How does the current change with charging time with the same resistance and capacitance in an RC circuit?

1. Introduction

I am a student who is really interested in Physics. I usually watch some short videos about science and technology on TikTok and YouTube to fulfill my curiosity. I once learned from a video that RC circuit is widely used in analog circuits and pulse digital circuits. As the circuit form, signal source and parameters of resistance and capacitance are different, RC circuit has a variety of applications: differential circuits, integral circuits, coupling circuits, filter circuits, and pulse voltage division circuits. RC circuit is one of the most important circuits in the world. I also learned that capacitors with extremely large capacitance could be used as charge banks. When I am charging my charge bank, I always hope that the charging rate will be faster, and I feel interested in the time needed to charge a charge bank. Therefore, I decided to construct an experiment to explore the relationship between the current of the RC circuit and the charging time of charging the capacitor. During the exploration, many failed experiments were conducted to select suitable equipment to reduce error for this experiment.

1.1 Background Information

A capacitor is a passive electronic component with two terminals that stores electrical energy in an electric field by accumulating charge on two closely spaced surfaces which are insulated from each other (as shown in Figure 1). The working principle of a capacitor is that charge will be forced to move in the electric field. When there is dielectric between conductors, it will hinder the movement of charge and make the charge accumulate on the conductor, resulting in the accumulation of charge storage. The dielectric may be air, paper, plastic, or any other substance that does not conduct electricity and prevents the two metal poles from coming into contact with each other. The metal plate attached to the negative electrode of the battery will absorb the electrons produced by the battery, and the metal plate attached to the positive electrode of the battery will release electrons to the battery. After charging, the capacitor will have the same voltage as the battery. The effect of a capacitor is called capacitance(C, unit: F), which can be calculated by the equation below:

$$C = \frac{Q}{V}$$

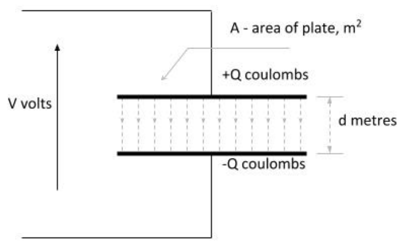


Figure 1.Sketch map of capacitor (Source: myElectrical Engineering)

The resistor is also a passive two-terminal electrical component that implements resistance to the circuit. It represents the leakage resistance of the capacitor. Time constant(τ) is the product of resistance(R) and capacitance(C). It has the units of seconds. It could be represented by the equation below:

$$\tau = R \times C$$

According to the Kirchhoff's circuit laws:

$$\varepsilon = V_R + V_c$$

And by using Ohm's Law $V = IR$ and $V = \frac{Q}{C}$, we can obtain:

$$\varepsilon = IR + \frac{Q}{C}$$

Q means the charge on the end of the capacitor after time t with the unit of coulomb(C). By using $\frac{dQ}{dt} = I$, we can get:

$$\varepsilon - \frac{Q}{C} = \frac{dQ}{dt} \times R$$

By integrating both sides of the equation and changing the positions of the variables on the two sides of the equation, we can obtain:

$$\int \frac{dQ}{\varepsilon - \frac{Q}{C}} = \int \frac{1}{R} dt$$

By solving the equation, we can obtain:

$$-C\ln\left|\varepsilon - \frac{Q}{C}\right| = \frac{t}{R} + \text{constant}$$

After making both sides be exponents of e and simplifying the equation, we can get:

$$\varepsilon - \frac{Q}{C} = e^{-\frac{t}{RC}} \times \text{constant}$$

There is no charge on the end of the capacitor initially. In other words, when t=0 sec, Q=0 coulombs. By putting these values into the equation above, we can obtain:

$$\text{constant} = \varepsilon$$

Q_0 represents the total charge of the RC circuit with the unit of coulomb(C). By combining the two equations above, we can obtain:

$$Q = Q_0(1 - e^{-\frac{t}{RC}})$$

By dividing both sides of the equation by t, we can determine that the current of the RC circuit (as shown in Figure 2 below) changes with the charging time:

$$I = I_0 e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}}$$

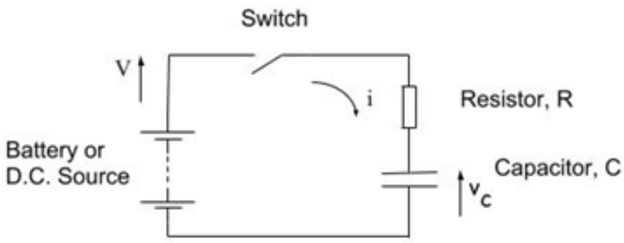


Figure 2.Sketch map of RC circuit (Source: myElectrical Engineering)

