Name: _____

Class: _____

Due Date: _____

C.1 Simple Harmonic Motion

Understandings

- Conditions that lead to simple harmonic motion.
- The defining equation of simple harmonic motion as given by $a = -\omega^2 x$.
- A particle undergoing simple harmonic motion can be described using time period *T*, frequency *f*, angular frequency ω, amplitude, equilibrium position, and displacement.
- The time period in terms of frequency of oscillation and angular frequency as given by $T = \frac{1}{f} = \frac{2\pi}{\omega}$.
- The time period of a mass-spring system as given by $T = 2\pi\sqrt{\frac{m}{k}}$.
- The time period of a simple pendulum as given by $T = 2\pi\sqrt{\frac{l}{g}}$.
- A qualitative approach to energy changes during one cycle of an oscillation.

Equations

$$a = -\omega^2 x$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Additional HL Understandings

- A particle undergoing simple harmonic motion can be described using phase angle.
- Problems can be solved using the equations for simple harmonic motion as given by
  - $x = x_0 \sin(\omega t + \phi)$
  - $v = \omega x_0 \cos(\omega t + \phi)$
  - $v = \pm\omega\sqrt{{x_0}^2 - x^2}$
  - $E_\mathrm{T} = \frac{1}{2} m\omega^2 {x_0}^2$
  - $E_\mathrm{p} = \frac{1}{2} m\omega^2 x^2$

Additional HL Equations

$x = x_0 \sin(\omega t + \phi)$

$v = \omega x_0 \cos(\omega t + \phi)$

$v = \pm\omega\sqrt{{x_0}^2 - x^2}$

$E_\mathrm{T} = \frac{1}{2} m\omega^2 {x_0}^2$

$E_\mathrm{p} = \frac{1}{2} m\omega^2 x^2$

If you are interested in learning more about waves then please read the book *Vibrations and Waves* by George C. King.

The solutions can be found on the YouTube channel Go Physics Go:

https://www.youtube.com/@gophysicsgo/playlists

Part 1: Use your favorite sources to answer the following questions

1. C: Define *oscillation.*

2. C: Give three examples of oscillations.

3. C: Define *periodic.*

4. C: Define *period.* Units?

5. C: Define *amplitude.* Units?

6. C: Define *frequency.* Units?

7. C: What is the mathematical relationship between the *frequency* and *period* of a wave?

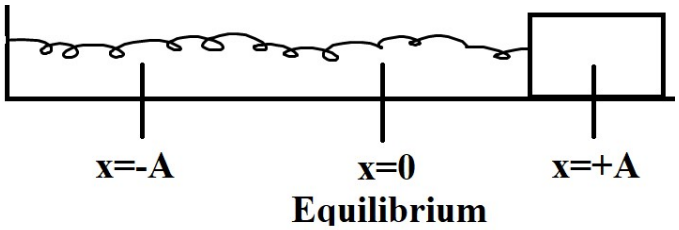
8. C: State the equation for the angular frequency for an object undergoing simple harmonic motion.

9. C: Topic A.1 Review: The slope of a displacement vs. time graph tells us the _____ of an object while the slope of a velocity vs. time graph tells us the _____ of an object.

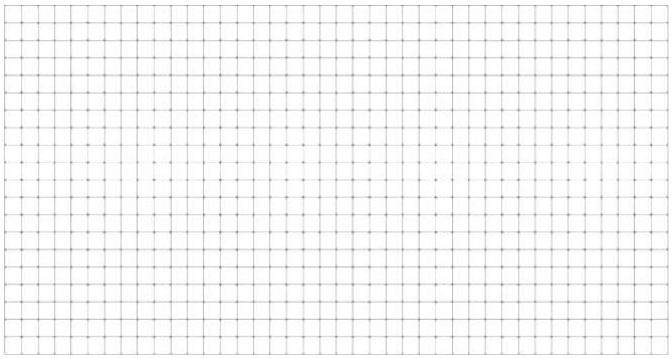
10.C: Topic A.2 Review: Define *equilibrium.*

11.C: Give the name, define, and give the units of each variable from Hooke's Law $\vec{F} = -k \times \Delta\vec{x}$.

