What is the Relationship Between the Rate Ice Melts and Its Surface Area?

Introduction

After learning about derivatives in mathematics and acceleration in physics, I have come to understand that the rate of change in speed for a function or a moving object does not have to be constant; it can vary over time. One example that particularly caught my attention is the melting of ice in a cup of cola. When a large cube of ice is placed in the cup, it melts slowly, often taking over an hour. However, if I add crushed ice of a similar size, the ice melts rapidly, causing my cola to lose its cold temperature quickly. I attribute this phenomenon to the disparity in surface area exposed to the air and the liquid. Fascinated by this observation, I have chosen it as the topic for my physics IA.

Rather than solely measuring the average melting speed by observing the time it takes for the ice to completely melt, I intend to regard the melting speed as a function. To achieve this, I will measure the change in speed and surface area at regular intervals of time.

illustration 1

Figure above shows how ice melt

Exploration

Method

It is important to note that the melting of ice refers to the transformation of a solid into a liquid state, which requires energy without any accompanying change in temperature. This property is critical for the forthcoming experiment.

illustration 2

This illustration shows the change of cubic ice when melting

Typically, the average speed of ice melting is determined by measuring the time taken for the ice to completely melt. However, the melting rate is a dynamic function that is neither constant nor linear. In order to explore this phenomenon, ice cubes and balls will be utilized, as their sizes and surface areas are easily measured. At regular intervals of time, the melting rate and surface area of the ice will be measured. By analyzing the data collected, the relationship between the melting rate and the surface area exposed to the air will be determined.

Lab Materials

All the material required for the experiment, including staff and devices, are listed and shown in the figure below.

1. Cubic and ball-shaped ice molds.
2. Cubic and ball-shaped ice
3. Timing clock
4. Ruler
5. Thermometer
6. Vernier Caliper

illustration 3

Hypothesis

First of all, suppose the ice I used in the experiment is of a traditional shape, such as a cube or a sphere, those shapes will have a center in which heat transferred from the outside is smallest at that point compared to all other coordinates inside the ice.

Now considering the equation for calculating the latent heat of fusion and Fourier's equation for heat transmission:

$$Q = mL_f \tag{1.1}$$

$$Q = \frac{kA(T_2 - T_1)}{d} \tag{1.2}$$

Where:

Q: The amount of heat transferred and absorbed by the ice.

m: The mass of the ice.

L_f: The latent heat of fusion of ice.

k: The thermal conductivity of ice.

A: The surface area of the ice in the experiment.

d: The average distance to the heat center of ice.

$T_2 - T_1$: The temperature difference across the ice and the environment.

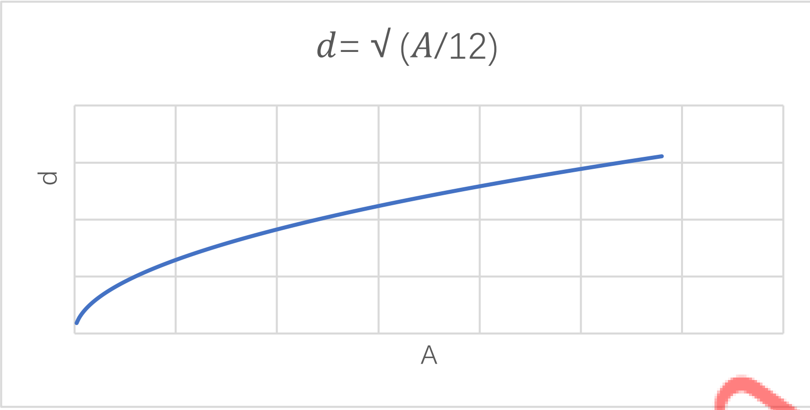
During the time interval of each measurement, I would assume the temperature of ice and the environment always remain the same at a normal level, the temperature of ice is always 0 °C, so the dependent variables are A, the surface area, and d, the distance to the heat center. As the shape of the ice, I used in the experiment is regular (cube and ball), the relationship between A and d can be modeled as

$$d = \sqrt{\frac{A}{12}} \qquad \text{(cube)(1.3)}$$

$$d = \sqrt{\frac{A}{4\pi}} \qquad \text{(ball)(1.4)}$$

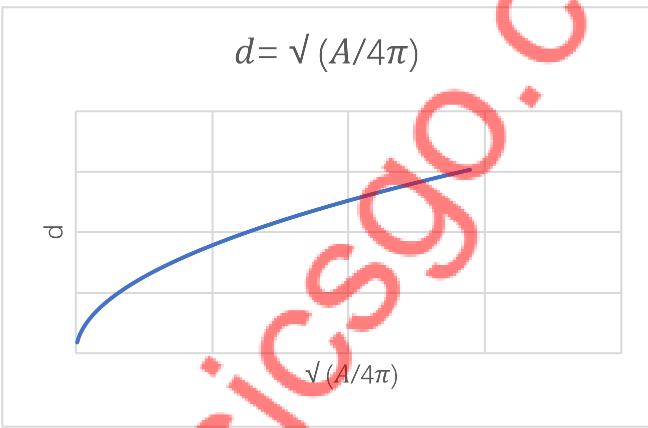
In the following, the illustration for above two equations are presented, all the data in the following graphs start measuring when ice is at about 0 °C.