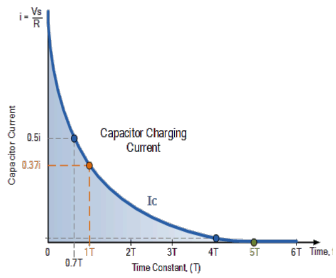


Figure 3.Schematic diagram of the relationship between the current and the charging time (Source: Electronic Tutorials)

I represents the current in the RC circuit after charging the capacitor for t with the unit of Ampere(A). I_0 represents the initial current of the RC circuit with the unit of Ampere(A). t is the charging time of the capacitor with the unit of seconds(s). R represents the resistance of the resistor R with the unit of Ohm(Ω). C represents the capacitance of the capacitor C with the unit of Farad(F).

1.2 Hypothesis

According to the law of conservation of charge, as the charging time increases, it means more and more charges are attached to the two surfaces of the capacitor, so the current of the circuit will decrease as the charging time increases. In this way, the current in the RC circuit can be calculated by $I = I_0 e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}}$, which means

$$ln(\frac{I}{I_0}) \propto t$$

1.3 Assumptions

1. There is no resistance in the wires of the RC circuit.
2. The battery is relatively new with constant output charge.
3. The dielectric between the two surfaces of the capacitor is uniform, and it has a uniform electric field.
4. Heat produced by the RC circuit is ignored.

2. Methodology

2.1 Variables

Table 1. Independent, dependent and controlled variables in the experiment

Independent variable	Charging time of the capacitor. Its unit is s, and I used a stop watch with uncertainty $\pm 0.01\text{s}$ to measure the charging time of the capacitor.	
Dependent variable	The current in the RC circuit after time *t*. It is measured by the ammeter with the unit of A. It has an uncertainty of $\pm 0.0001\text{A}$.	
Controlled variables	*Why to control*	*How to control*
Resistance of the resistor	According to the derived equation, if *R* increases, *I* will decrease. Therefore, controlling the resistance of the resistor in the experiment can avoid the change of the current	I used the same resistor in the experiment. Thus, the resistance was the same.
Capacitance of the capacitor	According to the derived equation, if *C* increases, *I* will decrease. Therefore, controlling the capacitance of the capacitor in the experiment can avoid the change of the current.	I used the same capacitor in the experiment. Thus, the capacitance was the same.

Note: As the theoretical equation I derived is $I = I_0 e^{-\frac{t}{RC}}$, I will use $\ln(\frac{I}{I_0})$ to find the relationship between I and t. Thus, it will be easier for me to calculate the uncertainties and errors of my data and graph.

2.2 Apparatus

- Resistor
- Capacitor
- Wires
- Power source
- Electric current sensor (± 0.0001)/A
- LabQuest

