Research Question: What is the relationship between the concentration of the same optically active molecule in water and the rotated angle of light?

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1. Introduction

Light, in my mind, is always a mysterious “thing”, having constant speed and unchangeable transmitting direction in one medium. Changing mediums and huge gravity can change the direction of light macroscopically. It makes me wonder if I can change light’s direction microscopically, which means changing the direction of the electromagnetic wave’s oscillation.

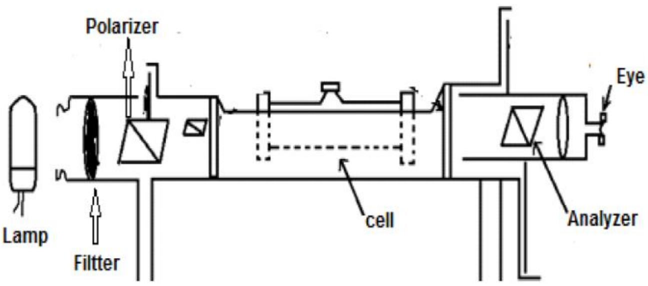
Jean-Baptiste Biot, a French physicist, discovered that compounds like sugar could rotate polarized light. It was not until 1874 that Dutch chemist Jacobus Henricus Van't Hoff proposed that carbon’s tetrahedral structure was responsible for the optical activity, the ability to rotate plane-polarized light, of many organic compounds.

Therefore, I found a way to rotate the oscillation direction of light waves: emit light through a tetrahedral-structure-solution. In order to visualize the rotation, I used one monitor and 2 polarizers to show the rotation. Since concentration of molecules is one of the factors of the rotated angle of light, I decided to explore the relationship between the rotated angle and the concentration of the same optically active molecule in water.

The above figure shows how the light rotates through a solution.

1.1 Background Information

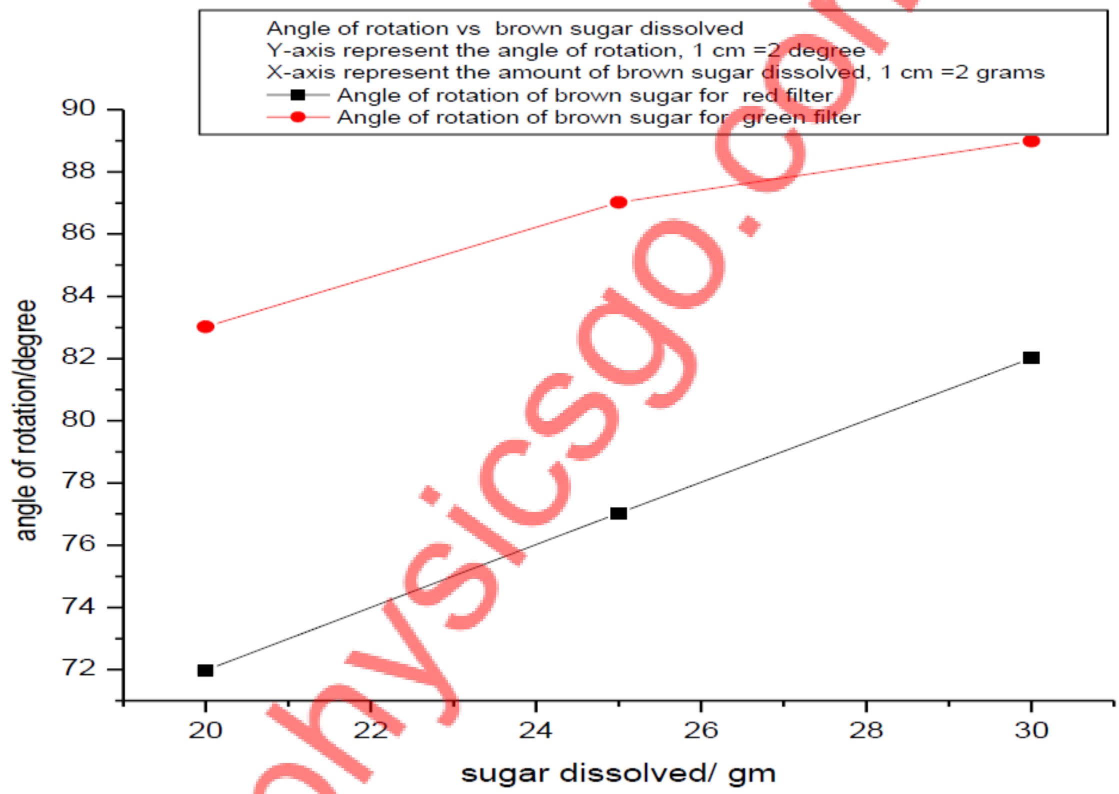
One typical example of carbon’s tetrahedral structure is sugar. Figure 1 shows one past experiment in 2019 that studies the relationship between concentration and solution and the rotated angle. An equation is concluded, showing the relationship between the rotational angle and factors that affect it:



