How does the change in length of the secondary pendulum affect the period of the pendulum as a whole?

Physics IA

1.1 Introduction

During my childhood, the yo-yo was always a popular toy in our community among kids. During that time, watching TV programs about yo-yos, were investigating the techniques of yo-yo ourselves. Among a lot of advanced techniques, lots of them involve an action which uses one of the fingers to block the string to form a secondary pendulum, resulting in a faster and fiercer flipping of the yo-yo. This is in the Fig.1 below ("Learn How to Do the Pinwheel Yoyo Trick"):

Fig.1 Basic motion creating a secondary pendulum from yoyotricks.com

Thus, from that time, I began thinking about whether a secondary pendulum like that would affect the period of the pendulum as a whole.

1.2 Background knowledge

According to the IB physics textbook, the formula for simple harmonic motion having a small angle within 10° is (Tsokos):

$$T = 2\pi\sqrt{\frac{l}{g}}$$

In this equation, T stands for the period in seconds; l for the length of the string in meters; π and g stands for two constants.

Therefore, from the equation, we can see that the mass hung under the string would not affect the period of the pendulum. The only factor would be the length of the string. But for the secondary pendulum, it has two half pendulums with two string lengths. So, it would go through two different half periods (see Fig.2)

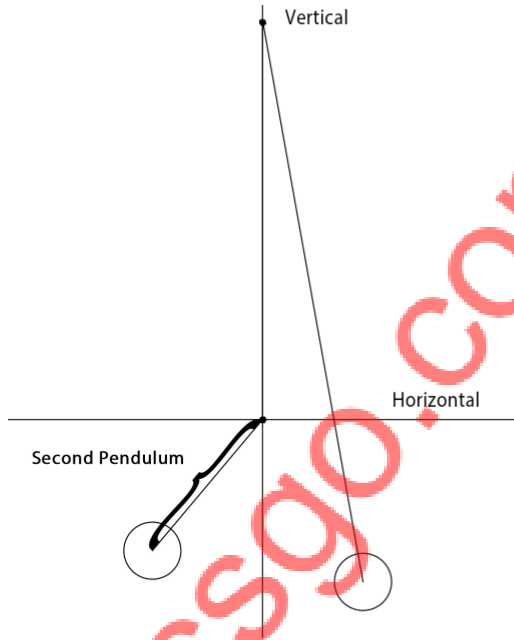


Fig.2 Secondary pendulum drawn in CAD

thereby, I would hypothesize the formula to be:

$$T = \pi\sqrt{\frac{l_1}{g}} + \pi\sqrt{\frac{l_2}{g}}$$

where l_1 is the length of the string as a whole and l_2 would be the length of the string within the secondary pendulum. For this experiment, since I am going to analyze the relationship between the secondary length and the period of the pendulum, I would control l_1 to be a constant, and only change l_2 to determine the trend in the period. Thus, the first part of the equation, $\pi\sqrt{\frac{l_1}{g}}$, is a constant.

As a result, I am hypothesizing the graph showing the relationship between l_2 and the T will follow the equation $y = a\sqrt{x} + b$.

1.3 Hypothesis

I am hypothesizing the graph showing the relationship between l_2 and the T will follow the equation $y = a\sqrt{x} + b$.

1.4 Research question

How does the change in length of the secondary pendulum affect the period of the pendulum as a whole?

2.1 Variables

The independent variable in the investigation is the length of the secondary pendulum. This can be changed by changing the contact point along the string.

The dependent variable would be the period of the pendulum. This can be measured by using a stop watch to measure the time for the pendulum to complete three period, and then, dividing the measurement by three, so that the period can be measured with less uncertainty.

The first controlled variable would be the length of the initial pendulum. This can be controlled through using a fixed 1m string. This should be controlled because the length of the whole pendulum would definitely affect the pendulum as a whole, affecting the experiment result. The second controlled variable is the mass hanging below the string. This is controlled by using the same mass throughout the trials. This is because in reality, there is air resistance. The more mass, the better it would be to combat air resistance, affecting the measured result of the period T. Thus, it should be controlled. The third variable which is controlled is the starting angle, or maximum displacement, of the pendulum. This is controlled because the mass would be going through different paths of air resistances if the angles are different, resulting in alternative results. Thus, the angle of the initial pendulum should be constant.

2.2 Materials

The materials I used are list below:

1. Microphone stand
2. Music stand
3. Microphone wire
4. Stop watch

The setup of the experiment is shown in Fig.3 below:

Fig.3 The experiment setup

2.3 Procedure

1. Hang the microphone wire 1.0 meter below the microphone stand, making the mass at the end of the wire the lowest point as a mass.

