  $\text{k}\Omega$  (kilo-Ohm), 24.0  $\text{k}\Omega$ , 29.9  $\text{k}\Omega$  and 52.4  $\text{k}\Omega$
- 3) A DC voltmeter in  $\text{V}$  ( $\pm 0.005\text{V}$ ) and a DC ammeter in  $\mu\text{A}$  ( $\pm 0.05\mu\text{A}$ )
- 4) 9.8 V (Volt) DC power battery
- 5) 3 toggle switches: K1, K2 and K3
- 6) 2 LEDs
- 7) Jumper wire
- 8) 1 wooden circuit board
- 9) Mobile phone, used for video record and stopwatch (Figure 7)



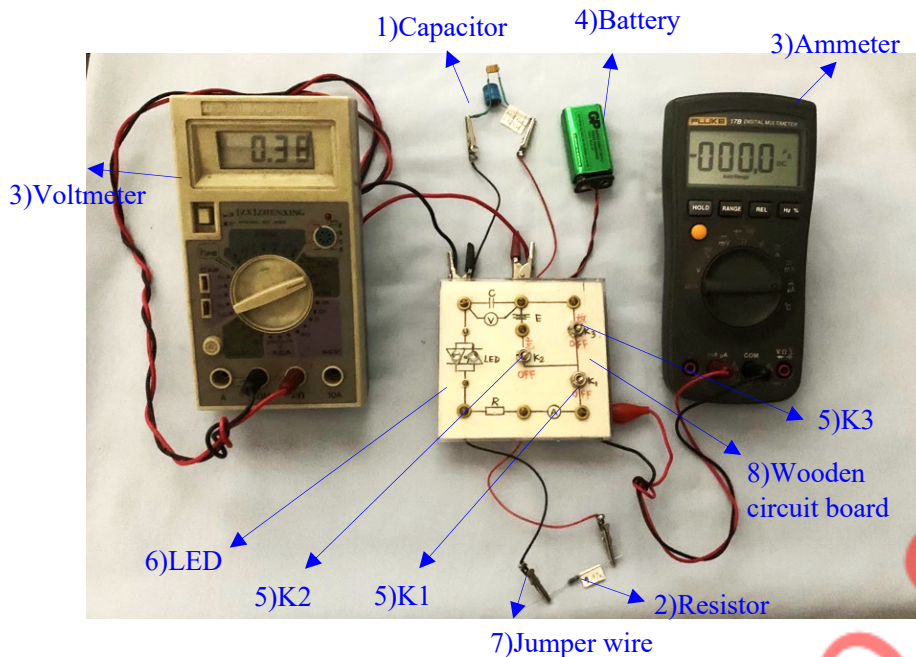


Figure 6. Experimental circuit

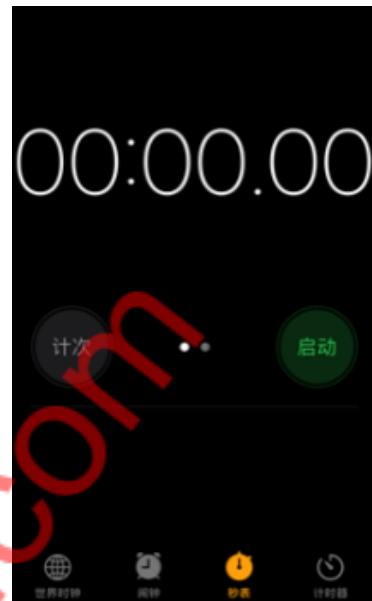


Figure 7. Mobile phone

## 5. Experiment Procedure and Data Collection

The charge and discharge experiment includes the following steps:

### 5.1 Charging

- 1) Use the first electric resistor.
- 2) Close all the three switches.
- 3) Open K2 and close K3.
- 4) Open K1.
- 5) Record the video of the charge process by using my mobile phone.
- 6) Read voltmeter and ammeter every 2 seconds until the readings become almost unchanged or change is too small.
- 7) Conduct the charge test three times to ensure repeatability.
- 8) Replace the resistor to the second, third and fourth resistor, and repeat steps 2) to 7).

### 5.2 Discharging

- 1) Use the first electric resistor.
- 2) Close all the three switches.
- 2) Open K1 and close K2.
- 3) Open K3.
- 5) Record the video of the discharge process by using my mobile phone.
- 6) Read voltmeter and ammeter every 2 seconds until the readings become almost unchanged or too small.
- 7) Conduct the discharge test three times to ensure repeatability.
- 8) Replace the resistor to the second, third and fourth resistor, and repeat steps 2) to 7).

## 6. Ethical and Safety Concerns

- 1) Turn off all the switches before all the devices are connected. This can protect all the devices and protect the user from electric shock.
- 2) Turn off the switch once the data is recorded. This is friendly to the electronic devices and the life of the devices will be durable.
- 3) Do not reverse the positive and negative pole of the electrical appliances. This is harmful to the appliances and might cause short circuit.
- 4) Reserve the electrical appliances, especially the battery in the insulation box to prevent electric leakage.

## 7. Raw Data

Total over 130 groups of  $I$  and  $V$  data were collected. Due to the limitation on the essay length, ten groups of data for each resistor are listed below. The remaining will be presented in the Appendix.

Table 2.  $I$  and  $V$  measured in charging and discharging (14.21 k $\Omega$  resistor)

Time ( $\pm 0.5$ s)	Experiment-1				Experiment-2				Experiment-3				Average				Standard deviation			
	Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)
2	598.0	1.48	449.0	6.54	571.0	0.60	505.0	7.71	505.0	2.16	487.0	6.94	558.0	1.41	480.3	7.06	47.8	0.78	28.6	0.59
4	379.0	4.20	345.0	5.19	394.0	4.10	387.0	5.63	387.0	4.21	355.0	5.09	386.7	4.17	362.3	5.30	7.5	0.06	21.9	0.29
6	281.9	5.69	246.7	3.55	292.4	5.62	268.3	4.15	268.3	5.97	253.3	3.48	280.9	5.76	256.1	3.73	12.1	0.19	11.1	0.37
8	202.6	6.76	164.5	2.44	209.7	6.71	180.2	3.07	180.2	7.16	168.1	2.51	197.5	6.88	170.9	2.67	15.4	0.25	8.2	0.35
10	146.9	7.71	118.1	1.82	151.8	7.68	139.2	2.12	139.2	7.68	121.0	1.78	146.0	7.69	126.1	1.91	6.4	0.02	11.4	0.19
14	79.7	8.72	62.0	0.89	86.8	8.63	75.3	1.10	78.8	8.75	63.4	0.94	81.8	8.70	66.9	0.98	4.4	0.06	7.3	0.11
20	35.5	9.44	28.8	0.37	35.4	9.42	31.5	0.45	32.9	9.39	29.4	0.39	34.6	9.42	29.9	0.40	1.5	0.03	1.4	0.04
28	15.5	9.71	9.0	0.12	16.8	9.70	11.1	0.15	14.0	9.65	9.2	0.12	15.4	9.69	9.8	0.13	1.4	0.03	1.2	0.02
34	10.8	9.78	4.5	0.06	10.9	9.79	5.2	0.07	9.3	9.72	4.8	0.06	10.3	9.76	4.8	0.06	0.9	0.04	0.4	0.01
36	9.9	9.79	3.5	0.05	9.3	9.80	3.6	0.06	8.3	9.74	3.6	0.06	9.2	9.78	3.6	0.06	0.8	0.03	0.1	0.01

Table 3.  $I$  and  $V$  measured in charging and discharging (24.0 k $\Omega$  resistor)