The graph of equation (1.3) is presented as graph 1 below, which visualize the calculated relationship between the surface area of the spherical ice and its measured height:



graph 1

The graph of equation (1.4) is presented as graph 2 below, which visualize the calculated relationship between the surface area of the spherical ice and its measured height:



graph 2

I will measure the surface area of ices by measuring their heights h, so there will be following equation converting my measurement into data for calculation:

For ice cubes:

$$A = 6h^2 \tag{1.5}$$

$$d = \frac{h}{\sqrt{2}} \tag{1.6}$$

For ice spheres:

$$A = \pi h^2 \tag{1.7}$$

$$d = \frac{h}{2} \tag{1.8}$$

By substituting equation (1.5)(1.6)(1.7)(1.8) into equation (1.2) and (1.1), all independent variables can be expressed in the term of h:

$$Q = mL_f = 6\sqrt{2}hk(T_2 - T_1)$$

$$m = \frac{6\sqrt{2}hk(T_2 - T_1)}{L_f} \quad \text{(cube)(1.9.1)}$$

$$m = \frac{2\pi hk(T_2 - T_1)}{L_f} \qquad \text{(ball)(1.9.2)}$$

In this way, considering equation (1.1) and (1.9), I would assume following factors will have influence on ice's melting speed:

1. The ambient temperature
2. The mass of ice = the volume of ice
3. The average distance to the center of ice
4. The surface area of ice (ice's shape)

Besides, other variables can also cause a variation to the results of the experiment since they have an influence on values of constants in the above equations, such as: The components of ice, and whether the ice is pure or not, like if some salts are dissolved.

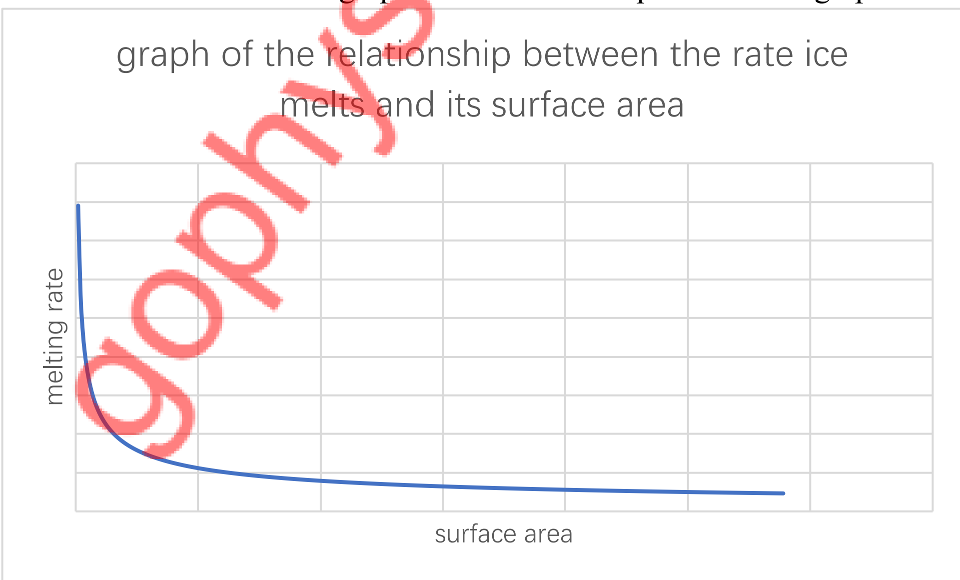
When the ice is in regular shapes like a cube or ball, then the above variables and correlations can be simplified into the linear relationship between the melting rate of ice and ice's height. Back to the research question, I would hypothesize the reciprocal-square-root's relationship between the surface area of ice and ice's melting rate, which is:

$$m = c\sqrt{A}$$

Then taking the derivative of m with respect to t

$$\frac{dm}{dt} = \frac{c}{\sqrt{A}}$$

Where c is a constant here. The graph of the relationship is shown in graph 3 below:



graph 3

Variable lists

Independent variable: A

A, the surface area of the ice. This value can be derived from the direct measurement of regular ice's height h.

Dependent variable: R

$R = \frac{dQ}{dt}$, which is the rate of the ice melting.