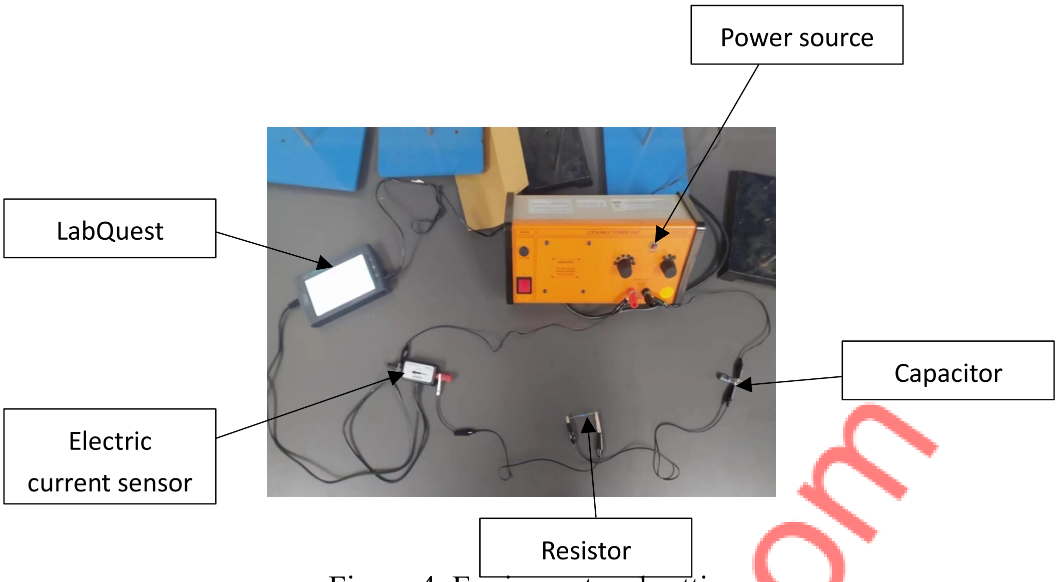


Figure 4. Equipment and setting

2.3 Safety consideration

Hazards: During the conduction of experiment, I need to avoid people touching the whole RC circuit. Also, I have to pay attention to the electric shock when I close the switch. Before the experiment, I should check the wires to prevent problems, such as short circuit.

Environmental issue: As we should save the resources to protect the environment, the wires, the resistor, the capacitor, the stop watch, the ammeter and the LabQuest can be kept after this experiment for reusing.

Ethical issue: There is no ethical consideration since I do not use any living creatures in my experiment.

2.4 Procedures

1. Build the RC circuit as seen in Figure 4
2. Turn on the LabQuest and connect it with the ammeter
3. Close the switch and start the stop watch at the same time
4. Wait for 5τ and close the switch
5. Clean up and return the equipment used back to the laboratory

Notes:

Why I need to wait for 5τ: it is impossible for us to make the capacitor be fully charged according to the derived equation. To solve this problem, I searched on the Internet and found out that 5τ is the time needed to nearly fully charge a 0.001F capacitor in the RC circuit. It is so nearly fully charged that it could be seen as fully charged.

3. Data Interpretation

3.1 Representations

Concept	Representations
Resistance of the resistor	R
Capacitance of the capacitor	C
Charging time	t
Initial current in the circuit	I_0
Current in the circuit after charging time t in the trial n	I_n
The potential difference between 2 ends of the power source	ε
Time constant	τ

3.2 Data collection

<u>Quantitative Data:</u>

Table 1. Raw data table

Trial #	Resistance of the resistor R (Ω)	Capacitance of the capacitor C (F)	Initial current in the circuit I_0 (A ± 0.0001A	Current in the circuit $I_{0.5}$ (A ± 0.0001A) (After charging time of 0.05s ± 0.01s	Current in the circuit I_1 (A ± 0.0001A (After charging time of 0.10s ± 0.01s)	Current in the circuit $I_{1.5}$ (A ± 0.0001A) (After charging time of 0.15s ± 0.01s)	Current in the circuit I_2 (A ± 0.0001A (After charging time of 0.20s ± 0.01s)	Current in the circuit $I_{2.5}$ (A ± 0.0001A (After charging time of 0.25s ± 0.01s	Current in the circuit I_3 (A ± 0.0001A (After charging time of 0.30s ± 0.01s)	Current in the circuit I_4 (A ± 0.0001A (After charging time of 0.40s ± 0.01s)	Current in the circuit I_5 (A ± 0.0001A (After charging time of 0.50s ± 0.01s)
1	120	0.001	0.0696	0.0406	0.0329	0.0218	0.0197	0.0092	0.0083	0.0011	0.0008
2	120	0.001	0.0792	0.0583	0.0208	0.0177	0.0108	0.0090	0.0089	0.0026	0.0002
3	120	0.001	0.0721	0.0560	0.0211	0.0197	0.0126	0.0111	0.0051	0.0018	0.0013
4	120	0.001	0.0733	0.0430	0.0205	0.0167	0.0129	0.0074	0.0036	0.0029	0.0012
5	120	0.001	0.0749	0.0463	0.0332	0.0229	0.0108	0.0106	0.0105	0.0032	0.0001

Table 2. Processed data table

Time constant τ (s)	Charging time t ($s \pm 0.01s$)	Average Current in the circuit I ($A \pm 0.0001A$)
0.12	0.00	0.0738
0.12	0.05	0.0488
0.12	0.10	0.0257
0.12	0.15	0.0198
0.12	0.20	0.0134
0.12	0.25	0.0095
0.12	0.30	0.0073
0.12	0.40	0.0023
0.12	0.50	0.0007

Note:

1. Average current in the circuit I can be calculated by the equation below:
$$I = \frac{I_a + I_b + I_c + I_d + I_e}{5}$$
2. The uncertainty for average current in the circuit I is:
$$\frac{0.0001\text{A} + 0.0001\text{A} + 0.0001\text{A} + 0.0001\text{A} + 0.0001\text{A}}{5} = 0.0001\text{A}$$
3. Time constant τ could be calculated by using the equation below:
$$\tau = R \times C = 120\Omega \times 0.001\text{F} = 0.12\text{s}$$

Qualitative data:

1. Before turning on the power source: The current shown on the lab quest is keeping in the range from 0.06A to 0.08A
2. After turning on the power source: the current shown on the LabQuest gradually decrease dramatically. Then the current will be much closer to 0A.

3.3 Data processing 1: qualitative data analysis

According to the qualitative data we got from the experiment, it shows that the more charging time is, the lower electric current becomes. Additionally, the value of electric current decreased at a faster rate at the start of charging time than at the end. So it decreased exponentially.

3.4 Data processing 2: deduction of the value of ($\ln(\frac{I}{I_0})$), uncertainty and error bars choose

The uncertainty of the value the electric current is $\pm 0.0001\text{A}$ because it is measured by the electronic(digital) device LabQuest and uncertainty of the values measured by such devices should be the smallest scale of the digital devices. The uncertainty of time is $\pm 0.01\text{s}$ for the same reason.

According to my hypothesis,

$$\ln(\frac{I}{I_0}) = kt$$

Thus, in order to find the relation between I and t, a graph of $\ln(\frac{I}{I_0})$ with respect to t needs to be drawn, and uncertainty of $\ln(\frac{I}{I_0})$ could be calculated as below:

For example, when I=0.0257A and t=0.10s, percentage uncertainty of I is:

$$\left|\frac{\Delta I}{I}\right| = \frac{0.0001A}{0.0257A} \times 100\% = 0.389\%$$

Percentage uncertainty of I_0 is:

$$\left|\frac{\Delta I_0}{I_0}\right| = \frac{0.0001A}{0.0738A} \times 100\% = 0.136\%$$

Thus, absolute uncertainty of $\ln(\frac{I}{I_0})$ is:

$$\Delta \ln\left(\frac{I}{I_0}\right) = \frac{\Delta\left(\frac{I}{I_0}\right)}{\left(\frac{I}{I_0}\right)} = \frac{\Delta I}{I} + \frac{\Delta I_0}{I_0} = 0.389\% + 0.136\% = 0.00525$$

Additionally, the value of $\ln(\frac{I}{I_0})$ is:

$$\ln\left(\frac{I}{I_0}\right) = \ln\left(\frac{0.0257\text{A}}{0.0738\text{A}}\right) = -1.055 \approx -1.06$$

Finally, the percentage uncertainty of $\ln(\frac{I}{I_0})$ is:

$$\left|\frac{\Delta \ln(\frac{I}{I_0})}{\ln(\frac{I}{I_0})}\right| = \left|\frac{0.00525}{-1.06} \times 100\%\right| = 0.5\%$$

Similarly, I can calculate the other values of $\ln(\frac{I}{I_0})$ and their corresponding percentage uncertainties, which are shown in the table 3 below:

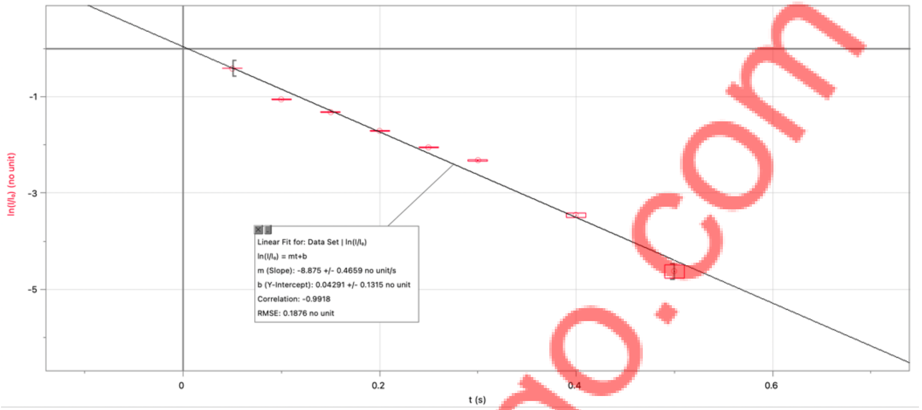
Table 3. Value and percentage error of $(\ln(\frac{I}{I_0}))$