Time ( $\pm 0.5$ s)	Experiment-1				Experiment-2				Experiment-3				Average				Standard deviation			
	Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)	I ( $\pm 0.05\mu$ A)	V ( $\pm 0.005$ V)
1	347.8	1.45	381.0	9.46	323.1	2.18	355.5	9.46	330.5	1.77	365.1	9.47	333.8	1.80	367.2	9.46	12.7	0.37	12.9	0.01
3	293.5	2.89	364.3	9.14	251.2	3.44	340.5	8.79	267.7	3.45	349.6	8.63	270.8	3.26	351.5	8.85	21.3	0.32	12.0	0.26
5	228.8	4.06	348.8	7.56	213.2	4.75	287.2	6.94	209.1	4.76	270.9	6.83	217.0	4.52	302.3	7.11	10.4	0.40	41.1	0.39
11	125.7	6.75	296.8	3.81	117.4	7.00	154.8	3.84	119.8	7.10	152.4	3.61	121.0	6.95	201.3	3.75	4.3	0.18	82.7	0.13
17	71.2	8.14	90.3	2.14	66.6	8.24	84.9	2.07	65.4	8.29	80.4	1.95	67.7	8.22	85.2	2.05	3.1	0.08	5.0	0.10
23	43.9	8.89	48.4	1.17	47.2	8.93	48.8	1.18	41.9	8.91	46.3	1.12	44.3	8.91	47.8	1.16	2.7	0.02	1.3	0.03
29	27.6	9.24	30.2	0.68	25.2	9.30	28.5	0.66	24.7	9.33	28.1	0.65	25.8	9.29	28.9	0.66	1.6	0.05	1.1	0.02
33	19.9	9.43	20.2	0.47	18.7	9.47	19.8	0.46	18.9	9.47	18.8	0.44	19.2	9.46	19.6	0.46	0.6	0.02	0.7	0.02
39	13.4	9.57	11.9	0.27	12.7	9.61	11.7	0.27	12.7	9.62	11.1	0.26	12.9	9.60	11.6	0.27	0.4	0.03	0.4	0.01
49	8.1	9.71	5.2	0.12	7.8	9.73	5.2	0.12	7.9	9.73	5.1	0.12	7.9	9.72	5.2	0.12	0.2	0.01	0.1	0.00

Table 4.  $I$  and  $V$  measured in charging and discharging (29.9 k $\Omega$  resistor)

Time ( $\pm 0.5$ s)	Experiment-1				Experiment-2				Experiment-3				Average				Standard deviation			
	Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V



	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)	(±0.05μ A)	(±0.005 V)
2	282.9	1.13	290.2	8.72	291.5	0.94	290.9	8.87	267.4	1.81	279.3	8.62	280.6	1.29	286.8	8.74	12.2	0.46	6.5	0.13
4	246.8	2.36	252.9	7.78	245.8	2.48	245.0	7.46	233.6	2.66	235.6	7.14	242.1	2.50	244.5	7.46	7.3	0.15	8.7	0.32
8	171.4	4.67	175.1	5.38	178.6	4.56	175.5	5.35	162.6	4.89	168.9	5.13	170.9	4.71	173.2	5.29	8.0	0.17	3.7	0.14
16	93.6	7.04	94.4	2.82	90.7	7.07	85.9	2.61	89.2	7.24	85.3	2.69	91.2	7.12	88.5	2.71	2.2	0.11	5.1	0.11
20	66.9	7.87	66.6	2.05	69.0	7.82	64.6	1.97	65.7	7.87	64.3	1.96	67.2	7.85	65.2	1.99	1.7	0.03	1.3	0.05
24	53.6	8.34	51.0	1.50	52.1	8.41	51.1	1.50	49.7	8.45	49.2	1.44	51.8	8.40	50.4	1.48	2.0	0.06	1.1	0.03
30	33.9	8.90	32.1	0.94	35.1	8.88	32.2	0.93	33.5	8.94	31.1	0.92	34.2	8.91	31.8	0.93	0.8	0.03	0.6	0.01
38	19.6	9.33	17.7	0.52	20.8	9.32	17.2	0.51	19.9	9.34	16.6	0.48	20.1	9.33	17.2	0.50	0.6	0.01	0.6	0.02
44	14.2	9.51	11.5	0.33	14.7	9.50	11.5	0.33	13.7	9.52	10.8	0.32	14.2	9.51	11.3	0.33	0.5	0.01	0.4	0.01
54	9.0	9.66	6.0	0.18	9.1	9.67	5.7	0.17	8.7	9.68	5.7	0.17	8.9	9.67	5.8	0.17	0.2	0.01	0.2	0.01

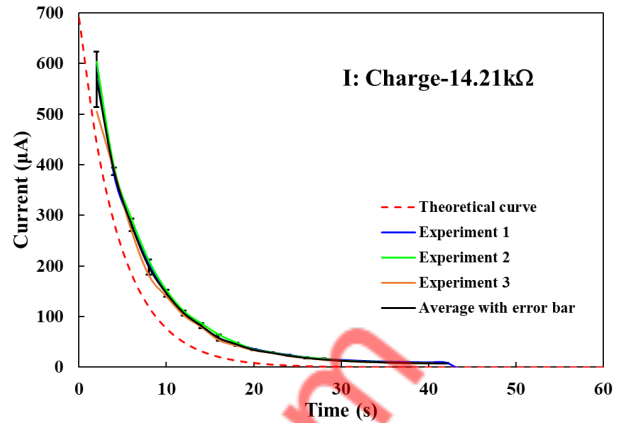
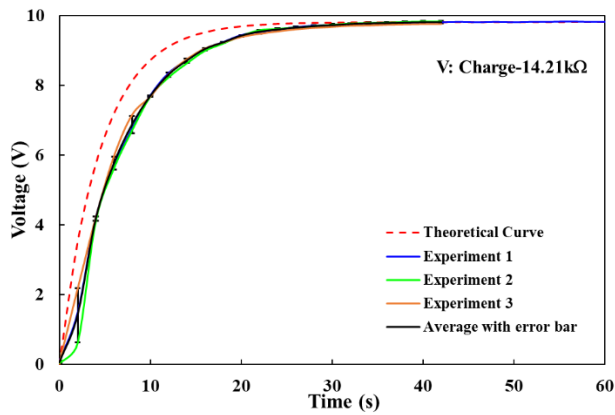
Table 5. I and V measured in charging and discharging (52.4 kΩ resistor)

Time (±0.5 s)	Experiment-1		Experiment-2				Experiment-3				Average				Standard deviation					
	Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)	I (±0.05μ A)	V (±0.005 V)
2	179.6	0.25	158.0	8.59	171.2	0.75	166.3	8.92	175.9	0.48	162.2	8.64	175.6	0.49	162.2	8.72	4.2	0.25	4.2	0.18
4	162.7	1.28	146.0	7.86	158.4	1.32	150.7	8.18	159.4	1.44	150.0	8.10	160.2	1.35	148.9	8.05	2.3	0.08	2.5	0.17
6	147.7	1.99	132.4	7.06	143.8	2.20	136.8	7.35	144.8	2.37	136.2	7.28	145.4	2.19	135.1	7.23	2.0	0.19	2.4	0.15
10	122.1	3.35	107.3	5.73	118.9	3.53	113.0	6.07	119.7	3.61	112.6	6.03	120.2	3.50	111.0	5.94	1.7	0.13	3.2	0.19
18	88.3	5.32	77.4	4.03	84.4	5.50	79.9	4.18	85.1	5.47	79.6	4.06	85.9	5.43	79.0	4.09	2.1	0.10	1.4	0.08
28	54.6	7.09	48.0	2.47	54.0	7.12	49.5	2.57	53.6	7.14	50.2	2.60	54.1	7.12	49.2	2.55	0.5	0.03	1.1	0.07
36	40.1	7.83	33.5	1.73	38.3	7.93	35.1	1.79	38.7	7.91	35.0	1.81	39.0	7.89	34.5	1.78	0.9	0.05	0.9	0.04
44	28.4	8.44	23.5	1.22	27.5	8.49	24.6	1.28	27.9	8.47	24.1	1.25	27.9	8.47	24.1	1.25	0.5	0.03	0.6	0.03
52	21.1	8.84	16.6	0.86	21.1	8.88	17.3	0.91	20.4	8.88	17.6	0.90	20.9	8.87	17.2	0.89	0.4	0.02	0.5	0.03
60	15.7	9.11	11.8	0.62	15.0	9.15	12.3	0.63	15.3	9.14	12.1	0.63	15.3	9.13	12.1	0.63	0.4	0.02	0.3	0.01

