Name: ______________________

Class: ______________________

Due Date: ______________________

# A.5 Galilean and Special Relativity

**Additional HL Understandings**

- Reference frames.
- Newton's laws of motion are the same in all inertial reference frames and this is known as Galilean relativity.
- In Galilean relativity the position $x'$ and time $t'$ of an event are given by $x' = x - vt$ and $t' = t$.
- Galilean transformation equations lead to the velocity addition equation as given by $u' = u - v$.
- Two postulates of special relativity.
- The postulates of special relativity lead to the Lorentz transformation equations for the coordinates of an event in two inertial reference frames as given by
  $x' = \gamma(x - vt)$
  $t' = \gamma(t - \frac{vx}{c^2})$
  where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.
- Lorentz transformation equations lead to the relativistic velocity addition equation as given by $u' = \frac{u-v}{1-\frac{uv}{c^2}}$.
- The space-time interval $\Delta s$ between two events is an invariant quantity as given by $(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$.
- Proper time interval and proper length.
- Time dilation as given by $\Delta t = \gamma \Delta t_0$.
- Length contraction as given by $L = \frac{L_0}{\gamma}$.
- The relativity of simultaneity.
- Space-time diagrams.
- The angle between the world line of a moving particle and the time axis on a space-time diagram is related to the particle's speed as given by $\tan\theta = \frac{v}{c}$.
- Muon decay experiments provide experimental evidence for time dilation and length contraction.

**Additional HL Equations**

$x' = x - vt$

$t' = t$

$u' = u - v$

$x' = \gamma(x - vt)$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$u' = \frac{u-v}{1-\frac{uv}{c^2}}$

$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$

$\Delta t = \gamma \Delta t_0$

$L = \frac{L_0}{\gamma}$

$\tan\theta = \frac{v}{c}$

**The solutions can be found on the YouTube channel Go Physics Go:**

https://www.youtube.com/@gophysicsgo/playlists

1. C: Define *observer*.

2. C: Define *reference frame*.

3. C: Define *inertial reference frame*.

4. C: Define *event*.

5. C: Define *simultaneous events*.

6. C: Define and state the equations for *Galilean transformations*.

7. C: Define *Galilean relativity*.

8. C: State *the two postulates of relativity.*

9. E: A train is traveling horizontally to the right with a speed of $18.0\ \frac{\text{m}}{\text{s}}$. A man on the train is walking to the right with a speed of $4.00\ \frac{\text{m}}{\text{s}}$. Determine the velocity of the man by an observer at rest outside the train.

10. E: A train is traveling horizontally to the left with a speed of $32.0\ \frac{\text{m}}{\text{s}}$. A man on the train is walking to the right with a speed of $4.00\ \frac{\text{m}}{\text{s}}$. Determine the velocity of the man by an observer at rest outside the train.

11. E: Usain Bolt and Florence Griffith-Joyner race each other. They both begin at the same time. Usain Bolt runs with a constant speed of $10.44\ \frac{\text{m}}{\text{s}}$ while Florence Griffith-Joyner runs with a constant speed of $9.53\ \frac{\text{m}}{\text{s}}$.

   a. What is the distance between Usain Bolt and the starting line after two minutes?

   b. According to Florence Griffith-Joyner, what is the distance to Usain Bolt after two minutes?

   c. Determine the speed of Usain Bolt as observed by Florence Griffith-Joyner.

12. E: Wayde van Niekerk and Marita Koch race each other. They both begin at the same time. Wayde van Niekerk runs with a constant speed of $9.30\ \frac{\text{m}}{\text{s}}$ while Marita Koch runs with a constant speed of $8.40\ \frac{\text{m}}{\text{s}}$.

   a. What is the distance between Wayde van Niekerk and the starting line after three minutes?

b. According to Marita Koch, what is the distance to Wayde van Niekerk after three minutes?

c. Determine the speed of Wayde van Niekerk as observed by Marita Koch.

13.C: What is the conclusion of the Michelson-Morley experiment?

14.C: State the *Lorentz transformation equations*. Define each variable.

15.C: **Use a pencil and ruler!** State and draw a *Lorentz factor vs. speed* graph.

