Research Question: how different temperature of a guitar string affects its frequency

Introduction

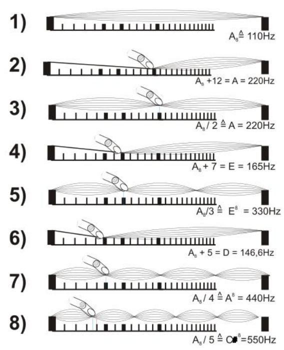
My mom invited me to play guitar in front of the family during the Chinese new year in 2022, but it was a "mess". I placed the guitar on the balcony, which suffers direct exposure to the scorching sunlight. When I played the guitar, I found out that the music sounded quite weird: every single syllable was off-key. Even though I was embarrassed on the stage, my family was satisfied with my performance because most of them do not know music. To avoid the same event happening again, I conducted some research online and learned that the strings should be placed in a cool and low temperature environment. A higher temperature will decrease tension, which affects the pitch of the song (LoPresto). Most guitarists are aware that a higher temperature on the strings results in a lower pitch, which depends on the change in the tension of the string. Unfortunately, the relationship behind these two factors are not clearly explained. I want to investigate how different temperature of a guitar string affects frequency.

Background

Guitar strings

The vibration of guitar strings is a typical example of standing waves, which are the result of harmonic waves reflecting from fixed ends. This means that the vibration can be seen as two waves having an identical frequency and amplitude and interfering with one another while traveling in the opposite direction ("The Physics Classroom Tutorial"). Nodes are fixed points in a wave due to destructive interference, and antinodes are positions where the wave vibrates with maximum amplitude. In this case, the frets on the sting are nodes and are added to form different harmonics, which is a positive multiple of the fundamental frequency (2π). For the first harmonic of a wave, there are only two nodes and one antinode, which indicates that the length of the string is equal to one half of the wavelength.

Figure 1 below shows the string harmonics with different frets ("String Harmonic").



A guitar has 6 acoustic strings, each with a higher pitches ranging from E-6th, A-5th,D-4th, G-3rd, B-2nd, E-1st. E-1st string has the sharpest pitch and the smallest width, which means that the sound is easy to distinguish and whirl. Thus, it is chosen in this experiment.

How temperature affects frequency

When a guitar string is attached to fixed points, changing the temperature causes some stress on the surface because the different temperatures change the frequency and magnitude of molecular motion that would cause stretching and contracting of the wire. Fixing the string restricts it to respond to this change largely, so stress is caused (Singh et al.). The value of the coefficient of thermal expansion varies for different materials and can determine whether the string expands or contracts when the temperature increases. For steel strings, the above description applies to the situation when the material is under tension and compression ("26.2: Stress and Strain in Tension and Compression").

When the temperature of a string raises, elastic potential energy is stored, and particles would have a higher kinetic energy, which means an increase in moving speed and the frequency of successful collision. It means that the string is expanded since more particles are free to move at a faster speed and have a longer distance to each other. This would decrease the velocity because particles need a longer period of time to complete a cycle. Thus, increasing temperature decrease the frequency of the wave produced by a steel guitar string.

Methodology

Table 1 below shows the meaning of each variable being used in the methodology.

Variables	Definition
V	the velocity of the wave
T	the tension of the string
μ	the mass per unit length
L	the length of a string
λ	the wavelength of the wave
t	the period of the wave
σ	the thermal stress, or tension
E	the Young's modulus
α	the thermal expansion coefficient
ΔTe	the change in temperature
f_0	the initial frequency of the string
f_1	the received frequency of the string

By using a sonometer app (i.e. Audacity) to directly measure the frequency, the temperature of the string can be heated by hot air released from a high power rate air dryer, which can be detected by a temperature probe. The mass per unit length can be measured by dividing the mass of a string with its total length. A ruler and an electrical balance can be used to measure these values.

According to the wave equation for a string (Fernandez-Guasti), the equation for calculating velocity is

$$V = \sqrt{\frac{T}{\mu}} \tag{1}$$

As mentioned, a guitar string vibrates as a standing wave. The fundamental wavelength of a standing wave follows the following formula

$$\lambda = 2L \tag{2}$$

In general, the equation of the velocity of a wave is

$$V = \frac{\lambda}{t} \tag{3}$$

By rearranging equation Eq.3 and substituting λ by Eq.2 and V by Eq.1, the equation now becomes

$$f = \frac{\sqrt{\frac{T}{\mu}}}{2L} \tag{4}$$

Eq.4 is a generalized formula for calculating the frequency based on the value of tension, mass per unit length, and the length of the string.