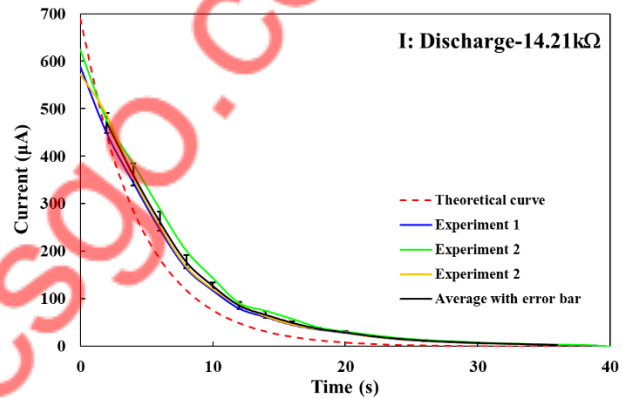
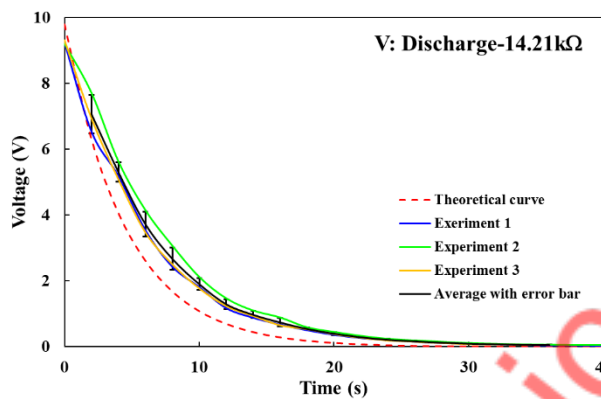
## 8. Data Processing

### 8.1 I-t and V-t plots

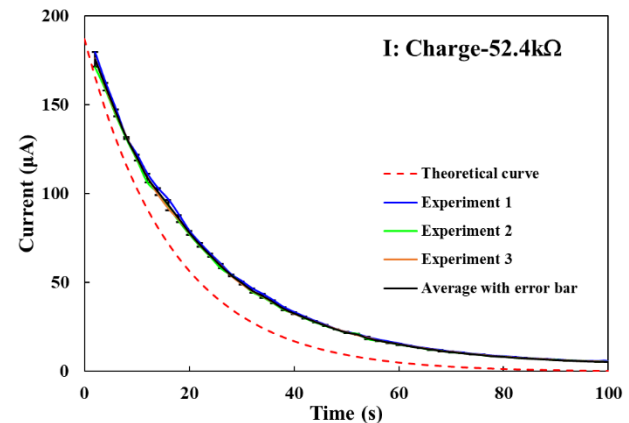
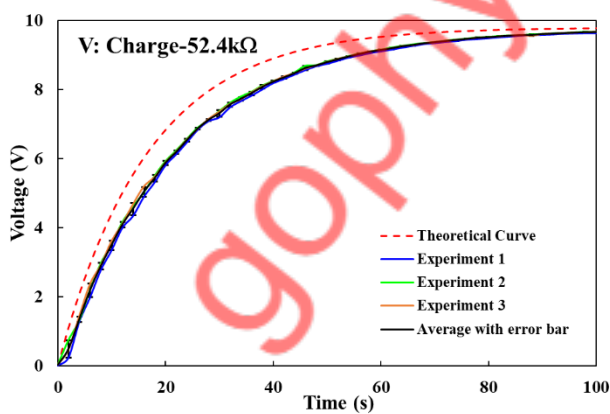
With K2 open, the measured voltage increased and the current decreased. The capacitor is initially uncharged, which means at  $t = 0$ , the voltage on the capacitor is nearly zero. With the experimental data, I plotted  $I - t$  and  $V - t$  diagrams drawn by using Excel to figure out the actual process how the capacitors are charged and discharged. In addition, error analysis was conducted to understand the difference in data of repeated experiments. Due to the limitation on the essay length, two sets of graphs, the smallest resistor and the largest resistor, were shown in the essay, and the other eight will be presented in the appendix. Figures 8-11 illustrate the experimental  $I - t$  and  $V - t$  diagrams during charging and discharging. For comparison purpose, the theoretical curves drawn based on the Kirchhoff's Law (Eq-6 to Eq-9) are included, as depicted by using the dashed red lines. The curves of average values of the three experiments are also included, with the error bar indicating the variation of the experimental data. The variation of experimental data is described by using standard deviation.



(a) (b)  
Figure 8. Voltage (a) and current (b) in charge process ( $R=14.21\text{ k}\Omega$ )



(a) (b)  
Figure 9. Voltage (a) and current (b) in discharge process ( $R=14.21\text{ k}\Omega$ )



(a) (b)  
Figure 10. Voltage (a) and current (b) in charge process ( $R=52.4\text{ k}\Omega$ )

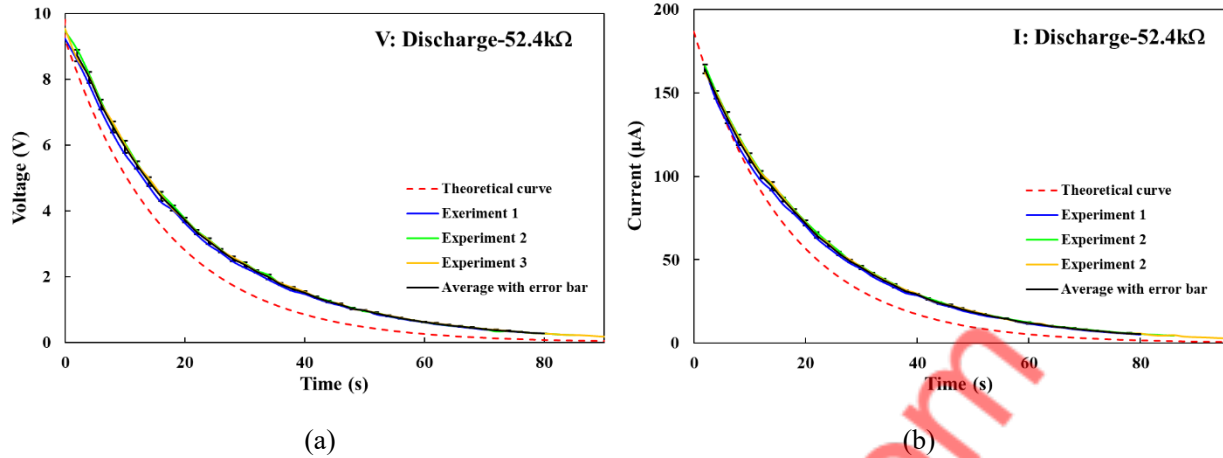


Figure 11. Voltage (a) and current (b) in discharge process ( $R=52.4 \text{ k}\Omega$ )

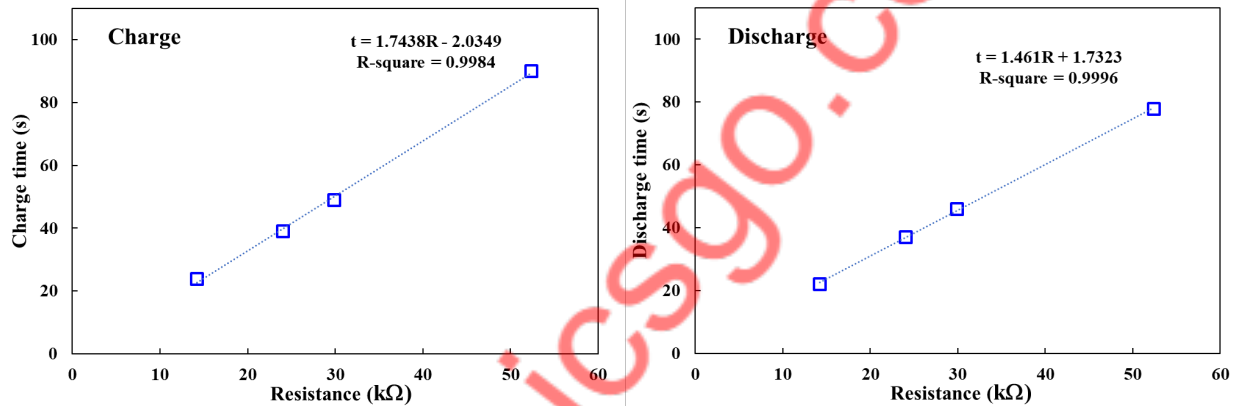


Figure 12. Relation of  $R$  and  $t_s$  in charge (a) and discharge (b) processes