2. Set the height of the music stand so that the contact point has a distance of 20cm right above the mass.

3. Hold the mass at the end of the wire 8 degrees away from the right angle with respect to the microphone stand.

4. Release the mass and start the stop watch.

5. Stop and record the stop watch after the mass completes 3 periods.

6. Repeat steps 3-5 for three more trials.

7. Repeat steps 2-5 with the contact point of 30cm, 40cm, 50cm, 60cm, 70cm, 80cm right above the mass.

2.4 Ethical and safety concerns

The height of the music stand should be judged carefully because it might fall to your hand without the tension provided by the screw.

There are no ethical concerns in this experiment.

3.1 Raw data table

Table 1 shows the time measured for completing 3 periods in different trials with changing pendulum length.

Length of the secondary pendulum (±0.1cm)	Time measured for 3 periods in trial 1 (±0.01s)	Time measured for 3 periods in trial 2 (±0.01s)	Time measured for 3 periods in trial 3 (±0.01s)	Time measured for 3 periods in trial 4 (±0.01s)
20.0	3.99	4.11	4.02	3.89
30.0	4.33	4.34	4.32	4.37
40.0	4.45	4.53	4.52	4.50
50.0	4.88	4.78	4.83	4.78
60.0	5.06	5.09	5.10	5.08
70.0	5.30	5.22	5.22	5.27
80.0	5.35	5.46	5.41	5.42

Table 1. Period with respect to the length of secondary pendulum

3.2 Data processing

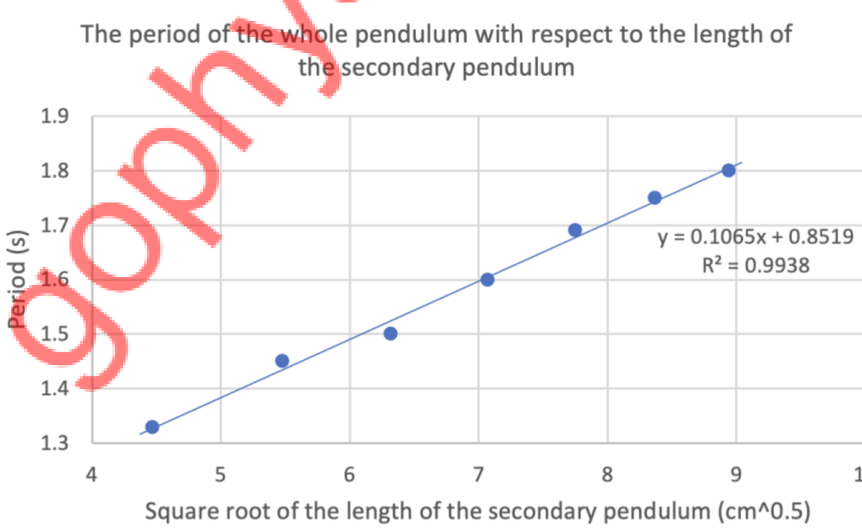
Table 2 shows the relationship between the square root of the secondary pendulum length and the average time taken for the pendulum to compete 1 period.

Length of the secondary pendulum ($\pm$0.1cm)	Average time taken for 3 periods ($\pm$0.01s)	Average time taken for 1 period ($\pm$0.01s)	$\sqrt{l_2}$
20.0	4.00	1.33	4.47
30.0	4.34	1.45	5.48
40.0	4.50	1.50	6.32
50.0	4.81	1.60	7.07
60.0	5.08	1.69	7.75
70.0	5.25	1.75	8.37
80.0	5.41	1.80	8.94

Table 2. Period with respect to the square root of the length of the secondary pendulum

3.2 Graph

The relationship between the period of the pendulum as a whole and the square root of the length of the secondary pendulum can be shown in graph 1 below:



Graph.1. The relationship between the period of the pendulum as a whole and the length of the secondary pendulum

Therefore, according to the graph, we can determine there is a linear relationship between the $\sqrt{l_2}$ and the period of the pendulum.

Additionally, more calculations can be taken to prove:

$$\pi\sqrt{\frac{l_1}{g}} = \pi\sqrt{\frac{1\text{m}}{9.81\text{N/g}}} = 1.0\text{s} \approx 0.85\text{s}$$

$$\pi\sqrt{\frac{1}{g}} = \pi\sqrt{\frac{1\text{m} \times 0.01\text{m/cm}}{g}} = 0.1003 \approx 0.1065$$

Therefore, this would prove my hypothesis that:

$$T = \pi\sqrt{\frac{l_1}{g}} + \pi\sqrt{\frac{l_2}{g}}$$

Where $\pi\sqrt{\frac{l_1}{g}}$ is the vertical axis and $\sqrt{l_2}$ is the independent variable.

3.3 Conclusion

In conclusion, the period of the pendulum has a linear relationship with the square root of the length of the secondary pendulum.