Based on the background information mentioned above, the expression for the thermal stress can be used to represent the tension.

$$\sigma = E\alpha\Delta Te \tag{5}$$

Dividing the initial frequency by the final frequency after the temperature, the expression is

$$\frac{f_1}{f_0} = \frac{\frac{1}{2L}\sqrt{\frac{T_1}{\mu}}}{\frac{1}{2L}\sqrt{\frac{T_0}{\mu}}} = \sqrt{\frac{T_1}{T_0}} = \sqrt{\frac{T_0 + \Delta T}{T_0}}$$

Squaring both sides gives

$$\left(\frac{f_1}{f_0}\right)^2 = \frac{T_0 + \Delta T}{T_0} = \frac{T - \mathrm{E\alpha\Delta Te}}{T} = 1 - \frac{\mathrm{E\alpha\Delta Te}}{T}$$

Based on Eq.4, T_i can be substituted by in terms of f_0

$$\left(\frac{f_1}{f_0}\right)^2 = 1 - \frac{E\alpha\Delta Te}{4\mu L^2 {f_0}^2}$$

$${f_1}^2 = {f_0}^2 - \frac{E\alpha\Delta Te}{4\mu L^2} \tag{8}$$

For Eq.8, two variables are measured and compared, f_1 and ΔTe. Other parameters are all constant through the experiment.

The independent variable in this experiment is the heating time of the heat lamp on the string ranging from 10 minutes, 20 minutes, 30 minutes, 40 minutes, and 50 minutes.

The dependent variable in this experiment is the frequency of the wave produced by the string by applying a different heating time of the heat lamp ranging from 10 minutes, 20 minutes, 30 minutes, 40 minutes, and 50 minutes.

Table 2 below shows 5 controlled variables in this experiment with their impacts and how they should be controlled.

Controlled variables	Impact of the variable	Methods of controlling the variable
The same high *E* guitar string is used throughout the experiment.	Different guitar strings ranging from standard *EADGBE* tuning have different tensions and thicknesses. Even strings with the same tuning slightly varies due to the composition of the material. Changing the string would be a systematic error affecting the accuracy of the frequency measured.	Using the same high *E* guitar string throughout this experiment is an effective method.
The manually adjusted tension applied to the string should be constant throughout the experiment.	Since different tensions would affect the frequencies of waves produced by guitar strings based on the background information mentioned above, it should be a constant value because the only independent variable is the temperature. Manually changing the tension would make the value of the frequency measured less accurate.	Keep the string loose and do not tune the string during the experiment since the change in temperature would affect the original tuning of the string.
The length of the guitar string (between two ends) should be the same.	The wavelength is directly related to the length of the string. Adding frets between the string will decrease the wavelength, and thus the frequency is increased. This is a systematic error affecting the accuracy of the frequency measured	No frets are added to the string and stablizers are used to fix the string on both ends to prevent the string from moving.
The distance between the sonometer and the string should be 5cm.	Changing the distance will affect the amplitude of the wave, which shows an inversely proportional relationship with the frequency. This is a sysmatic error affecting the accuracy of the model	Use a ruler to measure the distance between the sonometer and the string. Stick the receiver of the wave (phone) with tape.
The surrounding environment	Sounds from the surrounding environment will directly interfere with the sonometer, which affects the	Four insulation pads will be used to surround the equipment during the