$$[a]_T^\lambda = \frac{\emptyset}{L \times C}$$

(Figure 1)

$[a]_T^\lambda$ refers to the property of the dissolved compound in the liquid. λ is the wavelength and T is temperature in ℃. $\emptyset$ is the rotational angle. Its unit deg $mL\ g^{-1}dm^{-1}$, is always be shortened as deg. L is the length that the light travels in decimeters, and C is the concentration of the solution in g/ml. Since I will only use a sucrose-water solution, $[a]_T^\lambda$ is a constant. Therefore, figure 2 shows that when the controlled variable is L, the rotated angle will increase when the concentration of the solution increases.



(Figure 2)

2. Methodology

2.1 Table 1 shows the controlled, independent, and dependent variables; the units, why to control them and how to control.

	the variable	unit	Why to control	How to control
controlled variable	depth of the solution	cm	the depth of the solution affects the path length of the light. The longer the path length, the greater the rotated angle. Thus, in order to find the relationship between concentration and angle, depth should be constant	When all the solids dissolve, pour out extra solution until the solution is 100ml
	temperature	℃	temperature will change the solubility of the solution. Sugar's solubility is higher in higher temperature, so lowering the temperature may reduce the concentration of the solution	keep the AC temperature constant, and keep the door and windows close
	type of solution	N/A	different structures of different molecules have different ability to rotate light. Changing the dissolved substance changes this ability, making the experiment inaccurate	Use one type of sucrose from one brand
independent variable	concentration	g/100ml	To get one variable in the investigation	change the mass of dissolved sucrose
dependent variable	rotated angle	degree	/	/

(table 1)

According to the equation: $\emptyset = [a]_T^\lambda \times L \times C$, only C can be flexible so that I can find the relationship.

2.2 Apparatus

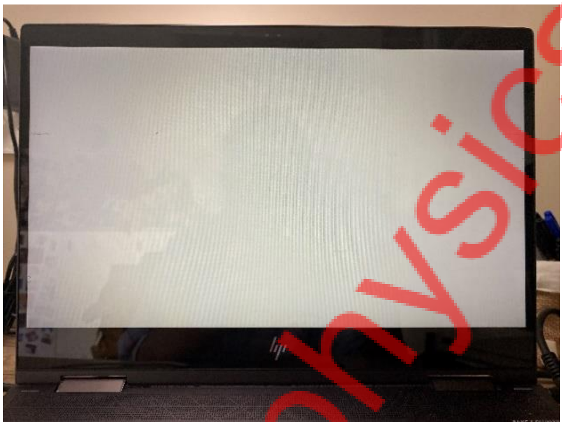
1) Beaker (figure 3)

(figure 3)

2)Two polarizers with degrees (figure 4)

(Figure 4)

3) Monitor (figure 5)



(Figure 5)

4) Ruler (figure 6)

(Figure 6)

5) Lab Quest (figure 7)

(Figure 7)

6) Light sensor (figure 8)

(Figure 8)

7) Cylinder (Figure 9)

(Figure 9)