Charging time t ($s \pm 0.01s$)	Value of $(\ln(\frac{I}{I_0}))$ (no unit)	Percentage uncertainty of $(\ln(\frac{I}{I_0}))$
0.05	-0.413	0.8%
0.10	-1.06	0.5%
0.15	-1.32	0.5%
0.20	-1.71	0.5%
0.25	-2.05	0.6%
0.30	-2.32	0.7%
0.40	-3.46	1%
0.50	-4.63	3%

All the values of $\ln(\frac{I}{I_0})$ take 3 significant figures according to my calculation process. Then I will draw graphs to try to find the relationship between $\ln\left(\frac{I}{I_0}\right)$ and *t* by using the software **LoggerPro**.

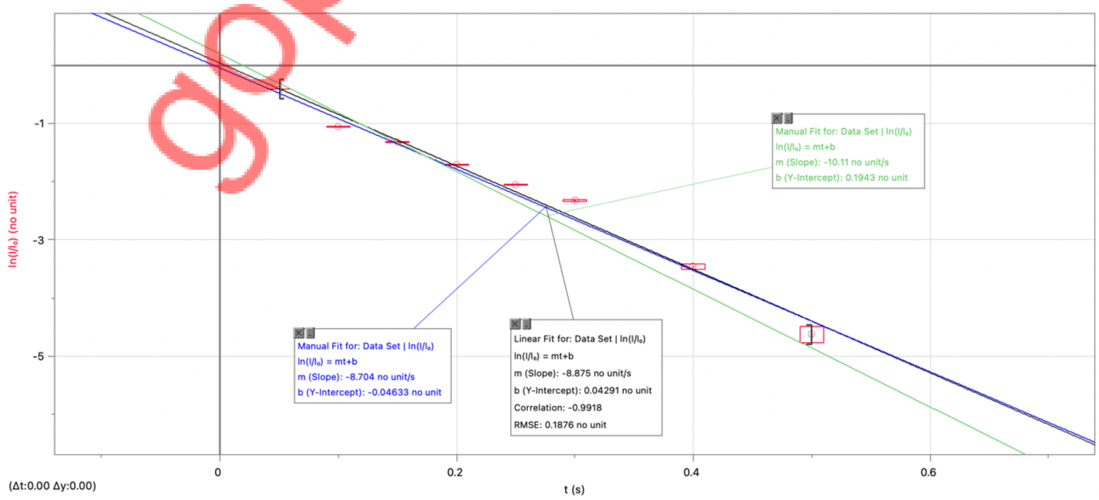
3.5 Graphs

Graph 1: Best-fit line of relationship between $\ln\left(\frac{I}{I_0}\right)$ and *t* (graph in Logger Pro)



The correlation coefficient of this best-fit line is -0.9918, which is very close to -1. This means that the relationship between $\ln\left(\frac{I}{I_0}\right)$ and *t* follows a really strong linear relationship. Since there are error bars and uncertainties of $\ln\left(\frac{I}{I_0}\right)$ and *t*, I also need to draw two worst-fit lines of the relationship between $\ln\left(\frac{I}{I_0}\right)$ and *t*, which are known as the steepest line and the shallowest line to go through error bars. They are shown below.

Graph 2: Worst-fit lines and best-fit line of relationship between $\ln\left(\frac{I}{I_0}\right)$ and *t* (graph in Logger Pro, the blue line and the green line are the worst-fit lines)



3.6 Data processing 3: calculate the coefficient k

According to the graph 2, I can deduce that:

$$\ln\left(\frac{I}{I_0}\right) = -8.875t + 0.04291$$

For the value 0.04291, there may be systematic errors or random errors that caused it, which I will explain later.

From the graph, we can know the 2 worst-fit lines are:

$\ln\left(\frac{I}{I_0}\right) = -8.704t - 0.04633$ and $\ln\left(\frac{I}{I_0}\right) = -10.11t + 0.1943$

Therefore, the percentage uncertainty for k is:

$$\text{Percentage uncertainty} = \frac{\left|\frac{(-8.704) + (-10.11)}{2} - (-8.875)\right|}{8.875} \times 100\% \approx 6\%$$

<u>Comment on percentage uncertainty:</u>

k, also known as my result, has a percentage error of 6%, which is not very big. Both random errors and systematic errors could cause such percentage error.