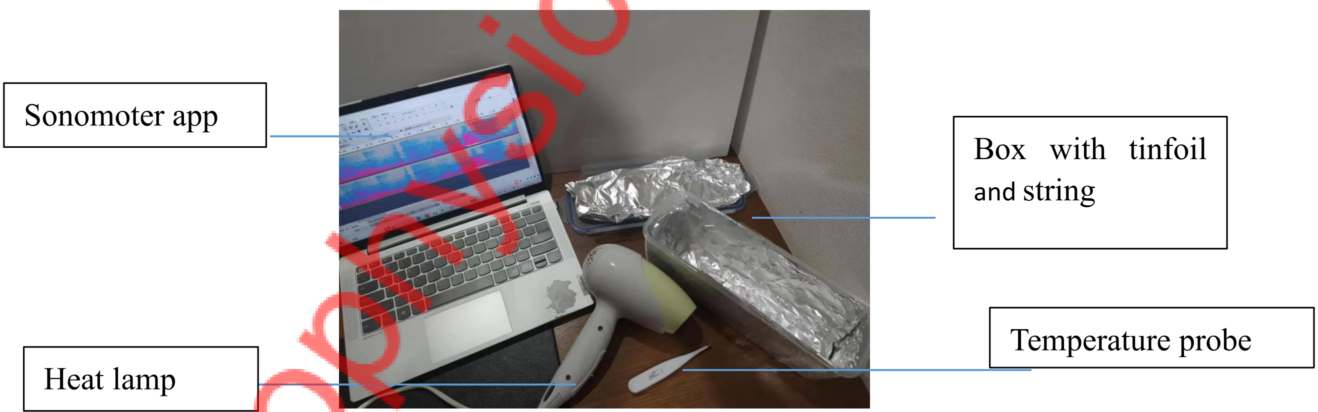
should have absolutely zero noise.	precision of the frequency from the wave created by the string	experiment.

Table 3 below shows the equipment which will be used to conduct this investigation with their corresponding quantity, size, and uncertainty.

Equipment	Quantity	Size	Uncertainty
High E guitar string	1	0.7m	±0.0005m
Sonometer app	1	/	±0.1Hz
Heat lamp	1	/	/
Insulation pads	4	1m*0.5m	/
A rectangular box	1	15cm*3cm*5cm	/
Ruler	1	1m	±0.0005m
Tap	1	/	/
tinfoil	1	20cm*8cm	/
Glue	1	/	/
Temperature probe	1	/	0.1°C
Guitar pick	1	/	/
Digital clock	1	/	±0.01s

Equipment setting up diagram

Figure 2 below shows the equipment setting up diagram of the experiment.



Procedure

1. Tightly stick the inner surface of the box with tinfoil, including the top cover
2. Fix both ends of the string to the tinfoil by tape
3. Surround the box with insulation pads
4. Measure the initial temperature by a temperature probe and record the vibration of the string 10 times.
5. Cover the box with a remaining hole for the heat lamp .
6. Change the lamp to the maximum power rate and heat the string for 10 minutes
7. Cover the box for 20 minutes after the heating process is finished
8. Measure the temperature of the string and record the vibration
9. Dispose the string until it returns to the original temperature

10. Repeat step 5 to step 9 with heating time of 20mins, 30mins, 40mins, 50mins.

Safety

This experiment has no negative impacts on the environment and does not include any ethical issues. However, strings with a high temperature are extremely hot and are dangerous when it direct contact with skin. Gloves are worn and a plastic guitar pick is used since it has good thermal insulation.

Data

Qualitative data

Before the experiment, tinfoil is tightly attached to the inner wall of the box with less obvious crumbles on the surface. The guitar string is tightened to be nearly straight. The pitch of vibration caused by the string is higher and more obvious.

Figure 3 below shows the adherence of tin foil by tape before the heating.

During the experiment, increased temperature slightly loosened the tap between the tin foil and the wall, and the fall of the attachment makes the string looser. There are more crumbles occurring on the surface, and the string slightly wraps. The tap for tightening the string to tinfoil is loosened. The pitch slightly decreases with increasing temperature. These effects become increasingly obvious at higher temperatures.

Figure 4 below shows the adherence of tin foil by tape after the heating.

After the experiment, the string becomes looser and slightly wrapped. The tape for attaching the wall, tin foil, and the string is slightly looser. The pitch of the string decreases. These effects continue when the temperature returns to room temperature.

It is interpreted that increasing the temperature would decrease the pitch of the string because of the thermal expansion of the wire and the looser stabilization of the string.

However, the effect of the latter is less dominate because it is observed that the loose of the attachment only slightly occur at higher temperature, and thus it can be neglected.

Quantitative data

Table 3 below shows the effect of different heating time by the lamp on the frequency of vibration caused by the string.