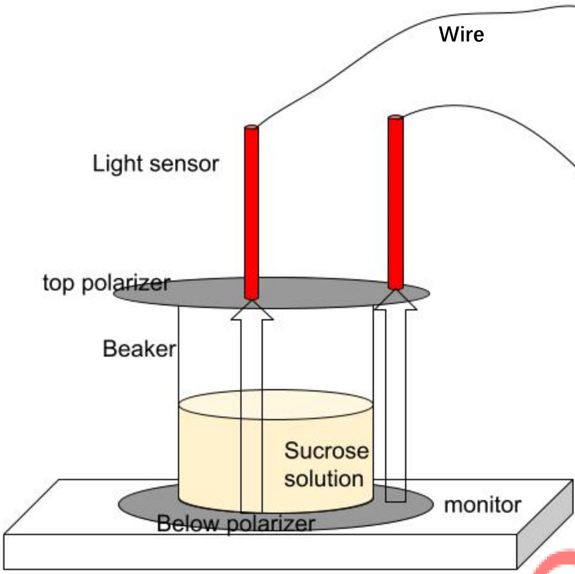
2.3 Safety consideration

Since the experiment doesn't contain dangerous actions, the whole investigation is safe.

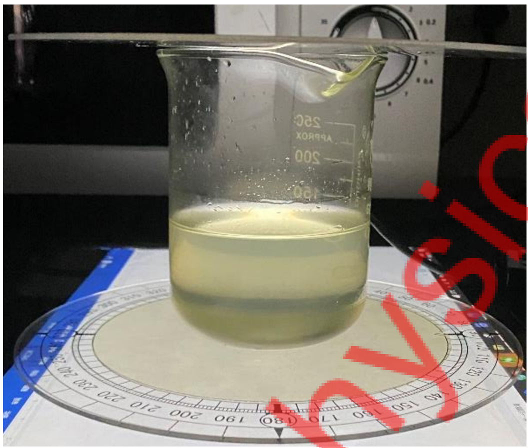
2.4 Procedures

1) Find a dark room with air conditioner
2) Turn off the lights and turn on the air conditioner to 26℃
3) Connect the light sensor with Lab quest
4) Turn on the monitor and adjust it to full spectrum (white screen)
5) Place one polarizer on the monitor
6) Measure 100ml of water using cylinder
7) Dissolve 25g sucrose in 100ml pure water in the beaker
8) Place the beaker over the first layer of polarizer
9) Place another polarizer on top of the beaker perpendicularly to the polarizer below (put 90℃ aligned with 180℃)
   *At this point, there is zero intensity of light crossing only two perpendicular polarizers. The two polarizers block all the electromagnetic waves from all directions. But light intensity observed from the light sensor is above zero crossing two polarizers and the sugar solution.
10) Rotate the top polarizer counter-clockwise until the light sensor detect the minimum value of light intensity through sugar solution
11) Align the thin edge of the ruler vertically with the polarizer rotated through 180 degrees above. Read values of angles of the polarizer below where the straightedge is aligned.
12) Find the difference between the reading value and 90 degrees on the bottom polarizer
13) Record this difference as rotated angle three times, and take the average.
14) Repeat steps 6-13 for another four times except changing the amount of sugar to 50g, 75g, 100g, and 125g (knowing sugar's solubility is 203.9g/100g water in room temperature

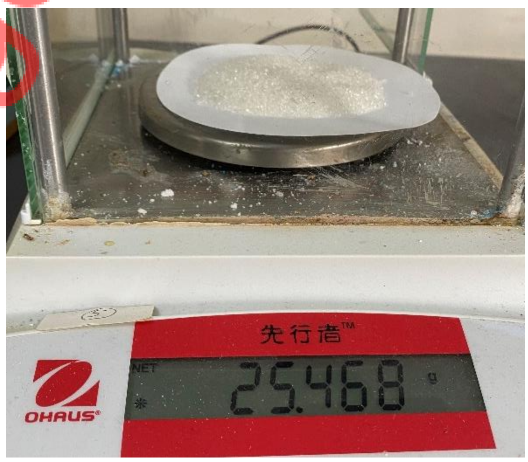
Figure 10, 10.1, and 10.2 shows the experiment setting.



(Figure 10)



(Figure 10.1)



(Figure 10.2)

2.5 Assumptions

Firstly, I need to put rotated angle and concentration to different sides of equation:

$$\emptyset = [a]_T^{\lambda} \times L \times C$$

$[a]_T^{\lambda}$ will be a constant because of the unchanging chemical and biological property of sucrose, and L will be a constant since I will make it as controlled variable in the experiment. $[a]_T^{\lambda} \times L$ =m (a constant). Therefore, the equation can be concluded as $\emptyset = mC$. My research question is to illustrate the relationship between concentration of the same optically active molecule in water and the rotated angle of light with a straight line and quantify the exact value of m.