Controlled variable

According to equation (1.1) and (1.2), the following variables will remain controlled:
m: The mass of the ice.
L_f: The latent heat of fusion of ice.
t: Time which passes from last measurement or from the start.
k: The thermal conductivity of ice.
T_1: The temperature of the ice when it's melting.
T_2: The environment temperature.
$T_2 - T_1$: The temperature difference across the ice and the environment.
Among the variables meationed above, some of them will remain constant to fulfill the requirements for controlling variables, and some of others are just constants that cannot be changed. Their constant values are listed in here:
$L_f = 225760$ J/kg
$k = 2.22$W/mK
$T_1 = 0$°C
$T_2 = 25$°C (Room temperature of the day at which I collected the data)
$T_2 - T_1 = 25$°C-0°C=25K

Experiment

Procedure to collect data

The following steps will be taken to conduct the experiment

1. Measure the initial height of the ice cube, the ice sphere, and the room temperature. (This step will be done before the ice is taken out since the size of ice is already

noted on the ice molds)
2. Remove the ice from the molds in the freezer and place them on a glass table.
3. Observe the ice samples.
4. After 5 minutes, measure the height of the ice samples at the moment and weigh the ice.
5. Repeat step 4 until the shape of the ice completely melts.
6. After the ice melts or become smaller than the smallest scale of the ruler, stop measuring the time and record the data of the size of the ice and surface area of the ice.

Raw Data

Table below is the raw data of heights of ice cubes and balls at different time from they are taken out from the refrigerator.

Table 1

Measurement of heights of ice

sequence	Height of the cubic ice (cm) ±0.5cm	Height of the spherical ice (cm) ±0.5cm	time interval (min) ±0.05s
1	6.5	6.0	origin
2	6.4	5.8	5
3	6.3	5.3	5
4	6.2	5.3	5
5	6.3	5.1	5
6	6.2	4.9	5
7	5.8	5.2	10
8	6.3	4.5	10
9	5.5	4.3	10
10	5.3	4.2	10
11	5.1	4.1	10

Table 2

Random error for raw data

sequence	Percent random error of the cubic ice height	Percent random error of the spherical ice height (cm) ±0.5cm	Percent random error of the time interval
1	7.69E-02	8.33E-02	/
2	7.81E-02	8.62E-02	1
3	7.94E-02	9.43E-02	1
4	8.06E-02	9.43E-02	1
5	7.94E-02	9.80E-02	1
6	8.06E-02	1.02E-01	1
7	8.62E-02	9.62E-02	0.5
8	7.94E-02	1.11E-01	0.5
9	9.09E-02	1.16E-01	0.5
10	9.43E-02	1.19E-01	0.5
11	9.80E-02	1.22E-01	0.5

Processed Data

Table 3 and Table 4 below is the processed data. The first column is the order of the data, which corresponds to the same order in the data table 1, it shows the time which has passed since the ice samples are taken out. The second line is $\frac{1}{\sqrt{A}}$ for ice cubes and balls at different time, where A is the surface area of the ice. The third column is the rate of change in the mass of the ice, this column of data is derived through Euler's method.

Table 3

Processed Data of Cubic Ice

sequence	$\frac{1}{\sqrt{A}}$ [cm^{-1}]	Random error of $\frac{1}{\sqrt{A}}$	$\frac{dt}{d\,kg}$ [kg/s] *	Random error of $\frac{dt}{d\,kg}$
1	6.28E-02	4.83E-03	/	/
2	6.38E-02	4.98E-03	-2.50E+00	-2.75E+01
3	6.48E-02	5.14E-03	-2.42E+00	-2.66E+01
4	6.58E-02	5.31E-03	-2.34E+00	-2.58E+01
5	6.48E-02	5.14E-03	2.34E+00	2.58E+01
6	6.58E-02	5.31E-03	-2.34E+00	-2.58E+01
7	7.04E-02	6.07E-03	-4.32E+00	-1.30E+01
8	6.48E-02	5.14E-03	5.49E+00	1.37E+01
9	7.42E-02	6.75E-03	-8.37E+00	-1.47E+01
10	7.70E-02	7.27E-03	-1.75E+00	-9.63E+00
11	8.00E-02	7.85E-03	-1.62E+00	-8.93E+00