The heating time (minutes) (±0.01s)	Temperature (°C) (±0.001°C)	Frequency(Hz) (±0.1Hz)		Average	Frequency^2 (±0.1Hz)
		Trials 1~5	Trials 6~10		
10	18.312	54.3	52.4	53±2	2900±200
		53.8	54.5		
		51.1	55.8		
		58.3	51.9		
		52.3	52.4		
20	24.691	52.6	53.1	52±2	2700±200
		50.8	49.7		
		53.9	50.6		
		52.3	52.3		
		51.4	52.8		
30	34.213	48.9	48.3	49±1	2400±100
		49.9	47.6		
		49.2	49.8		
		50.1	50.5		
		47.9	49.3		
40	39.975	46.2	46.7	46±2	2100±200
		48.9	45.9		
		46.6	47.1		
		44.5	46.4		
		46.4	45.3		
		46.2	46.7		
50	45.112	41.2	41.9	42±2	1800±200
		42.3	42.8		
		41.8	44.5		
		40.2	44.1		
		43.4	37.9		

Before any processing of data, the reliability of data should be ensured, which is to examine the existence of outliers. The formula of IQR is

$$\text{Lower Outlier}: f = Q_1 - 1.5 \times IQR$$

$$\text{Higher Outlier}: f = Q_3 + 1.5 \times IQR$$

For each temperature, the values of frequency with yellow marks are outliers, which is not considered in the following calculation.

The absolute uncertainty for the average frequency can be calculated by :

$$\frac{f_{Max} - f_{min}}{2}$$

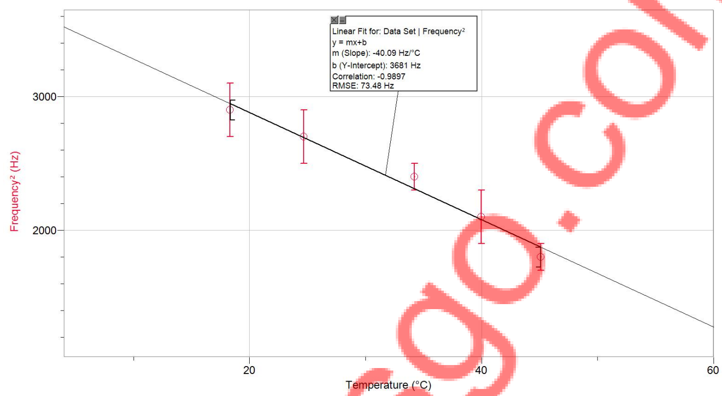
As each temperature has different values of frequencies, the value of Y also varies.

The absolute uncertainty for the squared value of frequency can be calculated by

$$(f \pm \Delta f)^2 = f^2 \pm \left| 2 \times \frac{\Delta f}{f} \right|$$

which is represented as the last column of Table 1.

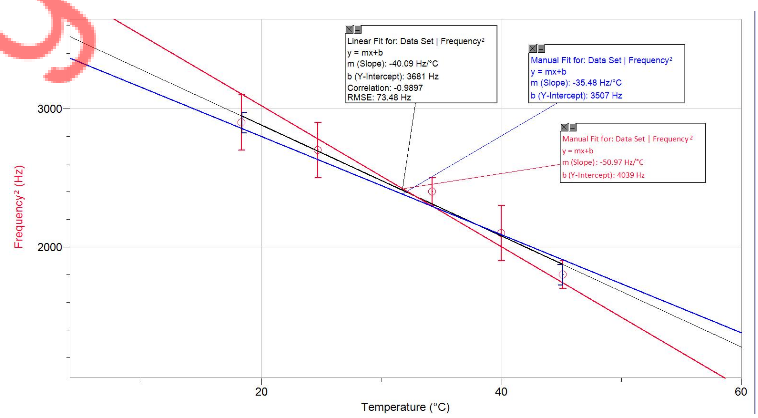
Graph 1 below shows the relationship between frequency squared and temperature



The relationship between the squared value of frequency and temperature is linear with an intercept of b for ${f_0}^2$ and a gradient value of m for $-\frac{E\alpha}{4\mu L^2}$. The relationship is supported by Eq.8. The R^2 for this line is -0.9897, which is strong enough to support that the correlation between two variables is linear.

The uncertainty of the gradient can be obtained by finding the differences between gradients of two worst-fit lines, where lines must pass the range of all error bars.

Graph 2 shows the best fit line and two worst fit lines for frequency squared and temperature



The gradients for the worst fit lines are -35.48 and -50.97, so the uncertainty for the best fit line is -15.49≈-20. Thus, the gradient of the best fit line is -40Hz±-20.

Sample calculation

Once Graph 1 is obtained, the final frequency can be calculated by given temperature.