4. Conclusion

According to my data collection and processing, I finally obtained the relationship between the current and charging time with the same resistance and capacitance in a RC circuit, which is: $\ln\left(\frac{I}{I_0}\right) = -8.875t + 0.04291$.

It can also be written as $I = I_0 e^{-8.875t+0.04291}$,which has a percentage uncertainty of $\pm6\%$. Thus, the longer time we charge the capacitor, the smaller current the RC circuit has. But according to my hypothesized equation, the relationship between the current and charging time with the same resistance and capacitance in an RC circuit should be: $I = I_0 e^{kt}$, so there may be systematic or random errors that will cause the value of 0.04291.

<u>Comment on results:</u>

There are be both systematic or random errors that will cause the value of 0.04291(non-zero error), as I stated before. That's because the equation I obtain now doesn't intersect with the y-axis at (0,0).

<u>Result justification:</u>

According to the theoretical equation I deduced in **1.1 Background Information** which is $I = I_0 e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}}$, the theoretical value of k should be $\left(-\frac{1}{RC}\right)$ or $\left(-\frac{1}{\tau}\right)$.

So according to my experiment, the theoretical value of k is:

$$k = -\frac{1}{RC} = -\frac{1}{\tau} = -\frac{1}{0.12s} \approx -8.333s^{-1}$$

Thus, the error of my result is:

$$\text{Error} = \left|\frac{-8.875 - (-8.333)}{-8.333}\right| \times 100\% \approx 6.5\%$$

My result is close to the theoretical result, and the error is caused by both systematic errors and random errors. Some of the factors are somewhat inaccurate values of resistance and capacitance, resistances of measuring instruments, and so on.

5. Evaluation

5.1 Strengths of my experiment

The strengths of my experiment are:

1. Simplicity: My experiment is simple and does not contain too many steps. This means that the uncertainties of my final result are relatively small. It is also convenient for me to repeat my experiments and obtain different values.
2. Cost-effective: The RC circuit I used is also very simple, and it does not have many electronic components. These components are also very common and available. Hence, the cost of my experiment is cheap.

However, as I stated before in **4. Conclusion**, the error of k is 6.5%, and there is a value "0.04291" which could be caused by the limitations of the experiment below.

5.2 Limitations of my experiment

5.2.1 Systematic errors

1. Inaccuracies of values of components: The components of my circuit have manufacturing tolerances, such as resistor and capacitor. Therefore, the actual values of these components may be different from the stated values. It will cause systematic errors and affect accuracy.

Solution: Use electronic components that are of high quality and low manufacturing tolerances. Thus, the values of these electronic components can be more authentic.

2. Additional resistance or capacitance: There may be additional resistance and capacitance existing in the measuring device—electric current sensor. This will cause the final result to be bigger than the theoretical value because with additional resistance or capacitance, $-\frac{1}{RC}$ will become bigger.

Solution: Measure the resistance and capacitance of the measuring device, and add them to the data. Thus, the results will be more accurate.

5.2.2 Random errors

1. Electrical noises: Electrical noises can be from external sources, such as electromagnetic interference, or internal electronic components. They can cause random fluctuations in my measurements, which will lead to random error.

Solution: Shield the circuit from external electric noises by using filtering or shielding barrier equipment. Thus, the circuit will be more stable.

2. Environmental factors: Unstable environmental temperature can impact the behaviors of electronic components. If the temperature increases, the wire may be

longer and thinner due to thermal expansion effect. If the resistor has a longer and thinner resistance wire, it will have a bigger resistance, which will further cause $-\frac{1}{RC}$ to be bigger.

Solution: Conduct the experiment in a controlled environment with a stable temperature. Thus, the resistance will be more stable.

5.3 Extension

In my exploration, I determined the relationship between the electric current and charging time of the capacitor in the RC circuit according to the coefficient k. In future exploration, I may research other factors that will impact the electric current in the RC circuit. For example, I can change the resistance or capacitance to find the relationship between them and the electric current. This makes me understand the basic theories of an RC circuit and how it works more clearly.

6. Bibliography:

McFadyen, Steven. "Capacitor Theory." MyElectrical, 12 Jun. 2013, myelectrical.com/notes/entryid/221/capacitor-theory.

"RC Charging Circuit." *ElectronicsTutorials*, www.electronics-tutorials.ws/rc/rc_1.html.