3. Data interpretation

3.1 Raw data

Table 2 shows the mass of sucrose dissolved in 100g water per trial. Every three trials is one group, which uses the same mass of sucrose. It also contains the measured angle of the rotation of the polarizer.

Groups	trials	M_s: mass of sucrose (g, $\pm 0.001g$)	M_w: mass of water (g, $\pm 12.5g$)	R: measured rotated angle (°, $\pm 0.5°$)
A	1	25. 000	100. 0	5. 0
	2		100. 0	4. 0
	3		100. 0	6. 0
B	4	50. 000	100. 0	18. 0
	5		100. 0	20. 0
	6		100. 0	22. 0
C	7	75. 000	100. 0	31. 0
	8		100. 0	32. 0
	9		100. 0	31. 0
D	10	100. 000	100. 0	29. 0
	11		100. 0	30. 0
	12		100. 0	29. 0
E	13	125. 000	100. 0	34. 0
	14		100. 0	35. 0
	15		100. 0	36. 0

(Table 2)

Every three continuous trials form a group has the same concentration of sucrose solution.

3.2 Sample calculation

I use group A (trials 1, 2, 3) to do the sample calculation. The same principle can be applied to the later 4 groups. The minimum index value of the polarizers is 1°, so the uncertainty of every measurement is 0.5°. To measure the average rotated angle of the first group of solution (trials 1-3):

$$\overline{R_a} = \frac{R_1 + R_2 + R_3}{3} = \frac{5.0 + 4.0 + 6.0}{3} = 5.0$$

$$R_1 + R_2 + R_3 = S_a = 15$$

$$\Delta S_a = \Delta R_1 + \Delta R_2 + \Delta R_3 = 0.5 + 0.5 + 0.5 = 1.5$$

$$\frac{\Delta R_a}{R_a} = \frac{\Delta S_a}{S_a} + 0$$

$$\Delta R_a = \frac{\Delta S_a}{S_a} \times R_a = \frac{1.5}{15} \times 5.0 = 0.5°$$

$$R_a = \overline{R_a} + \Delta R_a = 5.0 \pm 0.5°$$

*The constant 3's percentage uncertainty is 0.

The minimum index value of the electronic balance is 0.001, so the uncertainty of M_s is $\pm 0.001g$. The minimum index value of the beaker is 50ml, so the uncertainty of M_w is 12.5ml, or 12.5g. Then we need to get the concentration of the solution:

$$\overline{M_s} = \frac{M_{s1} + M_{s2} + M_{s3}}{3} = \frac{25.000 + 25.000 + 25.000}{3} = 25.000g$$

$$\Delta M_s = \frac{\Delta M_{s1} + \Delta M_{s2} + \Delta M_{s3}}{3} = \frac{0.001 + 0.001 + 0.001}{3} = 0.001g$$

$$M_s = \overline{M_s} + \Delta M_s = 25.000 \pm 0.001g$$

$$C(concentration) = \frac{M_s}{M_s + M_w} = \frac{25.000}{25.000 + 100.0} = 0.2 = 20\%$$

$$\frac{\Delta C}{C} = \frac{\Delta M_s}{M_s} + \frac{\Delta M_s + \Delta M_w}{(M_s + M_w)} = \frac{0.001}{25.000} + \frac{0.001 + 12.5}{(25.000 + 100.0)} = 0.100048 \approx 0.1$$

$$\Delta C = 0.1 \times 0.2 = 0.02$$

$$C_a = C \pm \Delta C = 0.20 \pm 0.02 = 20. \pm 2g/100ml$$

Therefore, when concentration of the solution is $20. \pm 2g/100ml$ water, the rotated angle is $5.0 \pm 0.5°$.