The length of the tested string is approximately 10cm±0.05cm, and given that the total length of the string is 3m with a mass of 0.007kg, the mass of the test string is estimated to be $\frac{10}{300} \times 0.007 = 0.000233kg$, so μ is 0.00233kg/m.

After inquiring about the raw material of the string and considering data online (Engineering ToolBox), the string is made of a copper alloy (yellow brass) with α of $11.2 \times 10^{-6} \frac{m}{mc}$, and the value of E is 117GPa.

Table 4 below shows the sample calculation of the final frequency for 24.691°C:

Calculation	Error propagation
$f_1{}^2 = f_0{}^2 - \frac{E\alpha\Delta Te}{4\mu L^2}$ $f_1 = \sqrt{f_0{}^2 - \frac{E\alpha\Delta Te}{4\mu L^2}}$ $\Delta\text{Te} = 6.379$ $f_1 = \sqrt{2900 - \frac{117 \times 11.2 \times 10^{-6} \times 6.379}{4 \times 0.00233 \times 0.1^2}}$ $f_1 = 53.0124$ $f_1 = 53Hz \pm 2Hz$	The uncertainty of length is $\frac{0.05}{10} = 0.5\%$ The uncertainty of μ is also 0.5% The uncertainty of $\frac{E\alpha\Delta Te}{4\mu L^2}$ is $2 \times 0.5\% + 0.5\% = 1.5\%$ The absolute uncertainty is $89.6983 \times 1.5\% = 1.34534$ The total uncertainty is 201.345 The uncertainty for f_1 is $\left(\frac{201.345}{2900}\right) \times 0.5 = 3.47\%$ $3.47\% \times 53.01 = 1.84$

To calculate the deviation between the experimental value and the calculated value, the error discrepancy is calculated, and the value is $\frac{\lvert 53-52 \rvert}{53} \times 100 = 1.89\%$, which is smaller than the error propagation.This suggests that it is possible for systematic errors to cancel each other out, so it does not affect the accuracy of the result significantly.

Table 5 below shows the calculated frequency and measured frequency:

Temperature(±0.001°C)	F_m	F_C	Error discrepancy
18.312	53	/	/
24.691	52	53	1.89%
34.213	49	51	3.92%
39.975	46	48	4.17%
45.112	42	45	6.67%

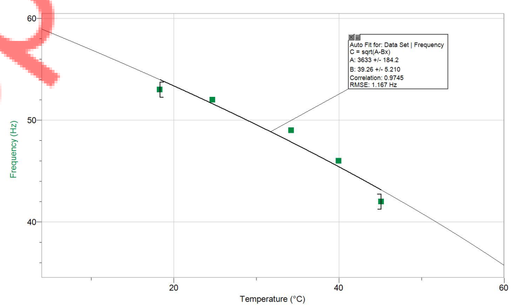
Based on Table 5, it is clear that as temperature increases, the error discrepancy rises. This is reasonable because, at higher temperatures, some limitations of methodology and equipment become more obvious, which will be mentioned in the evaluation.
It is observed that the calculated frequencies are always higher than the measured frequency. This may be because of the misuse of the parameter of Young's modulus and the thermal expansion coefficient. Meanwhile, it is observed that in data points at 45°C from Table 3, there is an increasing trend of frequency observed, despite the last outlier. This is because of the dominating effect of heat loss. When I was measuring the frequency, the string was directly exposed to room temperature, a temperature much lower than the temperature of the string. This effect is obvious at higher temperatures. Thus, the interval between the measurement of the temperature and the measurement of the frequency is long enough for the temperature of the string to drop. With a smaller temperature, the pitch would slightly increase, and thus the measured frequencies would show an increasing trend. However, since the effect is limited, it is only obvious at higher temperatures. Yes, that is part of the plan.

Evaluation

Conclusion

This experiment aims to how varying temperatures of a heat lamp with duration ranging from 10 minutes, 20 minutes, 30 minutes, 40 minutes, and 50 minutes of guitar strings affects its frequency. Based on Graph 1, the relationship between temperature and the square of the frequency is inversely proportional. This would indicate that the relationship between the frequency and temperature is exponential , which implies that frequency decreases more rapidly as temperature increases. The R^2 value is 0.9745, which is strong enough to prove the exponential relationship between these variables. It is also supported by Eq.8.