To explore the relation between a capacitor's charging or discharging with electric resistance, the critical time to charge the capacitor to 9.6 V or discharge to 0.3 V are identified in the measured data set. They are 24 s, 39 s, 49 s, 50 s for the four resistors during charging, and 22 s, 37 s, 46 s and 78 s during discharging. I drew the time – resistance plots as shown in Figure 12. The time – resistance relationship can be easily fitted by using a linear equation with a very high R-square, which means the relationship can be accurately described by the fitted equation.

## 8.2 Sample Calculation

I randomly select the measured voltage data in the group of charge-52.4 kΩ to demonstrate how the average values and standard deviation listed in Tables 2-5 were calculated.

The 3 voltage values at the 18th second are 5.32 V, 5.50 V and 5.47 V. Thus, their average value is  $\bar{x} = (5.32+5.50+5.47)/3=5.43 \text{ (V)}$ .

By evaluating the standard deviation, we can learn the variation of the data. The standard deviation is defined as

$$\sigma = \sqrt{\frac{\sum_i^N (x_i - \bar{x})^2}{N-1}} \quad (\text{Eq-10})$$

where  $\sigma$  is sample standard deviation,  $N$  is the number of data,  $x_i$  is the measured values of the sample, and  $\bar{x}$  is the mean value of the samples.

Therefore, the standard deviation for the above 3 voltage data is

$$\sigma = \sqrt{\frac{[(5.32-5.43)^2 + (5.50-5.43)^2 + (5.47-5.43)^2]}{3-1}} \approx 0.10 \text{ (V)} \quad (\text{Eq-11})$$

### 8.3 Data Analysis

From Figures 8-11 above, it can be found that the experimental results share a similar trend with the theoretical curves, which validates my experimental method. Moreover, the experimental curves are highly close to each other, implying good repeatability of the experiments. I also found that the capacitor voltage could reach 9.8 V after charging, which indicates battery drain is negligible. There are three kinds of errors: initial error, difference between theoretical curve and experimental results, and time difference between charging and discharging process.

### 8.4 Errors and limitations of the experiment

#### 8.4.1 Initial error

A sudden change can be observed at the beginning of most experiments. The standard deviation of the three experiments is larger at the beginning, and becomes smaller as the time increases. Initial error is mainly induced by human error, because turning on or off the switches leaves data recording in a rush. Furthermore, response time of circuit components and their insufficient synchronization also can be the causes of initial error.

Solution: The error occurs mainly because of the switch. To decrease the uncertainty of operation, I could change the switch to a button, so I only need to press the button down instead of toggling the switches. In this case, the turning on or off procedures will not leave data recording in a rush. On the other hand, I will select circuit components with high sensitivity and synchronization.

#### 8.4.2 Difference between experimental results and theoretical curves

An obvious difference exists between theoretical and experimental curves. According to Eq - 6 to Eq - 9, if the value of resistance increases, the  $I - t$  and  $V - t$  graphs will shift to the right. In practice, besides resistors, some other components also have resistance, such as LEDs and even wire.

Solution: Next time, I will try to measure the total value of resistance in the circuit so that the precision will be improved.

#### 8.4.3 Difference between charge and discharge time

I also found that the discharging process is faster than the charging process. It can be understood by comparing the final voltage of charging and the initial voltage of discharging. From the plots presented above, the voltage on the capacitor achieves 9.8 V at the end of the charging process, while the initial voltage of discharging process is always lower than 9.8 V. That means

power consumption occurs during the interval between charging and discharging experiments. This energy consumption was inevitable, because the experimental condition was not perfect.

Solution: Since it is a systematic error, it will be difficult to avoid. To improve, I will change the circuit component, for example, voltmeter of a higher quality should be used to reduce the power consumption which will affect the data recording in the experiment.

## **9. Conclusion**

Referencing to the analysis of the experimental data, my research question “What is the current vs. time graph and potential difference vs. time graph when a capacitor is charging and discharging” has been answered. From Graphs 17 to 24, it has been proved that the charging and discharging processes of capacitors follow the Kirchhoff's Law, and the voltage and current change in an exponential form. For the charging process, as time moves on, voltage will increase and current will decrease. For the discharging process, as time increases, both voltage and current will decrease. From Graph 25, it can be concluded that the charge and discharge time increases linearly with the increase of electric resistance. If the resistance in a circuit is close to zero, the charge time will be almost zero; if the resistance is infinite large, which is identical to a break circuit, the capacitor will be never charge/discharge.

Besides some limitations and errors that have been discussed in 8.4, the investigation also has significant strengths and gained success.

### **9.1 Strengths**

- 1) There is a lot of data collected. The sufficient data allows me to generate my plots for detailed analysis.
- 2) The three groups of data for each resistor are close to each other. This indicates that the experiment was set up successfully. The relationship between the independent variables and the dependent variables are reasonable.
- 3) The experiment was repeated several times. This reduced the random error and improved the precision of my data processing.

### **9.2 Extension**

The investigation informed me that charge and discharge of capacitors will be saturated. The increase in voltage will not become noticeable with time and there is a limit to the voltage. That is why my solar car could not travel further even though it was exposed to more time. On the other hand, the charge time will a decrease with decrease of resistance. This reminded me that people need to seek the capacitor with a huge capacitance and small resistance to make electrical devices work longer and be charged faster.

## Bibliography

- “All You Need to Know about Capacitors.” *Solectroshop.com*, 30 May 2020, solectroshop.com/en/blog/all-you-need-to-know-about-capacitors-n17.
- By. “History of the Capacitor – the Pioneering Years.” *Hackaday*, 12 July 2016, hackaday.com/2016/07/12/history-of-the-capacitor-the-pioneering-years/.
- Codepixer. “JAIVIC ELECTROMECH ENGINEERING PVT. LTD.” *Jaivicapacitor.com*, 2022, www.jaivicapacitor.com/HISTORY-OF-POWER-CAPACITORS.php.
- Five Years Out. “About Static4.Arrow.com/- /Media/Arrow/Files/Pdf/A/Arr\_broschuere\_automotive\_1701\_screen.pdf - Google Search.” [Http://Www.arrow.com/](http://Www.arrow.com/).www.google.com/search?q=About+static4.arrow.com/-/media/arrow/files/pdf/a/arr\_broschuere\_automotive\_1701\_screen.pdf&tbm=ilp&ilps=A DNMCi2kxxSWEj2z2dEHHU5pv3Nr4CXv1Q&biw=981&bih=653&dpr=2.
- Keithley, Joseph F. *The Story of Electrical and Magnetic Measurements : From 500 BC to the 1940s*. New York, Ieee Instrument & Measurement Society, 1999.
- Kumar, Nithish, et al. *Design of Battery Management System for Low Voltage Stand Alone System*. 2021.
- Tsokos, K. A. *Physics for the IB Diploma*. 1998. 6th ed., Cambridge, Cambridge University Press, 2014.

## Appendix

Complete raw data.