16.E: An event occurs at position $x = 4.00 \times 10^3$ m and time $t = 6.00$ s. Where and when did the event occur according to a rocket traveling horizontally in the positive direction at a speed of $v = 0.400c$? The origins of both reference frames are zero when both clocks read zero.

17.E: An event occurs at position $x = -6.00 \times 10^3$ m and time $t = 4.00$ s. Where and when did the event occur according to a rocket traveling horizontally in the positive direction at a speed of $v = 0.100c$? The origins of both reference frames are zero when both clocks read zero.

18.E: A rocket moves to the right past an IB school with a speed of $0.600c$. The IB school is located at the origin. Determine the time reading of the rocket if $x = 300.$ m. The origins of both reference frames are zero when both clocks read zero.

19.E: A rocket moves to the right past an IB school with a speed of $0.255c$. The IB school is located at the origin. Determine the time reading of the rocket if $x = 425$ m. The origins of both reference frames are zero when both clocks read zero.

20.C: Describe *time dilation* and *proper time interval*.

21.C: Describe *length contraction* and *proper length*.

22.C: Describe the *twin paradox*.

23.E: A man at rest sees two objects traveling towards each other each with a speed of $0.500c$. What is the speed of one object as seen by the other object? What would this value be using classical physics?

24.E: A man at rest sees two objects traveling towards each other each with a speed of $0.900c$. What is the speed of one object as seen by the other object? What would this value be using classical physics?

25.E: Romeo is behind Juliet and is running towards her with a speed of $0.600c$ as seen by a man at rest. Juliet is running away from Romeo with a speed of $0.400c$ as seen by the same man at rest. What is the speed of Romeo as observed by Juliet? What would this value be using classical physics?

26.E: Clyde is behind Terry and is running towards him with a speed of $0.355c$ as seen by a man at rest. Terry is running away from Clyde with a speed of $0.255c$ as seen by the same man at rest. What is the speed of Clyde as observed by Terry? What would this value be using classical physics?

27.C: A rocket is moving in a straight line with a constant speed $v$ in space. It sends out a pulse of light ahead of it. How fast does the pulse of light move relative to an observer at rest on Earth?

28.E: A rocket at rest on Earth is measured to have a length of 45.0 m and a diameter of 5.60 m. The rocket is then launched toward Alpha Centauri, which is 4.30 light years from the Earth, at $2.20 \times 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}$.

a. What will be the length of this rocket as measured by an observer on the Earth?

b. What will be the length of the rocket as measured by an astronaut on board the rocket?

c. What will be the diameter of the rocket as measured by an observer on the Earth?

d. What will be the distance to Alpha Centauri as measured by the astronaut on board the rocket?

29.E: A rocket at rest on Earth is measured to have a length of 65.5 meters and a diameter of 7.65 meters. The rocket is then launched toward Betelgeuse, which is 427 light years from the Earth, at $1.25 \times 10^8 \ \frac{\text{m}}{\text{s}}$.

a. What will be the length of this rocket as measured by an observer on the Earth?

b. What will be the length of the rocket as measured by an astronaut on board the rocket?

c. What will be the diameter of the rocket as measured by an observer on the Earth?

d. What will be the distance to Alpha Centauri as measured by the astronaut on board the rocket?

30.E: The distance to the star Epsilon Indi, as measured from the Earth frame of reference, is $1.07 \times 10^{17}$ m.

a. What is this distance in light years?

b. What will be the distance in light years to Epsilon Indi as measured by an observer on a rocket heading toward this star at 85.0% of the speed of light?

c. How long in years will it take for this rocket to reach Epsilon Indi according to an observer on the Earth?

31.E: The maximum distance to the planet Pluto, as measured from the Earth frame of reference, is approximately $7.50 \times 10^{9}$ m.

a. What is this distance in light years?

b. What will be the maximum distance in light years to Pluto as measured by an observer on a rocket heading toward this star at 80.0% of the speed of light?

c. How long in seconds will it take for this rocket to reach Pluto according to an observer on the Earth?