Graph 3 below shows the exponential relationship between frequency and temperature:



Strength and good practices

<u>The first strength</u> is that the length of the tested string is shortened in this experiment. Since the transfer of heat by the heat lamp is by convection and the rate of the heat lamp is low, increasing the temperature of the string is not very effective. Thus, the length is shortened, which allows for the air outlet of the heat lamp to fully cover the string. This would cause the reaction between the heated air and the string more

violent and thus increasing the efficiency. Meanwhile, if the string to become is too long, some part of it would not be exposed under the outlet. Thus, there would be varying temperatures along the string. Shortening the string allows every aspect of the test string to absorb nearly the same amount of heat, so that during the vibration, every part of the string is affected by the temperature, and thus the accuracy of the measurement of frequency would increase.

The second strength is that the inner surface, including the back of the cover of the box, is covered with layers of tin foil. If it is not done then the heat would be slightly lost to the surrounding environment by means of convection since there is a temperature difference between the environment and the inside of the box. If tinfoil is added then the heat can be better preserved because the surface of the tinfoil can reflect radiation and it has high absorptivity, so it can lock heat inside the box. This would ensure that no dramatic heat loss occurs during the waiting time and thus the accuracy of temperature change during the waiting time is improved. This is more important at higher temperatures because the effect of heat loss is more dramatic.

The third strength is that During the heating process, a cover is placed on the small hole for the heat lamp. Even though it would block the eyesight and the qualitative data becomes hard to observe, it would preserve most of the heat. No significant heat loss would occur, and thus the accuracy of the change in temperature during the heating time is improved.

The fourth strength is that ten trials are taken for each temperature. Since the reception of the vibration is sensitive to the surrounding noise, more trials are taken to decrease this random error, and some mathematical methods are also adopted.

Limitations

Limitations	Improvement
During the heating process, the viscosity of the tape is weakened, so the tinfoil and string would fall. Thus, the length of the string is increased when the temperature increases. Since the variable L is considered in the equation, raising the temperature would increase L and thus decrease the value of the final frequency. This is a systematic error which affects the accuracy, which partially accounts for why the experimental value is always lower than the calculated value as shown in Table 4.	A possible improvement would be setting two heat insulated walls right before the ends of the string, blocking the transfer of the heat to the tape. Thus, the tape is not affected, and heat is locked between the walls.
The key equation, Eq.8, is based on the assumption that when both ends of the string are fixed, the temperature change causes stress. However, since the surface area of the string is too small, it is hard to tightly adhere the wire to the tinfoil with tape. Thus, the string may slide during the heating process, and the precision of length is affected, which is a random error. This would disprove the assumption, and the equation may not be suitable.	A possible method is to whirl the ends of the wire to form circles, so that the surface area is increased, and there are more sites on which the string can contact the tape. Thus, the string is fixed more tightly.

When the tinfoil falls, the vibration of the string is affected since it is directly attached to it. The collision between the tinfoil and the inner wall of the box produces a lower pitch sound than the vibration of the string which is harder to distinguish. The noise would affect the pitch of the string, causing it to be lower than the actual value, which is a systematic error. This makes the experimental value lower than the calculated value.	An improvement would be attaching the string to the box instead of the wire. Drill a hole in the tinfoil and pass the string through it so the tinfoil is directly in front of the ends of the string, which can also lock the heat.
An assumption for μ is that the distribution of particles is evenly distributed throughout the string, so that the mass for that tested string is an accurate portion of the total mass. However, due to the limitations of the production process, the particles in the string is not perfectly distributed, which means the estimated mass may deviate from the true value. This random error would affect the precision of the final frequency based on Eq.8	Calculate the mass of the string based on Mersenne's laws, which states three equations related to the frequency of a stretched string("Mersenne's Laws | Physics").
Since ten trials are taken for each temperature, the temperature change may slightly change during the process because of heat loss to the surrounding environment. This systematic error is obvious at higher temperatures. The temperature of the string would drop more significantly during the vibration process, so the value of ΔTe decrease, and the final frequency would be higher than the true value.	Place the entire setup of the equipment into a heat insulted box and raise the temperature of the air within the box, so the heat loss is not obvious. Also, increase the room temperature as much as possible by using an air-conditioner.

Extension

In this experiment, some values are set such as the thermal expansion coefficient and Young's modulus. By delving into both concepts, an extension would be to improve the methodology and find two values specifically for the low E guitar string. The measurement of the coefficient can be done by using dilatometry and interferometry. The measurement of Young's modulus can be done by finding the stress and the strain of a string and dividing them, which is a specific form of Hooke's law.

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