Table 1. Experimental data of 14.21 k $\Omega$  resistor

Time (s)	Experiment-1				Experiment-2				Experiment-3			
	Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I ( $\mu$ A)	V (V)	I ( $\mu$ A)	V (V)	I ( $\mu$ A)	V (V)	I ( $\mu$ A)	V (V)	I ( $\mu$ A)	V (V)	I ( $\mu$ A)	V (V)
0	612.0	0.10	589.0	9.24	603.0	0.03	625.0	9.20	594.0	0.05	573.0	9.31
2	598.0	1.48	449.0	6.54	571.0	0.60	476.0	7.71	505.0	2.16	487.0	6.94
4	379.0	4.20	345.0	5.19	394.0	4.10	385.0	5.63	387.0	4.21	355.0	5.09
6	281.9	5.69	246.7	3.55	292.4	5.62	289.9	4.15	268.3	5.97	253.3	3.48
8	202.6	6.76	164.5	2.44	209.7	6.71	202.2	3.07	180.2	7.16	168.1	2.51
10	146.9	7.71	118.1	1.82	151.8	7.68	144.6	2.12	139.2	7.68	121.0	1.78
12	107.7	8.35	80.0	1.18	110.8	8.23	91.8	1.47	101.6	8.32	87.3	1.24
14	79.7	8.72	62.0	0.89	86.8	8.63	75.3	1.10	78.8	8.75	63.4	0.94
16	56.5	9.05	45.4	0.68	64.6	8.99	58.4	0.89	52.8	9.06	46.3	0.66
18	45.4	9.24	35.5	0.51	48.7	9.20	40.2	0.59	42.3	9.23	36.2	0.54
20	35.5	9.44	28.8	0.37	35.4	9.42	31.5	0.45	32.9	9.39	29.4	0.39
22	28.8	9.53	20.9	0.27	29.8	9.58	24.2	0.33	28.0	9.48	20.7	0.28
24	23.6	9.60	15.3	0.21	23.6	9.62	18.2	0.24	21.6	9.55	15.6	0.20
26	18.6	9.67	11.7	0.16	19.9	9.66	13.8	0.19	17.5	9.61	11.8	0.16
28	15.5	9.71	9.0	0.12	16.8	9.70	11.1	0.15	14.0	9.65	9.2	0.12



30	13.7	9.73	7.1	0.10	13.4	9.74	8.6	0.11	12.2	9.68	7.2	0.09
32	12.1	9.76	5.9	0.08	11.9	9.77	6.5	0.09	10.3	9.71	5.9	0.08
34	10.8	9.78	4.5	0.06	10.9	9.79	5.2	0.07	9.3	9.72	4.8	0.06
36	9.9	9.79	3.5	0.05	9.3	9.80	4.2	0.06	8.3	9.74	3.6	0.06

Table 2. Experimental data of 24.0 kΩ resistor

Time (s)	Experiment-1				Experiment-2				Experiment-3			
	Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)
0	379.4	0.06	381.0	9.48	383.9	0.51	371.5	9.46	393.1	0.07	381.8	9.50
1	347.8	1.45	364.3	9.46	323.1	2.18	355.5	9.46	330.5	1.77	365.1	9.47
3	293.5	2.89	348.8	9.14	251.2	3.44	340.5	8.79	267.7	3.45	349.6	8.63
5	228.8	4.06	296.8	7.56	213.2	4.75	287.2	6.94	209.1	4.76	270.9	6.83
7	186.7	5.24	238.5	5.99	174.2	5.59	233.0	5.77	170.9	5.61	220.1	5.43
9	159.1	5.99	194.2	4.77	142.8	6.45	189.7	4.60	140.1	6.45	186.8	4.53
11	125.7	6.75	152.1	3.81	117.4	7.00	154.8	3.84	119.8	7.10	152.4	3.61
13	103.6	7.36	129.4	3.19	100.6	7.45	126.5	3.22	98.8	7.46	139.6	3.02
15	82.6	7.83	105.9	2.55	80.2	7.90	103.5	2.57	81.8	7.92	98.0	2.42
17	71.2	8.14	90.3	2.14	66.6	8.24	84.9	2.07	65.4	8.29	80.4	1.95
19	61.5	8.44	71.3	1.80	55.6	8.53	69.8	1.73	56.5	8.53	68.8	1.64
21	51.0	8.67	58.8	1.45	49.1	8.77	57.5	1.40	48.2	8.78	54.5	1.32
23	43.9	8.89	48.4	1.17	47.2	8.93	48.8	1.18	41.9	8.91	46.3	1.12
25	38.3	8.99	42.4	0.95	34.8	9.07	41.5	0.96	35.3	9.08	39.4	0.90
27	31.4	9.17	36.4	0.84	28.6	9.23	34.4	0.78	29.9	9.21	33.9	0.77
29	27.6	9.24	30.2	0.68	25.2	9.30	28.5	0.66	24.7	9.33	28.1	0.65
31	23.0	9.34	24.2	0.56	21.6	9.39	23.7	0.54	21.9	9.39	23.4	0.53
33	19.9	9.43	20.2	0.47	18.7	9.47	19.8	0.46	18.9	9.47	18.8	0.44
35	17.3	9.47	16.8	0.39	15.9	9.54	16.5	0.38	16.4	9.52	15.8	0.36
37	14.8	9.55	13.6	0.32	14.3	9.57	13.4	0.31	14.8	9.57	13.2	0.31
39	13.4	9.57	11.9	0.27	12.7	9.61	11.7	0.27	12.7	9.62	11.1	0.26
41	11.9	9.63	10.0	0.23	11.3	9.65	9.8	0.23	11.3	9.65	9.4	0.22
43	11.0	9.65	8.5	0.19	10.2	9.67	8.4	0.19	10.2	9.68	8.0	0.18
45	9.7	9.66	7.2	0.17	9.3	9.69	6.9	0.16	9.2	9.70	6.8	0.16
47	8.9	9.68	6.0	0.14	8.3	9.71	6.0	0.14	8.3	9.72	5.8	0.14
49	8.1	9.71	5.2	0.12	7.8	9.73	5.2	0.12	7.9	9.73	5.1	0.12
51	7.6	9.72	4.5	0.10	7.3	9.74			7.2	9.75	4.5	0.11
53	7.2	9.73			6.8	9.75			6.8	9.76		
55	6.8	9.73			6.6	9.76			6.4	9.76		
57	6.4	9.75			6.2	9.76			6.0	9.77		
59	6.2	9.75			5.9	9.77			5.8	9.78		
61	5.9	9.75			5.6	9.78						
63	5.8	9.76										

Table 3. Experimental data of 29.9 kΩ resistor

Time (s)	Experiment-1				Experiment-2				Experiment-3			
	Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)
0	314.1	0.04	300.6	9.45	312.6	0.03	301.3	9.49	318.1	0.03	309.1	9.54
2	282.9	1.13	290.2	8.72	291.5	0.94	290.9	8.87	267.4	0.94	279.3	8.62
4	246.8	2.36	252.9	7.78	245.8	2.48	245.0	7.46	233.6	2.48	235.6	7.14
6	208.8	3.40	206.6	6.46	208.0	3.50	207.1	6.43	197.8	3.50	199.2	6.16
8	171.4	4.67	175.1	5.38	178.6	4.56	175.5	5.35	162.6	4.56	168.9	5.13
10	145.8	5.38	148.5	4.49	160.1	5.10	144.1	4.46	142.9	5.10	143.3	4.44
12	124.2	6.11	122.2	3.75	127.9	6.03	122.5	3.73	121.8	6.03	121.8	3.71
14	106.1	6.72	110.9	3.37	105.9	6.69	104.2	3.12	104.1	6.69	103.6	3.10
16	93.6	7.04	94.4	2.82	90.7	7.07	85.9	2.61	89.2	7.07	85.3	2.69