Table 4

Processed Data of Spherical Ice

sequence	$\frac{1}{\sqrt{A}}$ [cm^{-1}]	Random error of $\frac{1}{\sqrt{A}}$	$\frac{dt}{d\,kg}$ [kg/s]s*	Random error of $\frac{dt}{d\,kg}$
1	8.33E-02	7.84E-03	/	/
2	8.62E-02	8.39E-03	-2.19E+00	-7.92E+00
3	9.43E-02	1.00E-02	-4.84E+00	-9.92E+00
4	9.43E-02	1.00E-02	0.00E+00	Unable to derive
5	9.80E-02	1.08E-02	-1.70E+00	-6.15E+00
6	1.02E-01	1.17E-02	-1.57E+00	-5.68E+00
7	9.62E-02	1.04E-02	1.20E+00	2.70E+00
8	1.11E-01	1.39E-02	-2.59E+00	-3.24E+00
9	1.16E-01	1.53E-02	-6.08E-01	-1.90E+00
10	1.19E-01	1.60E-02	-2.84E-01	-1.63E+00
11	1.22E-01	1.68E-02	-2.71E-01	-1.55E+00

*: Rate of change in the volume is measured by Euler's Method.

Risk Assessment

There might be some electronic devices around the lab, since solid ice will convert to liquid water when it is melting, it is necessary to notice the risk of having water leakage which may lead to electric shock. Besides, with water on the floor, also pay attention not to have a wet floor.

Conclusion

Evaluation

Between "$\frac{1}{\sqrt{A}}$ [cm^{-1}]" and "$\frac{d\,t}{d\,kg}$ [kg/s]", The calculated PMCC values (Product Moment Correlation Coefficient, measures the strength of the correlation between two set of statistics) for cubic ice and spherical ice are -0.37748 and 0.288198 (it's a measure of correlation so it doesn't need a unit). They show converse results of their linear relationship and the relationships are also too weak. In this way, I would say they really deviate from my previous prediction. However, considering my process of theoretical deduction, I don't think there will be mistakes causing the deviation to such scale, the main factor probably still be the limitations in experiment, which lead to overlarge random errors.

Errors & Limitations

I think there are following problems within my first experiment which makes this lab failed:

1. The ice is deformed after a period of time exposed in the air. This makes the calculation of surface area no longer matched with the measurement of height. And the deformed ice also makes the measurement of height more difficult and unprecise since the shape of ices are no longer regular.
2. There are melted water beneath the ice, and the whole experiment is conducted on a table, which means its base is not exposed to the air. This will make different parts of the ice absorb heat at different rates, this will also let to ices' deformation.
3. The method of measuring ices' height is not very precise: when putting the ruler one the ice, the metal ruler will also absorb heat and melt the ice, thus the ruler will form a dented scratch. This phenomenon will make the ruler not really parallel to the side of the ice.
4. The time interval is too short between each measurement, this will cause bigger random errors on the final results of calculating the rate of ices' melting. Because the minimum random error, which is the smallest scale of the ruler, is fixed. With smaller time interval, the measured change in the heights of ices are also smaller, thus the percentage error of the change in heights get bigger, and the error of the final results is also magnified, thus lead to the failure in the first experiment.

Methods of Improvement

To address above issues, I propose the following improvements for the next experiment:

1. Utilize a metal plate: Placing the ice on a metal plate would facilitate efficient heat

transfer. Metals are known for their high thermal conductivity, allowing them to transfer heat effectively to the bottom of the ice, similar to the heat transfer from the surrounding air.

2. Discard melting water prior to each measurement: Emptying the melting water before taking measurements serves multiple purposes. Firstly, it prevents the ice from shifting or sliding during the measurement process, ensuring more accurate readings. Additionally, it enables the bottom of the ice to absorb heat at the same rate as the other parts, minimizing any potential discrepancies.
3. Employ various sizes of cubic and spherical ice: Prepare ice blocks of different sizes and shapes for measurement during the melting process. This approach allows each ice piece to be monitored for a specific duration, focusing on measuring its height over time. As the larger ice blocks gradually diminish in size, they will eventually match the initial size of smaller ice blocks. By implementing this improvement, each piece of ice will only need to melt for a relatively short period, reducing the likelihood of deformations occurring.
4. Increase time intervals between measurements: Adjusting the time intervals between each measurement can yield more significant differences in the measured heights. By extending the duration between measurements, any changes in the melting rate become more pronounced, enabling clearer deductions and conclusions to be drawn.

Implementing these proposed improvements in the next experiment will enhance the accuracy and reliability of the results, thereby enabling a more robust analysis of the relationship between the rate of ice melting and its surface area.

Conclusion

In conclusion, as the entire experiment was conducted by myself, there may be some systematic errors present in my procedures that could be eliminated through more precise operations. I firmly believe that addressing these shortcomings and improving upon them can reduce the calculated random errors in the final results, enabling me to draw stronger and more positive conclusions. Nevertheless, despite these limitations, I maintain confidence in the validity of my hypothesis. It is based on accurate deductions derived from correct and proven equations. Moving forward, I will focus on rectifying the issues highlighted in my reflection section to enhance the precision of my measurements. By doing so, I aim to determine the viability of my theory.

Reference

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