According to the GFG, the formula for pendulums can be derived through the equation of motion (Han):

$$T - mgcos\theta = mv^2L$$

Where T is the torque in Nm; m is the mass hung from the string; g is the gravity constant; θ is the angle of the pendulum from its equilibrium; v is the velocity of the mass; L is the length of the string.

Additionally, the pendulum can be seen as a lever that is motivated by a torque on the string. Therefore, the formula of the torque that motivates the pendulum to its equilibrium can be given as:

$$T = mgL \times sin\theta = mgsin\theta \times L = I \times a$$

Where I is the momentum of inertia; and the a is the circular acceleration. According to Han, for small angles of oscillations (Han):

$$sin\theta = \theta$$

In this way, the equations can be simplified as:

$$Ia = -mgL\theta$$

$$a = -\frac{mgL\theta}{I}$$

$$-\omega_0^2\theta = -\frac{mgL\theta}{I}$$

$$\omega_0 = \sqrt{\frac{mgL}{I}}$$

Then, since:

$$I = mL^2$$

So:

$$\omega_0 = \sqrt{\frac{g}{L}}$$

Take this equation into the formula (Tsokos):

$$T = \frac{2\pi}{\omega_0}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Therefore, the formula for the period of a single pendulum is derived, adding a node would result this formula to merely add another repetitive part:

$$T = \pi\sqrt{\frac{l_1}{g}} + \pi\sqrt{\frac{l_2}{g}}$$

As a result, the period of the pendulum has a linear relationship with the square root of the length of the secondary pendulum.

4.1 Evaluation

Throughout the experiment, some advantageous efforts were made to reduce the errors.

Firstly, I would say the trials were repeated sufficiently to reduce the error. Because I repeated each trial four times and calculated the averages time on them, I would say this greatly reduces the random error.

Secondly, measuring the time taken for three periods should be considered as an advantage. Because human eyes cannot be precise when measuring one period, even deciding whether the period ends or not can be very difficult for human eyes. Therefore, taking the measurement of 3 periods can greatly reduce the random errors caused by human eyes.

Thirdly, I am going talk about my graph analysis using the square root of the length of the secondary pendulum as an independent variable. This is because directly using the length as an independent variable can be very confusing, which means it would be hard for human eye to decide which type it should be. But for the square root one, we can directly prove it through the calculation to the components of the equation, reducing random error caused by human brain.

Some errors are present during the investigation as well.

Firstly, the hardness of the microphone can bring some systematic error. Due to the thickness of it, the wire can give a force of resistance while bending around over the contact point. (Shown in Fig.4) In this way, the period measured would be smaller than it should be, involving systematic error. This error can be improved through the usage of softer strings with less thickness and mass, so that they would provide a less resistance, thereby, the reduced systematic error.

Fig.4

Secondly, the mass hanging from the wire. This is because I was directly using the plug from the wire instead of really hanging a mass from it, the mass of the plug might be too small, presenting a greater influence caused by the air resistance. In this way, the systematic error brought by air resistance might increase the period we measured. This can be improved by hanging a greater mass from the string, so that it can be better combating air resistance, and thereby the decreasing systematic error.

Thirdly, I would also like to mention about the contact point. During the experiment, I could tell that in some of the trials, the contact surface between the music stand and the wire was not exactly perpendicular with each other, resulting in a slip during the oscillation. In this way, it might involve random error. This can be improved by using an angled board so that the string would not be so flexible while reaching the contact point, reducing the random error.

Works Cited

Basumatary, Beny. “Sound Waves: Meaning, Characteristics, Types, Echo, Applications.” *Testbook Learn*, 10 June 2021, testbook.com/learn/physics-sound-waves/. Accessed 4 Nov. 2022.

BYJU'S. “Sound Waves - Nature, Speed, Reflection of Sound with Formulas.” *BYJUS*, byjus.com/physics/sound-waves/#:~:text=What%20Is%20sound?. Accessed 4 Nov. 2022.

Han, Sanskrutiud. “Simple Pendulum - Definition, Formulae, Derivation, Examples.” *GeeksforGeeks*, 20 Mar. 2022, www.geeksforgeeks.org/simple-pendulum-definition-formulae-derivation-examples/#:~:text=Derivation%20of%20the%20potential%20energy%20of%20the%20simple. Accessed 6 Dec. 2022.

Tsokos, K A. *Physics for the IB Diploma Exam Preparation Guide*. Cambridge, Cambridge University Press, 2016.

Vedantu. “Resonance in Physics | Types and Examples of Resonance.” *Www.vedantu.com*, www.vedantu.com/physics/resonance.

---. “Speed of Sound Formula.” *VEDANTU*, www.vedantu.com/formula/speed-of-sound-formula.

Wolfe, Joe. “Helmholtz Resonance.” *Www.phys.unsw.edu.au*, www.phys.unsw.edu.au/jw/Helmholtz.html#name1. Accessed 7 Nov. 2022.