18	77.8	7.49	78.0	2.36	80.2	7.44	75.7	2.35	74.2	7.44	75.3	2.26
20	66.9	7.87	66.6	2.05	69.0	7.82	64.6	1.97	65.7	7.82	64.3	1.96
22	58.9	8.18	58.2	1.73	60.9	8.14	56.5	1.66	59.3	8.14	57.1	1.71
24	53.6	8.34	51.0	1.50	52.1	8.41	51.1	1.50	49.7	8.41	49.2	1.44
26	46.3	8.57	43.7	1.27	46.4	8.59	43.7	1.26	44.3	8.59	42.2	1.21
28	39.0	8.77	37.5	1.07	39.2	8.74	36.4	1.07	38.4	8.74	36.2	1.02
30	33.9	8.90	32.1	0.94	35.1	8.88	32.2	0.93	33.5	8.88	31.1	0.92
32	29.6	9.05	27.6	0.79	30.6	9.03	27.6	0.81	29.2	9.03	27.5	0.81
34	25.9	9.16	23.1	0.67	26.8	9.13	23.8	0.69	24.9	9.13	23.0	0.67
36	23.4	9.29	20.5	0.61	23.5	9.24	20.6	0.58	22.5	9.24	19.8	0.56
38	19.6	9.33	17.7	0.52	20.8	9.32	17.2	0.51	19.9	9.32	16.6	0.48
40	17.8	9.39	15.3	0.46	18.4	9.39	14.9	0.44	17.8	9.39	14.8	0.42
42	15.9	9.46	13.3	0.39	16.8	9.45	13.3	0.39	15.3	9.45	13.2	0.39
44	14.2	9.51	11.5	0.33	14.7	9.50	11.5	0.33	13.7	9.50	10.8	0.32
46	13.0	9.54	10.0	0.30	13.2	9.54	10.0	0.29	12.6	9.54	9.7	0.28
48	11.8	9.58	8.8	0.25	11.9	9.57	8.7	0.25	11.4	9.57	8.5	0.24
50	10.7	9.61	7.6	0.22	10.8	9.61	7.4	0.22	10.2	9.61	7.4	0.22
52	9.8	9.64	6.9	0.20	9.9	9.64	6.5	0.19	9.5	9.64	6.3	0.18
54	9.0	9.66	6.0	0.18	9.1	9.67	5.7	0.17	8.7	9.67	5.7	0.17
56	8.3	9.68	5.2	0.15	8.8	9.69	5.0	0.15	1.9	9.69	5.0	0.15
58	7.6	9.70			7.7	9.71	4.4	0.13	7.4	9.71		
60	7.1	9.72			7.2	9.72			7.0	9.72		
62	6.6	9.74			6.8	9.73			6.5	9.73		
64	6.3	9.74			6.4	9.74			6.2	9.74		
66	6.0	9.75			6.1	9.75			5.8	9.75		
68	5.7	9.76			5.8	9.76			5.6	9.76		
70	5.5	9.77			5.5	9.77						

Table 4. Experimental data of 52.4 kΩ resistor

Time (s)	Experiment-1				Experiment-2				Experiment-3			
	Charge		Discharge		Charge		Discharge		Charge		Discharge	
	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)	I (μA)	V (V)
0	183.9	0.02	164.8	9.24	181.9	0.03	176.7	9.47	179.5	0.03	175.8	9.54
2	179.6	0.25	158.0	8.59	171.2	0.75	166.3	8.92	175.9	0.48	162.2	8.64
4	162.7	1.28	146.0	7.86	158.4	1.32	150.7	8.18	159.4	1.44	150.0	8.10
6	147.7	1.99	132.4	7.06	143.8	2.20	136.8	7.35	144.8	2.37	136.2	7.28
8	131.7	2.78	118.3	6.35	130.7	2.97	124.3	6.61	131.6	2.92	123.8	6.69
10	122.1	3.35	107.3	5.73	118.9	3.53	113.0	6.07	119.7	3.61	112.6	6.03
12	111.2	4.00	97.7	5.26	106.3	4.16	102.8	5.47	109.0	4.11	102.4	5.43
14	103.2	4.35	91.5	4.75	100.5	4.61	94.5	4.93	99.4	4.68	95.9	5.00
16	96.8	4.90	83.3	4.28	92.5	5.02	87.7	4.54	91.5	5.19	87.4	4.51
18	88.3	5.32	77.4	4.03	84.4	5.50	79.9	4.18	85.1	5.47	79.6	4.06
20	79.2	5.81	70.6	3.64	77.1	5.92	72.8	3.78	77.7	5.89	72.5	3.75
22	72.4	6.13	63.1	3.28	70.4	6.23	66.4	3.41	71.0	6.20	64.9	3.38
24	66.2	6.48	57.6	2.97	64.4	6.57	60.5	3.14	65.0	6.55	60.3	3.12
26	60.6	6.84	52.6	2.74	57.9	6.89	55.2	2.84	59.5	6.86	54.0	2.82
28	54.6	7.09	48.0	2.47	54.0	7.12	49.5	2.57	53.6	7.14	50.2	2.60
30	50.9	7.20	44.6	2.28	49.5	7.37	46.0	2.42	49.1	7.35	45.9	2.40
32	46.7	7.49	40.1	2.11	44.6	7.61	42.0	2.19	45.1	7.58	41.9	2.17
34	43.6	7.67	36.6	1.91	41.7	7.79	38.4	2.06	42.1	7.71	38.3	2.00
36	40.1	7.83	33.5	1.73	38.3	7.93	35.1	1.79	38.7	7.91	35.0	1.81
38	36.2	8.01	30.1	1.57	34.6	8.11	31.5	1.65	35.1	8.09	32.0	1.68
40	33.4	8.19	28.8	1.48	32.4	8.24	29.3	1.53	32.3	8.23	29.8	1.55
42	30.8	8.31	26.1	1.34	29.8	8.35	26.8	1.39	30.2	8.37	26.3	1.38
44	28.4	8.44	23.5	1.22	27.5	8.49	24.6	1.28	27.9	8.47	24.1	1.25
46	26.2	8.57	21.9	1.13	25.4	8.68	22.5	1.16	25.8	8.60	22.8	1.18
48	23.8	8.68	19.7	1.02	23.5	8.70	20.6	1.06	23.8	8.69	20.9	1.03
50	22.1	8.75	18.1	0.99	21.7	8.81	18.6	0.96	22.0	8.79	19.2	0.99

52	21.1	8.84	16.6	0.86	21.1	8.88	17.3	0.91	20.4	8.88	17.6	0.90
54	19.2	8.93	15.5	0.80	18.1	8.96	15.9	0.82	19.0	8.95	15.9	0.83
56	17.9	8.99	14.8	0.73	17.1	9.04	14.6	0.75	17.3	9.03	14.5	0.76
58	16.8	9.06	12.9	0.67	16.1	9.10	13.4	0.69	16.2	9.10	13.1	0.69
60	15.7	9.11	11.8	0.62	15.0	9.15	12.3	0.63	15.3	9.14	12.1	0.63
62	14.5	9.16	10.9	0.56	14.0	9.21	11.3	0.58	14.2	9.19	11.9	0.59
64	13.5	9.22	10.0	0.52	13.1	9.26	10.2	0.53	13.3	9.25	10.3	0.54
66	12.7	9.27	9.4	0.48	12.1	9.30	9.6	0.50	12.5	9.28	9.6	0.50
68	11.9	9.32	8.5	0.44	11.5	9.34	8.8	0.46	11.5	9.33	8.8	0.46
70	11.1	9.35	7.8	0.40	10.6	9.37	8.1	0.42	11.0	9.36	8.1	0.42
72	10.6	9.38	7.2	0.38	10.2	9.41	7.5	0.35	10.3	9.40	7.5	0.39
74	9.9	9.41	6.7	0.35	9.6	9.44	6.9	0.36	9.7	9.43	6.9	0.36
76	9.4	9.45	6.1	0.32	9.0	9.47	6.3	0.33	9.2	9.46	6.3	0.33
78	8.8	9.47	5.7	0.30	8.4	9.50	5.8	0.30	8.6	9.49	5.8	0.30
80	8.5	9.49	5.3	0.28	8.1	9.52	5.4	0.28	8.2	9.51	5.9	0.28
82	8.0	9.52			7.6	9.54	5.0	0.26	7.8	9.53	4.9	0.26
84	7.6	9.54			7.3	9.56	4.6	0.24	7.4	9.55	4.6	0.24
86	7.2	9.56			7.0	9.57	4.3	0.23	7.1	9.57	4.9	0.23
88	6.9	9.58			6.6	9.59			6.8	9.55		
90	6.6	9.59			6.4	9.60			6.4	9.60		
92	6.3	9.60			6.1	9.62			6.2	9.61		
94	6.0	9.62			5.9	9.63			5.9	9.63		
96	5.8	9.63			5.6	9.65			5.7	9.64		
98	5.7	9.64			5.4	9.66			5.5	9.65		
100	5.5	9.64			5.2	9.66			5.3	9.66		

Complete set of graphs.

