Name: ______________________

Class: ______________________

Due Date: ______________________

D.1 Gravitational Fields

Understandings

- Kepler's three laws of orbital motion.
- Newton's universal law of gravitation as given by $F = \frac{Gm_1m_2}{r^2}$ for bodies treated as point masses.
- Conditions under which extended bodies can be treated as point masses.
- Gravitational field strength g at a point is the force per unit mass experienced by a small point mass at that point as given by $g = \frac{F}{m} = G\frac{M}{r^2}$.
- Gravitational field lines.

Equations

$$F = \frac{Gm_1m_2}{r^2}$$

$$g = \frac{F}{m} = G\frac{M}{r^2}$$

Additional HL Understandings

- The gravitational potential energy E_p of a system is the work done to assemble the system from infinite separation of the components of the system.
- The gravitational potential energy for a two-body system as given by $E_\mathrm{p} = -G\frac{m_1m_2}{r}$ where r is the separation between the center of mass of the two bodies.
- The gravitational potential V_g at a point is the work done per unit mass in bringing a mass from infinity to that point as given by $V_\mathrm{g} = -G\frac{M}{r}$.
- The gravitational field strength g as the gravitational potential gradient as given by $g = -\frac{\Delta V_\mathrm{g}}{\Delta r}$.
- The work done in moving a mass m in a gravitational field as given by $W = m\Delta V_\mathrm{g}$.

- Equipotential surfaces for gravitational fields.
- The relationship between equipotential surfaces and gravitational field lines.
- The escape speed v_{esc} at any point in a gravitational field as given by $v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}$.
- The orbital speed v_{orbital} of a body orbiting a large mass as given by $v_{\mathrm{orbital}} = \sqrt{\frac{GM}{r}}$.
- The qualitative effect of a small viscous drag due to the atmosphere on the height and speed of an orbiting body.

Additional HL Equations

$$E_{\mathrm{p}} = -\frac{Gm_1m_2}{r}$$

$$V_{\mathrm{g}} = -\frac{GM}{r}$$

$$g = -\frac{\Delta V_{\mathrm{g}}}{\Delta r}$$

$$W = m\Delta V_{\mathrm{g}}$$

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}$$

$$v_{\mathrm{orbital}} = \sqrt{\frac{GM}{r}}$$

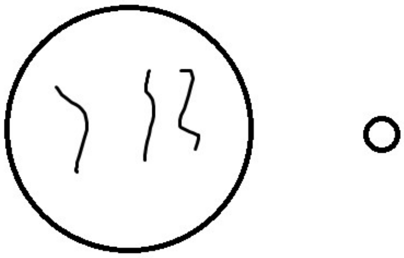
The solutions can be found on the YouTube channel Go Physics Go:

https://www.youtube.com/@gophysicsgo/playlists

1. C: State the names and describe the laws of Kepler’s three laws of orbital motion.

2. C: Define *Newton’s Law of Gravitation* $\vec{F}_{\mathrm{g}} = m_1\vec{g} = \frac{Gm_1m_2}{r^2}$. Units?

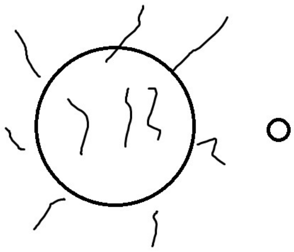
3. C: A satellite in space moves in a circle around the Earth with a constant speed at a radius r from the center of the Earth. Label the direction of the velocity, force, and acceleration of the satellite in the diagram below.



4. E: The force of gravity between a satellite circling the Earth at a distance r at a constant speed is F_g. What will happen to the magnitude of the force of gravity between the satellite and the Earth if the satellite moves a distance

a. $2r$. b. $3r$. c. $4r$. d. $r/2$. e. $r/3$. f. $r/4$.

5. C: A satellite is moving in a circle with a constant speed around the sun.



   a. Use Newton's second law of motion to obtain an equation for the speed of the satellite in terms of the mass of the sun M_{sun}, the mass of the satellite $M_{\text{satellite}}$, the distance of the satellite to the sun *r*, and the gravitational constant *G*.

   b. Use your solution to obtain an equation for the period *T* of the satellite.

6. E: The mass of the Sun is approximately 1.99×10^{30} kg. The Earth is approximately 1.50×10^{11} m from the Sun. Use this information to determine

   a. the speed of the Earth in m/s and

   b. the period of the Earth in days.