12.C: A mass lying on a smooth horizontal surface is attached to a spring and is stretched from its equilibrium position. It is then released. Label the forces on the mass.



13.C: **Use a pencil and ruler!** Draw and label an *acceleration vs. displacement* graph for simple harmonic motion.



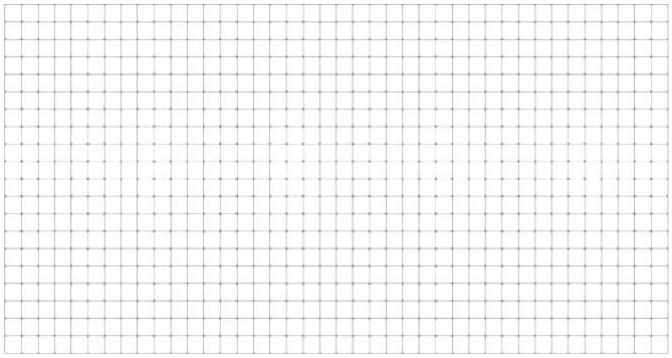
14.C: Derive the defining equation of simple harmonic motion $a = -\omega^2 x$.

15.C: What are the main characteristics of simple harmonic motion?

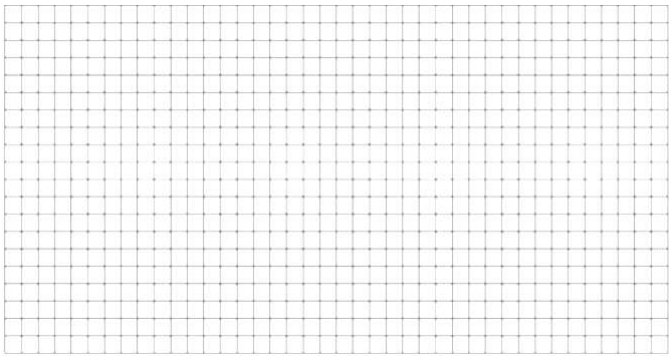
16.C: Derive the equation for the time period of a mass-spring system $T = 2\pi\sqrt{\frac{m}{k}}$.

17.C: State the equation for the time period of a simple pendulum.

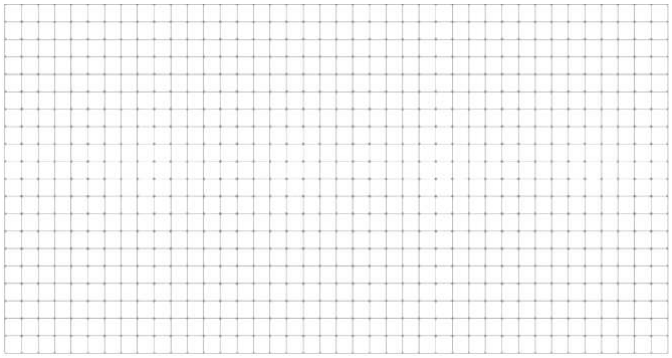
18.C: **Use a pencil and ruler!** Draw two waves which are *in phase*.



19.C: **Use a pencil and ruler!** Draw two waves which are *out of phase by 180°.*

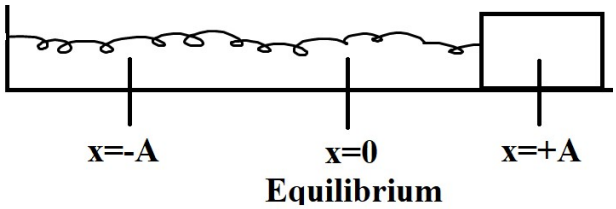


20.C: **Use a pencil and ruler!** Draw two waves which are *out of phase by 90°*.