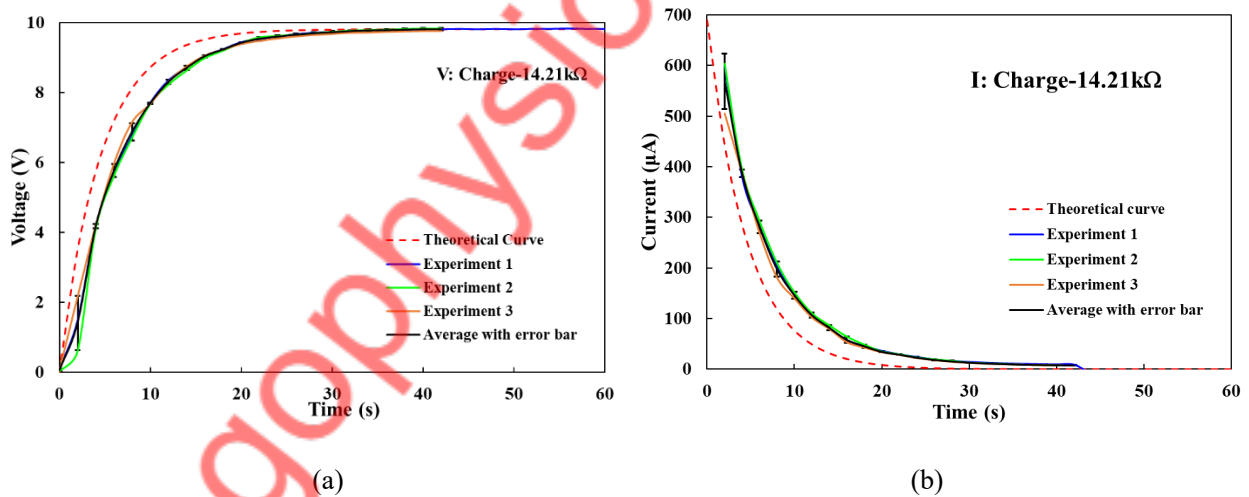
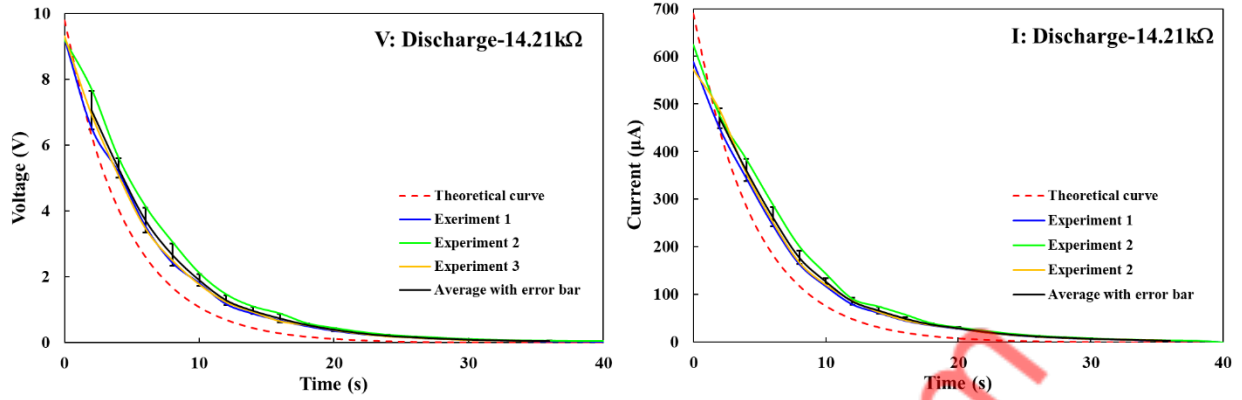
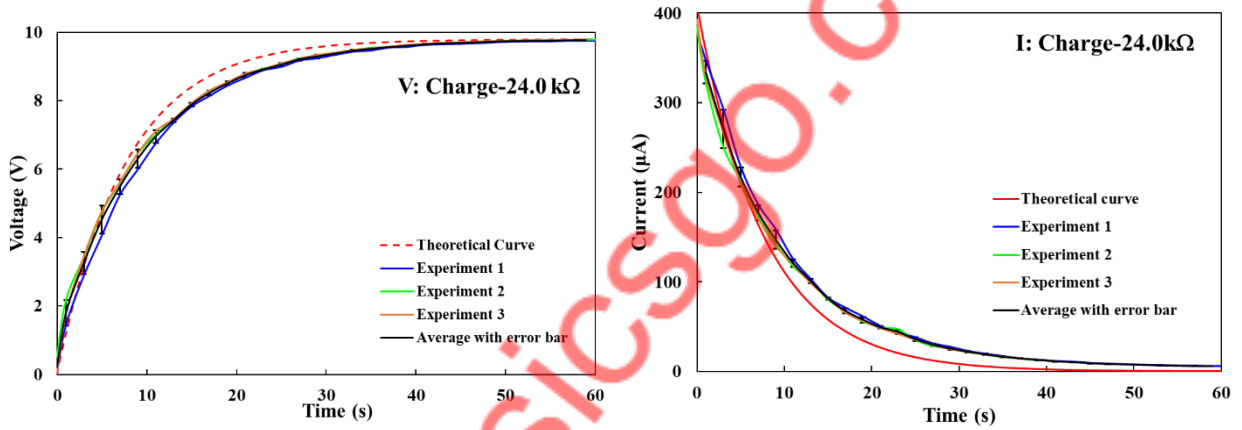


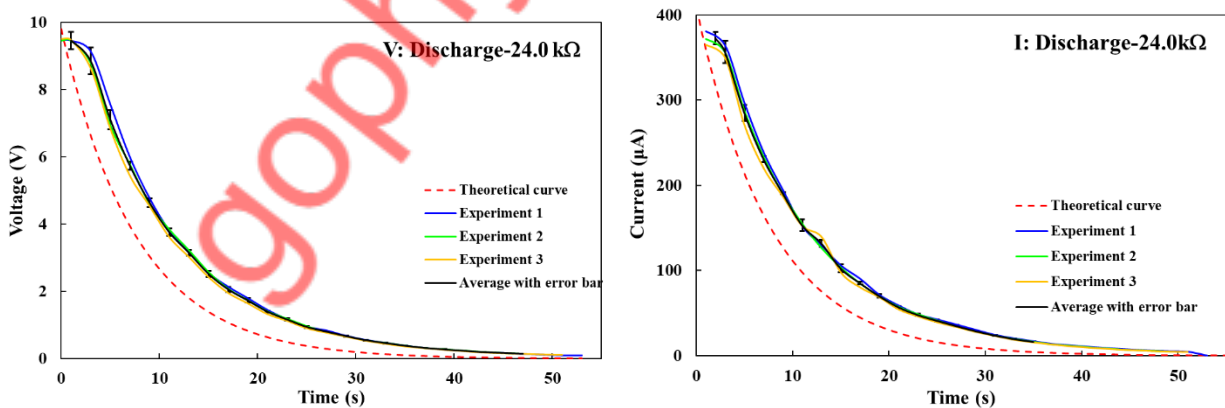
Figure 1. Voltage (a) and current (b) in charge process ( $R=14.21\text{ k}\Omega$ )