3.3 Processed data

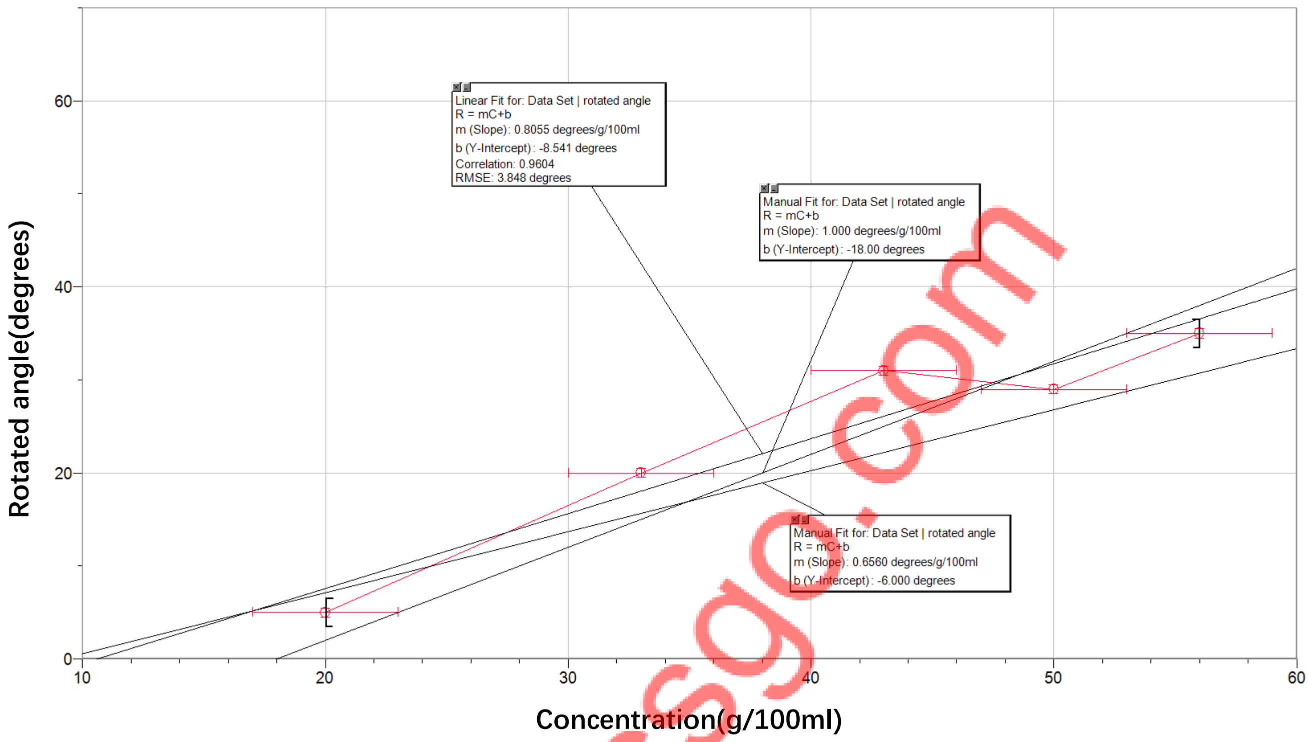
Using the above calculations and equations as the model for group B, C, D, and E, I come up with table 2. The table shows each trial's concentration and rotated angles. This helps the generation of the line graph, which will show the relationship between concentration and rotated angle more directly.

group	trials	Concentration (C, g/100ml)	rotated angle (R, °)
A	1	20.±2	5.0±0.5
	2		
	3		
B	4	33±3	20.0±0.5
	5		
	6		
C	7	43±3	31.3±0.5
	8		
	9		
D	10	50±3	29.3±0.5
	11		
	12		
E	13	56±3	35.0±0.5
	14		
	15		

(Table 2)

3.4 Data analysis

By entering the values into LoggerPro, I get graph 1, illustrating each groups concentration and rotated angle and their overall quantitative relationship.