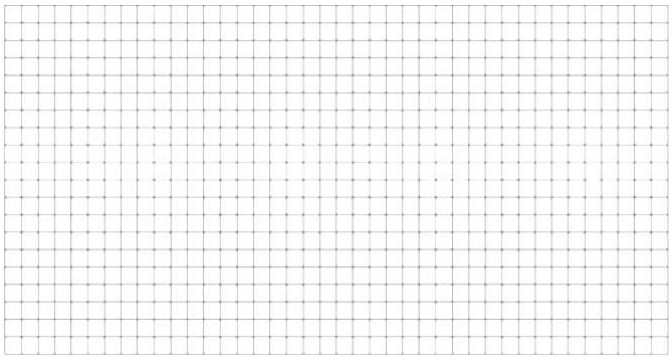
21.C: **Use a pencil!** The total energy of a simple harmonic oscillator is given by the equation $E_{\mathrm{T}} = E_{\mathrm{k}} + E_{\mathrm{p}} = \frac{1}{2}mv^2 + \frac{1}{2}k\Delta x^2 = \text{constant}$. In the figure below label the locations of

$E_{\mathrm{k,max}}$	$E_{\mathrm{k,min}} = 0\ \mathrm{J}$	$E_{\mathrm{p,max}}$	$E_{\mathrm{p,min}} = 0\ \mathrm{J}$
a_{max}	$a_{\mathrm{min}} = 0\ \frac{\mathrm{m}}{\mathrm{s}^2}$	v_{max}	$v_{\mathrm{min}} = 0\ \frac{\mathrm{m}}{\mathrm{s}}$

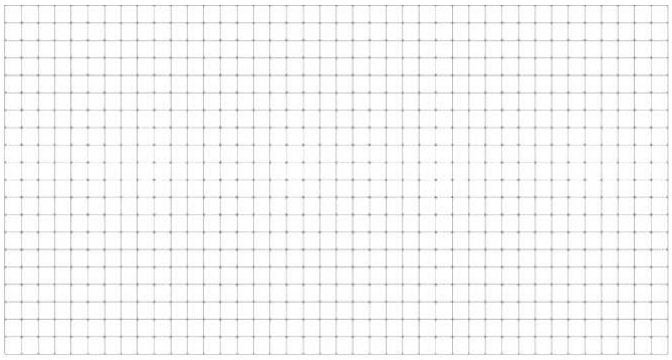


22.C: What is the mathematical relationship between the energy and amplitude of an object in simple harmonic motion?

23.C: **Use a pencil and ruler!** On the graph below draw an energy vs. displacement graph for a mass on a spring with three curves: a potential energy vs. displacement curve, a kinetic energy vs. displacement curve, and a total energy vs. displacement curve.



24.C: **Use a pencil and ruler!** On the graph below draw an energy vs. time graph during one oscillation for a mass on a spring with three curves: a potential energy vs. displacement curve, a kinetic energy vs. displacement curve, and a total energy vs. displacement curve.



25.E: A pendulum makes 22.0 oscillations in 42.0 seconds. Determine the period and frequency of the pendulum.

26.E: A 600. g mass is hung at the end of a vertical spring. The spring stretches 22.5 cm.

a. Determine the spring constant.

b. How much further will it stretch if a 900. g mass is hung from it?

27.E: A 497 g mass is attached to a spring with a spring constant $k = 9.87\ \frac{\mathrm{N}}{\mathrm{m}}$ and oscillates horizontally on a frictionless surface. The mass is displaced 6.54 cm from its equilibrium position and released.

a. Determine the maximum speed of the mass.

b. Determine the speed of the mass when it is 3.00 cm from its equilibrium position.

c. Determine the acceleration of the mass when it is 3.00 cm from its equilibrium position.

28.E: A 40.0 g mass undergoes simple harmonic motion at the end of a spring. Its maximum displacement from equilibrium is 18.0 cm and period is 1.20 s.

a. Determine the frequency.

b. Determine the spring constant.

c. Determine the maximum speed.

d. Determine the maximum acceleration.

e. Determine the speed of the mass when its displacement is 8.00 cm.

f. Determine the acceleration of the mass when its displacement is 8.00 cm.

29.E: Six vertical springs, each with a spring constant of 2570 N/m, are individually hung on a wall. Each spring hangs a 35.5 kg mass. Determine the period of the mass when it oscillates.