7. E: A satellite, which has a mass of 550. kg and a radius of 2.20 meters, is orbiting the Earth at an altitude of 375 km.

   a. What will be the magnitude of the gravitational force between this satellite and the Earth?

   b. What must the velocity of this satellite be in order for the satellite to remain in a stable orbit?

   c. What will be the magnitude of the centripetal acceleration of this satellite?

   d. How long, in seconds, will it take for this satellite to orbit the Earth once?

8. C: The following problem refers to *gravitational field strength*.

   a. Define g*ravitational field strength*. Is it a scalar or a vector?

   b. What is the equation and what are the units for *gravitational field strength*? Define and give the units of each variable.

c. Where is the *gravitational field strength* zero? Where is the *gravitational field strength* maximum?

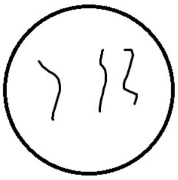
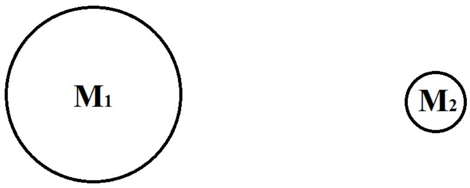
d. What are the mathematical limits of *gravitational field strength*? Can *gravitational field strength* be positive? Negative? Zero?

9. E: The mass of the Earth is approximately 5.97×10^{24} kg and its radius is approximately 6.38×10^{6} m. Use the equation $g = \frac{GM}{r^2}$ to determine the acceleration of gravity near the Earth's surface.

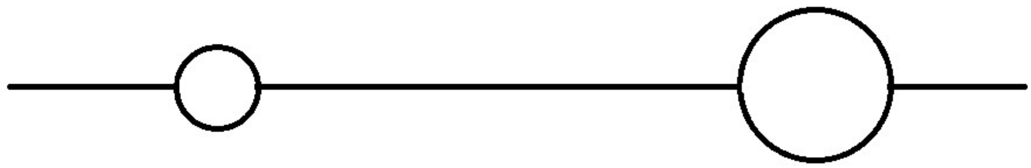
10. C: Draw a *gravitational field strength vs. distance* graph for a planet with a radius *r*.

11. C: List some rules in drawing gravitational field lines.

12.C: **Use a pencil and ruler!** Draw gravitational field lines for each figure.

<table>
<tr><td>A fixed uniform spherical mass
</td><td>Two uniform spherical masses with equal mass and equal radius
</td></tr>
<tr><td colspan="2">Two fixed uniform spherical masses $M_1 \gg M_2$
</td></tr>
</table>

13.E: The center of a planet of mass of $m_1 = 4M$ is a distance of $5d$ from the center of a larger planet of mass $m_2 = 9M$. Assume that the mass of the planets can be concentrated at their centers.



What will be the gravitational field strength (both magnitude and direction) at a point

a. $2d$ to the left of mass m_1?

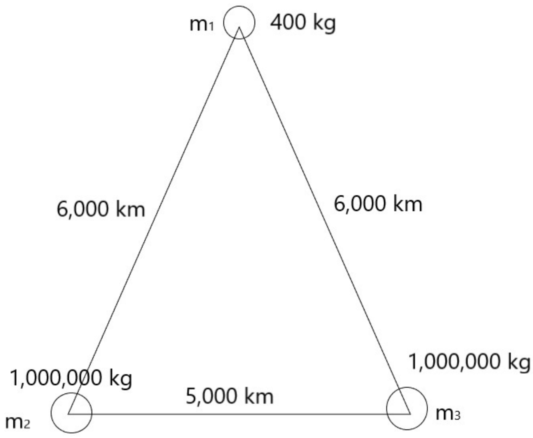
b. $2d$ to the right of mass m_1?

c. $2d$ to the left of mass m_2?

d. $2d$ to the right of mass m_2?

14. E: Earth has a mass of approximately 5.97×10^{24} kg while Mars has a mass of approximately 6.42×10^{23} kg. Both planets are separated by approximately 2.28×10^{8} km and can be taken to be point particles. How many meters from Mars does a 3.00×10^{3} kg white rhino have to be placed to feel no force?

15.E: A rock in space, which is initially at rest, has a mass $m_1 = 400.$ kg and is 6.00×10^3 km away from two fixed rocks, each with a mass of 1.00×10^6 kg, as shown in the image below. What is the acceleration of m_1 at the moment when it is released from rest?