(Graph 1)

I get the best fit line and two worst fit lines, which are found by connecting points that consists the flattest and the steepest straight lines. I will use these three lines' gradients and interception points to find an average one:

$$m = \frac{m_1 + m_2 + m_3}{3} = \frac{0.8055 + 1 + 0.6560}{3} = \frac{0.8205degress}{g}/100ml$$

$$b = \frac{b_1 + b_2 + b_3}{3} = \frac{(-8.541) + (-18) + (-6)}{3} = -10.847degrees$$

Therefore, I can obtain the quantitative relationship between concentration and the rotated angle:

$$A(angle) = 0.821C(concentration) - 10.847$$

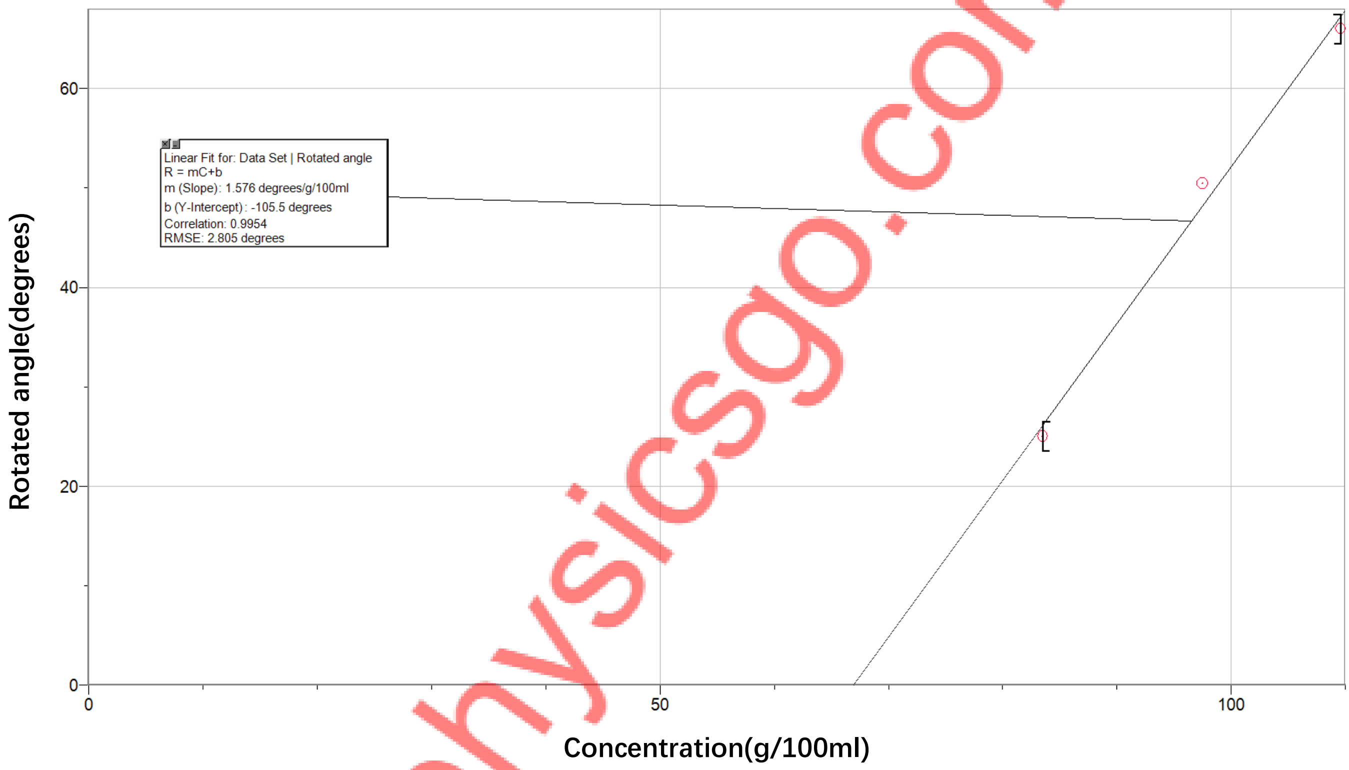
4. Conclusion

The relationship I found is specifically under the condition of white full spectrum light. According to a more professional research study, they study the relationship between rotated angle and red wavelength and green wavelength, which are wavelength that have specific and accurate quantified value of wavelength. Compared to red and green wavelengths, full spectrum white light, the combination of all wavelengths, has a more flexible value. Table 3 state the data from the research paper: the rotation angle for red filter and green filter in condition of different concentrations.