30.E: On Planet X an IB student wants to determine the acceleration of gravity near its surface by using a pendulum. The time to complete 20.0 oscillations is 35.0 s on a 0.825 m long massless string. Determine the acceleration of gravity on Planet X.

31.E: A 34.5 g mass is attached to a horizontal spring. The mass performs simple harmonic motion according to the equation $y(t) = (0.975\ \text{m})\cos(12t)$.

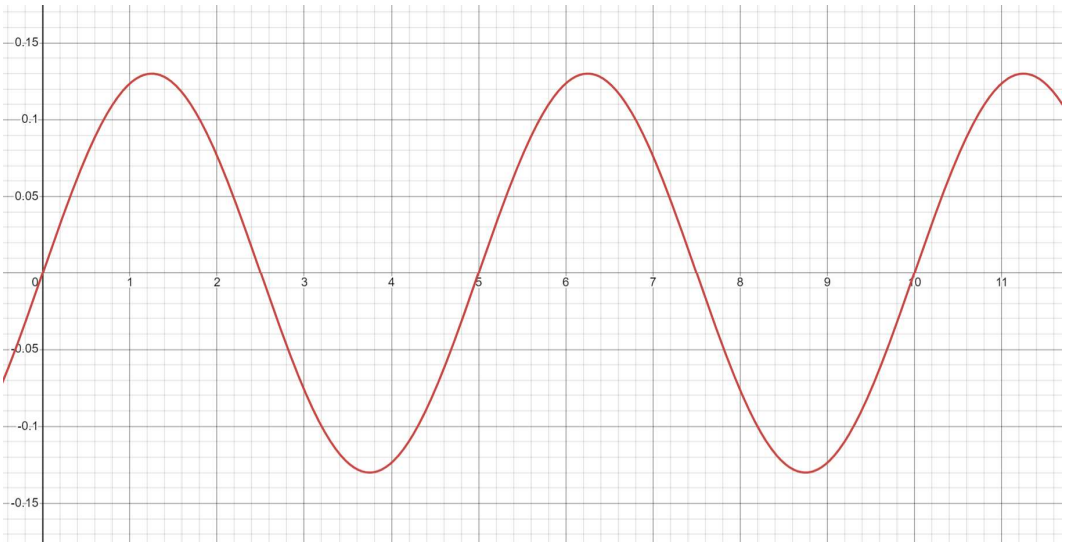
a. Determine the amplitude.

b. Determine the frequency.

c. Determine the position of the mass at $t = 0.682\ \text{s}$.

d. Determine the spring constant.

32.E: The *position vs. time* graph of a pendulum is given below:



a. What is the period of this pendulum?

b. What is the maximum displacement of this pendulum from the equilibrium position?

c. What is the angular velocity of this pendulum?

d. Write the equation which describes the position of this pendulum as a function of time.

e. What will be the position of this pendulum 4.5 seconds after it is released?

f. What will be the position of this pendulum 11.5 seconds after it is released?

g. What will be the maximum speed of this pendulum?

h. Write the equation predicting the velocity of this pendulum as a function of time.

i. What will be the velocity of this pendulum 4.50 seconds after it has been released?

j. What will be the velocity of this pendulum 3.75 seconds after it has been released?

k. What will be the maximum acceleration of this pendulum?