Additional HL Understandings

16.C: The following problem refers to *gravitational potential energy* E_P.

a. Define *gravitational potential energy* E_P. Is it a scalar or a vector?

b. What is the equation of the *gravitational potential energy* E_P of an object when it is near the surface of a planet? What is the general equation of the *gravitational potential energy* E_P? What is the significance of the negative sign? What are the units of *gravitational potential energy* E_P?

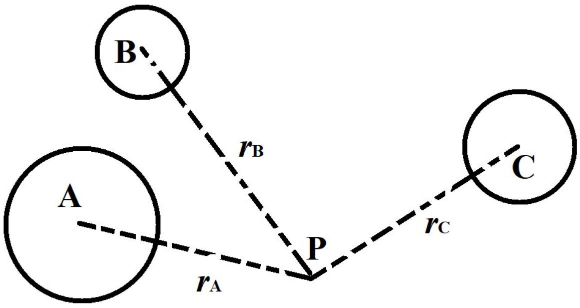
17.E: What will be the gravitational potential energy of a 5.20×10^5 kg rocket orbiting Saturn at an altitude of 1.00×10^4 km? Saturn has a mass of approximately 5.68×10^{26} kg and a radius of approximately 6.00×10^7 m.

18.C: The following problem refers to *gravitational potential* V_g.

a. Define *gravitational potential* V_g. Is it a scalar or a vector?

b. What is the equation for *gravitational potential* V_g? What are the units of *gravitational potential*?

19.E: Determine the gravitational potential at point P from three massive objects.



$m_A = 8.67 \times 10^{15}$ kg
$m_B = 5.30 \times 10^{15}$ kg
$m_C = 9.36 \times 10^{15}$ kg
$r_A = 2.70 \times 10^3$ m
$r_B = 1.30 \times 10^3$ m
$r_C = 6.50 \times 10^3$ m

20.C: What is the relationship between the gravitational field strength g and gravitational potential V_g?

21.C: What is constant in a *gravitational equipotential surface*?

22.C: How much work is done in moving a mass along the same *equipotential surface*?

23.C: How much work is done in moving a mass along two different *equipotential surfaces*? State the equation.

24.C: What is the relationship between an objects *gravitational equipotential surfaces* and *gravitational field lines*?

25.C: **Use a pencil and ruler!** Draw gravitational field lines and equipotential surfaces for each figure.

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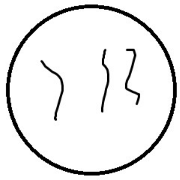</td><td>Two uniform spherical masses with equal mass and equal radius
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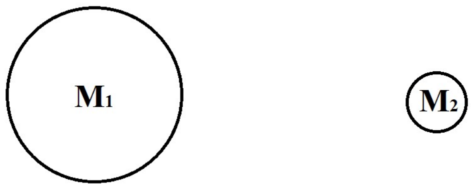</td></tr>
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26.C: The following problem refers to *escape speed*.

a. Define *escape speed*. This is also called *escape velocity*.

b. What are some assumptions made when defining *escape speed*?

c. For which objects does *escape speed* apply to?

d. For which objects does *escape speed* not apply to?

e. Use the law of conservation of energy $E_{\text{initial}} = E_{\text{final}}$ to solve for the equation of the *escape speed* of an object leaving the gravitational pull of a planet. What is the minimum speed needed for an object to escape the gravitational pull of the Earth?

27.C: You might want to derive this equation with your teacher: A planet is orbiting in a circular motion with a constant speed around a star (like the Sun).

a. Use Newton's Law of Gravitation $F_\text{g} = m_\text{planet} a_\text{planet} = \frac{G m_\text{star} m_\text{planet}}{r^2}$ and the equation for centripetal acceleration $a_\text{planet} = \frac{{v_\text{planet}}^2}{r}$ to solve for the speed squared ${v_\text{planet}}^2$ of a planet moving around a star. Place a box around your answer. In both equations r is the distance from the planet to the star.

b. Take your solution from part a and multiply both sides by $\frac{m_\text{planet}}{2}$. This new equation is equal to the kinetic energy of the planet. Place a box around your answer.

c. Substitute your answer from part b to the total energy of the orbiting planet $E_\text{total} = E_\text{k} + E_\text{g} = \frac{1}{2} m_\text{planet} {v_\text{planet}}^2 - \frac{G m_\text{planet} m_\text{star}}{r}$. Place a box around your answer. Your answer should have only one fraction.

d. Is the total energy positive, negative, or zero? Why?

e. Draw an *energy vs. distance* graph. On this graph draw the kinetic energy vs. distance, potential energy vs. distance, and total energy vs. distance.