Concentration g/mL)(	Rotation angle (°) for red filter	Rotation angle (°) for green filter
0.835	25.03	25.01
0.975	50.45	56.85
1.096	66.01	75.96

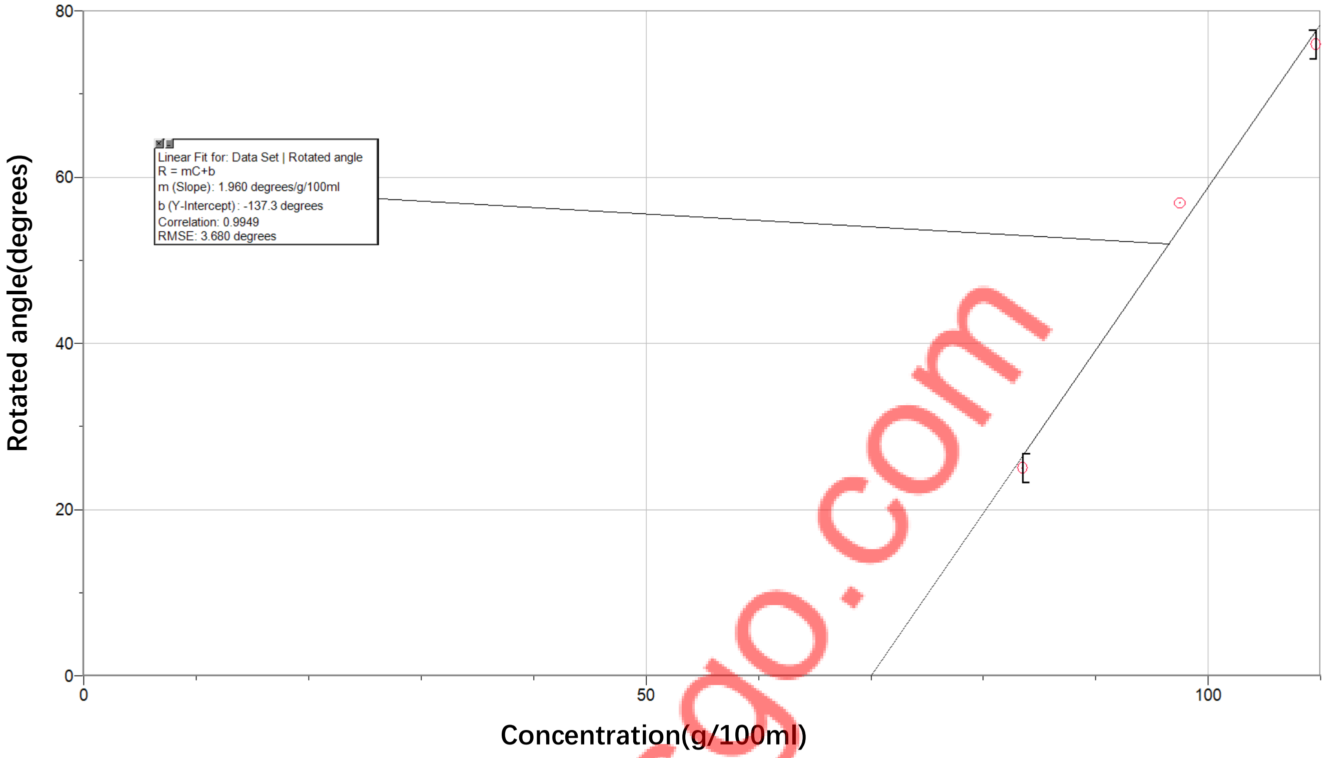
(Table 3: "Rotation of Plane-Polarized Light" 127)

Graph 2 illustrate the relationship between rotated angle of red wavelength and the concentration of sugar in water. We can see from the graph that the gradient is 1.576.



(Graph 2)

Graph 3 illustrates the relationship between rotated angle of green wavelength and the concentration of sugar in water.



(Graph 3)

I can see from the graph that the gradient is 1.960. 1.960 is greater than 1.576. Red lights wavelength is longer than green lights. Therefore, I can conclude that the longer the wavelength, the smaller the gradient. The wavelength of full spectrum light must be smaller than the average wavelength of green and red light, so the gradient should be greater than $\frac{1.960+1.576}{2}=1.768$. However, the calculated value is 0.821, which means I underestimate the value.