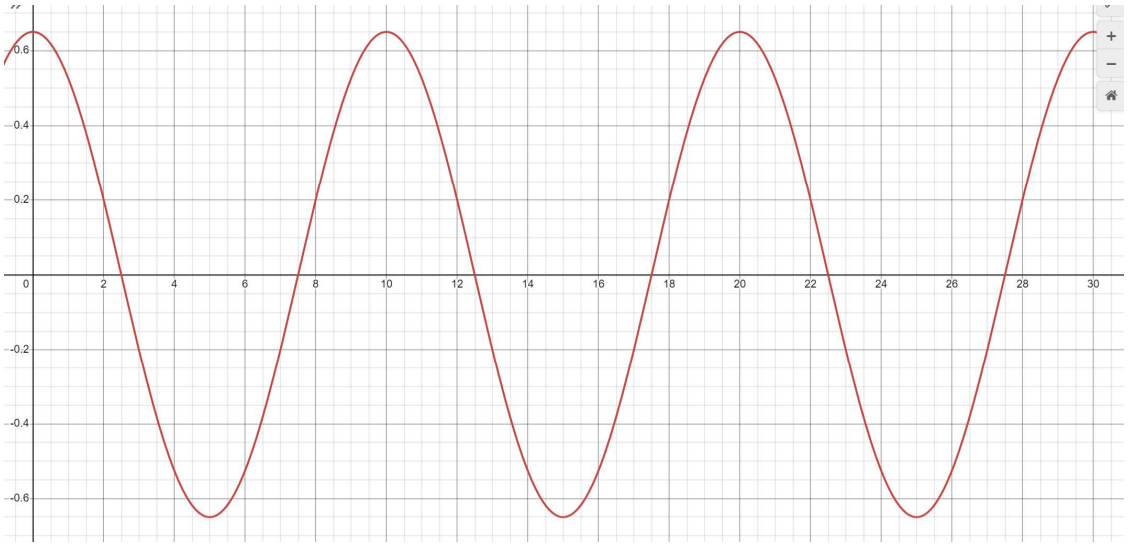
l. What will be the acceleration of this pendulum 4.50 seconds after it has been released?

m. What is the length of this pendulum?

n. Based on the graph, at which times is the velocity of this pendulum zero?

o. Based on the graph, at which times is the acceleration of this pendulum zero?

33.E: The *position vs. time* graph of a pendulum is given below:



a. What is the period of this pendulum?

b. What is the maximum displacement of this pendulum from the equilibrium position?

c. What is the angular velocity of this pendulum?

d. Write the equation which describes the position of this pendulum as a function of time.

e. What will be the position of this pendulum 18.5 seconds after it is released?

f. What will be the position of this pendulum 44.5 seconds after it is released?

g. What will be the maximum speed of this pendulum?

h. Write the equation predicting the velocity of this pendulum as a function of time.

i. What will be the velocity of this pendulum 18.5 seconds after it has been released?

j. What will be the velocity of this pendulum 25.0 seconds after it has been released?

k. What will be the maximum acceleration of this pendulum?

l. What will be the acceleration of this pendulum 18.5 seconds after it has been released?

m. What will be the acceleration of this pendulum 25.0 seconds after it has been released?

n. Based on the graph, at which times is the velocity of this pendulum zero?

o. Based on the graph, at which times is the acceleration of this pendulum zero?

Additional HL Understandings

34. C: Math review: Describe the significance of the variables *A*, *B*, *C*, and *D* in the equation $y = A\sin(Bx + C) + D$.

35. E: An object is undergoing simple harmonic motion with a period of 0.255 s, a maximum displacement of 5.28 cm, and a phase angle of $\frac{\pi}{4}$.

    a. Determine the displacement of the object after 1.25 s.

    b. Determine the velocity of the object after 2.50 s.

    c. Determine the maximum speed of the object.

36.C: Derive the equations of motion, energy, and speed for simple harmonic motion.

37.E: A mass of 0.765 kg undergoes simple harmonic motion with a maximum displacement of 0.232 m and a frequency of 0.652 Hz.

a. Determine the period of the motion.

b. Determine the total energy.

c. Determine the potential energy of the mass when it is 0.100 m from its equilibrium position.

d. Determine the kinetic energy of the mass when it is 0.100 m from its equilibrium position.

e. Determine the speed of the mass when it is 0.100 m from its equilibrium position.

f. Determine the maximum speed of the mass.

Optional for math lovers

The small angle approximation ($\theta < 10°$) for the period of a pendulum is given in the physics data booklet: $T = 2\pi\sqrt{\frac{l}{g}}$

The exact solution is given from the video below:

Exact Solution of the Nonlinear Pendulum
Flammable Maths
https://www.youtube.com/watch?v=efvT2iUSjaA

Watch and take notes from the video above.