28. C: An object is orbiting a planet. It encounters a small viscous drag due to the atmosphere. Describe the subsequent motion of the object.

29. E: A rocket with a mass of 6.40×10^4 kg is orbiting Jupiter's moon Callisto. Callisto has a radius of 2.40×10^6 m and a mass of 7.35×10^{22} kg.

a. What would the velocity of the rocket have to be in order for it to orbit Callisto at an altitude of 4.60×10^3 km?

b. What would be the gravitational force between this rocket and Callisto while orbiting at this altitude?

c. What would be the kinetic energy of this rocket while orbiting Callisto at this altitude?

d. What would be the gravitational potential energy of this rocket while orbiting Callisto at this altitude?

e. What would be the total energy of this rocket while orbiting Callisto at an altitude of 4.60×10^3 km?

f. What would be the total energy of this rocket while sitting at rest on the surface of Callisto?

g. How much kinetic energy would you have to give to this rocket while sitting on the surface of Callisto in order to put the rocket into orbit around Callisto at an altitude of 4.60×10^3 km?

h. With what velocity would this rocket have to be launched from the surface of Callisto in order to go into orbit around Callisto at an altitude of 4.60×10^3 km?

i. With what velocity would this rocket have to be launched from the surface of Callisto in order for the rocket to escape the gravitational effects of Callisto?

30.E: A rocket, which has a mass of 3.80×10^4 kg, is initially sitting at rest on the surface of the planet Venus. Venus has a radius of 6.05×10^6 m and a mass of 4.87×10^{24} kg.

a. What is the total energy content of this rocket while sitting at rest on the surface of Venus?

b. What velocity would be required for this rocket to orbit Venus at an altitude of 550. km?

c. What total energy is required if this rocket is to orbit Venus at an altitude of 550. km?

d. With what velocity should this rocket be launched from the surface of Venus in order to go into orbit around Venus at an altitude of 550. km?

e. With what minimum velocity should this rocket be launched from the surface of Venus in order to escape the gravitational effects of Venus?

f. What will be the velocity of this rocket when it is very far from Venus if the rocket is launched from the surface of Venus with a velocity of $1.40 \times 10^4\ \frac{\mathrm{m}}{\mathrm{s}}$?

31.E: A rocket, which has a mass of 6.40×10^4 kg, is sitting on the surface of Neptune. Neptune has a mass of approximately 1.03×10^{26} kg and a radius of approximately 2.43×10^7 m. This rocket is to be launched from Neptune's surface with the intention of going into orbit around the planet Neptune at an altitude of 1.00×10^4 km.

a. With what minimum velocity should this rocket be launched from Neptune's surface in order to go into orbit around Neptune at the given altitude?

b. What will be the total energy content of this rocket while orbiting Neptune at the given altitude?

c. How much kinetic energy must be added to this orbiting rocket if it is to escape the gravitational effects of Neptune?

d. What velocity must this orbiting rocket attain in order for it to escape the gravitational effects of Neptune?

e. Suppose that somehow the planet Neptune were to change into a black hole. What would the maximum radius of Neptune have to be in order for it to become a black hole? This is not in the IB physics syllabus.

32.E: A 3.20×10^4 kg rocket is orbiting the planet Jupiter at an altitude of 2.00×10^4 km. Jupiter has a mass of approximately 1.90×10^{27} kg and a radius of approximately 7.14×10^7 m.

a. What is the velocity of this rocket while orbiting Jupiter at this altitude?

b. What is the total energy content of this rocket while orbiting at this altitude?

c. How much additional energy must this rocket acquire in order to leave orbit and escape the gravity of Jupiter?

d. Suppose that this orbiting rocket is given an additional 3.20×10^{13} J of energy. What will be the resulting velocity of this rocket when it is very far from Jupiter?

e. What would the maximum radius of Jupiter have to be if it was to become a black hole? This is not in the IB physics syllabus.

33.E: A rocket, which has a mass of 1.80×10^{4} kg, is moving through space with a velocity of $1.25 \times 10^{4}\ \frac{\text{m}}{\text{s}}$ when it begins its approach to Saturn. This rocket would like to go into orbit around Saturn at an altitude of 8.20×10^{3} km. Saturn has a mass of approximately 5.68×10^{26} kg and a radius of approximately 6.00×10^{7} m.

a. What is the initial total energy of this rocket before it approaches Saturn?

b. What total energy is required in order for the rocket to go into orbit around Saturn at the given altitude?