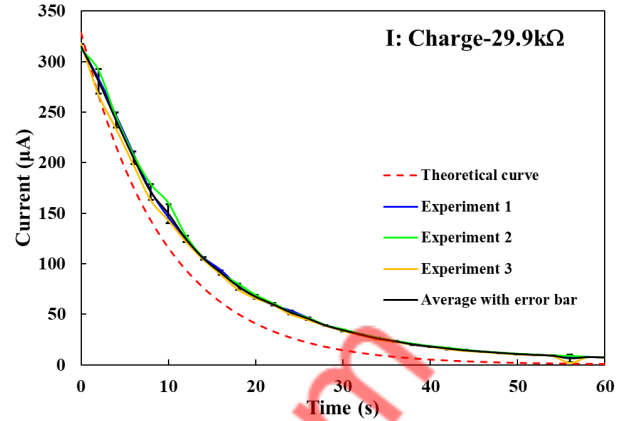
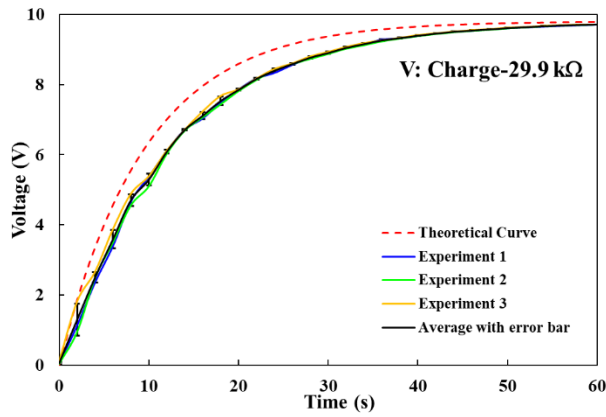
(a) (b)  
Figure 2. Voltage (a) and current (b) in discharge process ( $R=14.21\text{ k}\Omega$ )



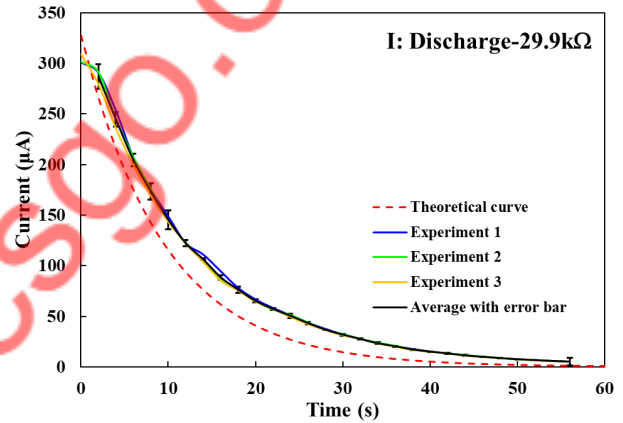
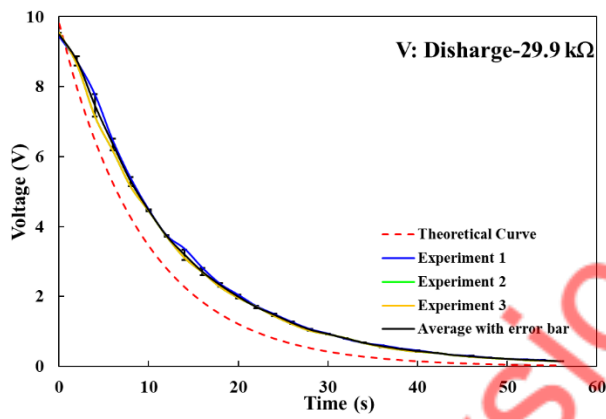
(a) (b)  
Figure 3. Voltage (a) and current (b) in charge process ( $R=24.0\text{ k}\Omega$ )



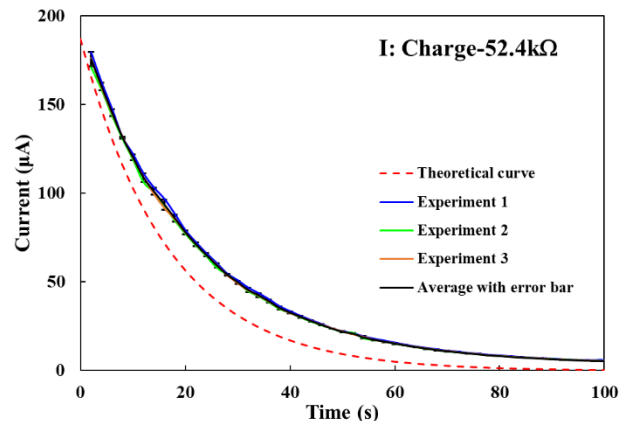
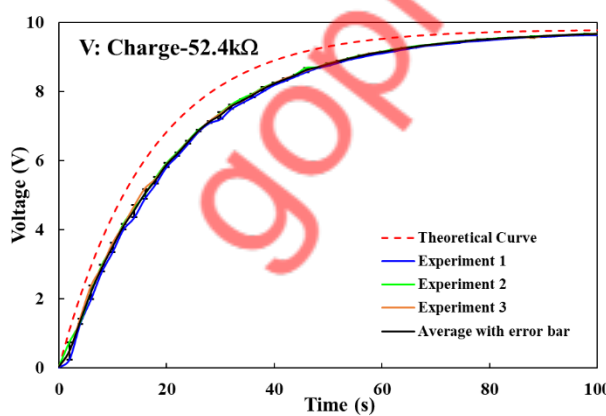
(a) (b)  
Figure 4. Voltage (a) and current (b) in discharge process ( $R=24.0\text{ k}\Omega$ )



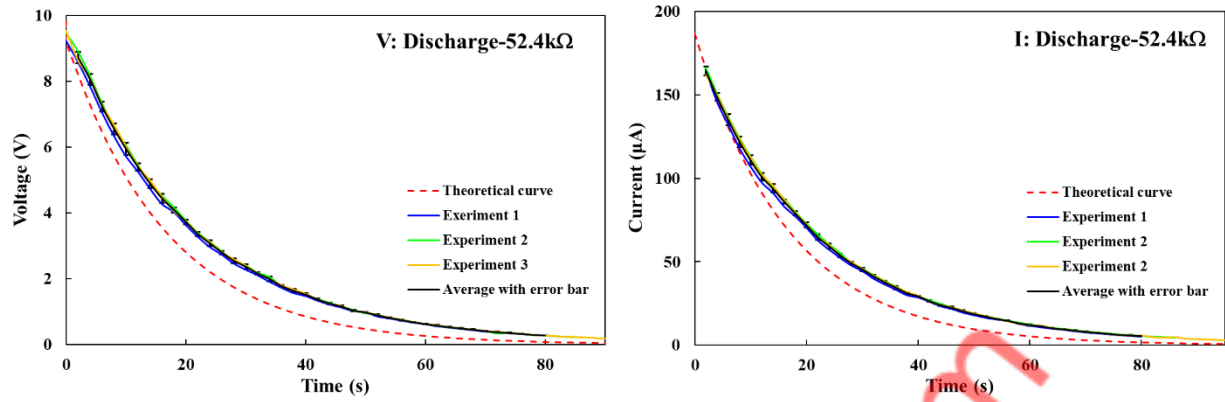
(a) (b)  
Figure 5. Voltage (a) and current (b) in charge process ( $R=29.9\text{ k}\Omega$ )



(a) (b)  
Figure 6. Voltage (a) and current (b) in discharge process ( $R=29.9\text{ k}\Omega$ )



(a) (b)  
Figure 7. Voltage (a) and current (b) in charge process ( $R=52.4\text{ k}\Omega$ )



(a) (b)  
 Figure 8. Voltage (a) and current (b) in discharge process ( $R=52.4 \text{ k}\Omega$ )

gophysicsgo.com