5. Evaluation

5.1 Strengths and weaknesses of experiment

Strengths

1. Firstly, my experiment is not complex. I didn't include many complicated equipment and the procedure to obtain my results is pretty easy. In fact, I just need one step of calculation each to get my raw data in two variables: find the difference between the angles on the polarizers, and read the numbers on the electronic balance. Though these steps still contain uncertainties, they don't include secondary calculation to get raw data.
2. I minimize the background intensity. The I did the experiment in a room with black curtains, so that I can minimize the background light intensity. Black curtains have

high emissivity, which is the surface of objects effectiveness in emitting energy as thermal radiation and varies between 0.0 and 1.0. The darker the curtain, the less light passes through the window, and the background light intensity is minimized.

Systematic errors

1. Although I tried to minimize the background light intensity, there were still 1.2 lux of light intensity in the background because the monitor must be bright so that it radiates wavelength. But this error won't affect my result because my experiment and conclusion does not involve exact values of light intensity. Instead, I just need to record the angle when light intensity is largest and lowest, which means only relative value is needed.

   <u>Solution:</u> Conduct the experiment in an all-black box. A white full-spectrum light is struck outside the experimental environment and this light is shone on a piece of white paper inside the experimental environment, replacing the original display with the reflected light from the diffuse light of the white paper. The diffuse light will be less bright than the direct light because the white paper is 75% albedo, so that the ambient light intensity will also be 75% of the direct light originally used. This means that the ambient light intensity is $1.2\times 0.75\% = 0.9lux$. In this way background light intensity is reduced to the lowest possible level.

2. When I conducted my group C, which is when the dissolved sugar is 75g, I dissolved the solids about 12 hours before I conducted the actual experiment. The sugar was not precipitated as the solution was not saturated. Therefore, during this time, some amount of water is evaporated, which in turn making the concentration of the sugar solution greater than I originally planned. Since the conclusion is "the higher the concentration, the greater the rotated angle", the rotated angle I measured in group three are all greater than the actual results. This is also why the point on the graph representing group three results are slightly off the straight line. Indeed, the evaporation rate of bare water is 2.0–2.5 mm/h, which means 2-2.5 millimeters of water depth is lost per day.

   <u>Solution:</u> Conduct the experiment before the water evaporates too much and sugar concentration increases. In fact, conduct the experiment immediately after all the sugar is dissolved to avoid the underestimation in sugar concentration.

3. The maximized intensity is reduced in several ways. This reduction reduces the accuracy of the data because the minimum intensity is always 0, meaning the range of data is smaller but the minimum index value is still 1 degree. Theoretically, when the range of data is smaller, the minimum index should be smaller too.

   The beaker is made of glass, which reduces some intensity when the light pass through. If the light is vertically passed through the glass, then the reduced intensity can be ignored. However, in the experiment, the monitor (light source) is greater than the beaker, so the light which does not pass through the beaker will reflect some of the light. This makes the intensity of light smaller than the intensity measured when it passes through pure sugar solution.

The sugar solution has a yellow color itself, therefore, its color absorbs part of the light intensity from the sensor.

Solution: select the sensor that has the same shape and size of the beaker and let the light only vertically pass through the beaker to avoid reflection, and choose the sugar which has the least color.

Random errors

1. When using a ruler to match the angles on two polarizers, I slightly touched them. This makes one or both of the polarizers rotate clockwise or anti-clockwise. Then the angle measured is less accurate.

   Solution: use two iron stands to fix two polarizers. When reaching the minimum intensity, remove the beakers, use the fixers on the iron stand to move two polarizers together to measure the angle without slightly rotating them.

5.2 Extension

Since white light has a range of wavelengths, I will use monochromatic light sources to conduct the experiment again so that I can obtain a more accurate and specific relationship in fixed wavelength. As I obtain my new results, these relationships can be used to determine the concentration of the sugar solution. I will also conduct experiments to determine the relationship between the rotated angle and another variable, the path length of the light, by changing the depth of the solutions in the beaker and keeping the concentration constant. Finally, I will try other types of solutions which can also rotate the polarized light. In this way, I can find how the properties of solutes affect the rotated angle.

Bibliography

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