32.E: A rocket is moving toward the star Delta Pavonis with a velocity of $2.40 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$. The distance to Delta Pavonis as measured from the rest frame on Earth is measured to be $1.88 \times 10^{17}$ m.

a. What will be the velocity of this rocket as a decimal fraction of the speed of light?

b. What is the distance to Delta Pavonis in light years?

c. What will be the distance to Delta Pavonis as measured by an astronaut on board the rocket?

33.E: A rocket is moving toward the star Eta Cassiopeiae with a velocity of $2.00 \times 10^{7}\ \frac{\mathrm{m}}{\mathrm{s}}$. The distance to Eta Cassiopeiae as measured from the rest frame on Earth is measured to be $1.82875 \times 10^{17}$ m.

a. What will be the velocity of this rocket as a decimal fraction of the speed of light?

b. What is the distance to Eta Cassiopeiae in light years?

c. What will be the distance to Eta Cassiopeiae as measured by an astronaut on board the rocket?

34.E: As measured from the Earth the distance to Tau Ceti is determined to be 11.8 light years. How fast in $\frac{m}{s}$ must a rocket be moving toward Tau Ceti so that the distance to Tau Ceti is reduced to 1.18 light years?

35.E: As measured from the Earth the distance to AD Leonis is determined to be 16.1939 light years. How fast in $\frac{m}{s}$ must a rocket be moving toward AD Leonis so that the distance to AD Leonis is reduced to 12.0 light years?

36.E: How fast must a rocket be moving in order for its length to be reduced to 50.0% of its rest length?

37.E: How fast must a rocket be moving in order for its length to be reduced to 1.00% of its rest length?

38.E: According to an observer on the Earth the time it should take to reach a certain star is determined to be 7.50 years. How long will it take to reach this star according to an observer on board a rocket moving toward this star with a velocity of $0.985c$?

39.E: According to an observer on the Earth the time it should take to reach a certain star is determined to be 22.0 years. How long will it take to reach this star according to an observer on board a rocket moving toward this star with a velocity of $0.800c$?

40.E: The distance to Barnard's star is measured to be 6.00 ly by an observer at rest on the Earth.

a. Assuming that a rocket is moving toward this star at 0.980c, how long in years will it take for this rocket to reach Barnard's star according to an observer on the Earth?

b. How long in years will it take to reach this star according to an observer on board the rocket?

c. What will be the distance in light years to Barnard's star according to an observer on board the rocket?

d. Assuming that the astronaut was 21.0 years old when she left the Earth in 1988, in what year will the astronaut arrive at Barnard's star according to an observer on Earth?

e. How old will the astronaut be when she arrives at Barnard's star according to an observer on board the rocket?

f. Suppose that the astronaut has a normal heart rate of 65.0 beats per minute when measured while at rest on the Earth. What will be the astronaut's heart rate as monitored by an observer on board the rocket with the astronaut?

g. What will the astronaut's heart rate be while on the rocket moving toward Barnard's star as monitored by an observer on the Earth?

41.E: The distance to Groombridge 34 A is measured to be 11.6191 ly by an observer at rest on the Earth.

a. Assuming that a rocket is moving toward this star at 0.822c, how long in years will it take for this rocket to reach Groombridge 34 A according to an observer on the Earth?

b. How long in years will it take to reach this star according to an observer on board the rocket?

c. What will be the distance in light years to Groombridge 34 A according to an observer on board the rocket?

d. Assuming that the astronaut was 32 years old when she left the Earth in 1988, in what year will the astronaut arrive at Groombridge 34 A according to an observer on Earth?

e. How old will the astronaut be when she arrives at Groombridge 34 A according to an observer on board the rocket?

f. Suppose that the astronaut has a heart rate of 62 beats per minute when measured while at rest on the Earth. What will be the astronaut's heart rate as monitored by an observer on board the rocket with the astronaut?

g. What will the astronaut's heart rate be while on the rocket moving toward Groombridge 34 A as monitored by an observer on the Earth?

42. E: Two astronauts play a game of chess on a rocket moving with a velocity of 0.999c away from the Earth. According to the astronauts the game takes 2.50 hours. How long, in hours, does the game take according to an observer at rest on Earth?

43. E: Two astronauts are taking a very boring online IB HL physics class on a rocket moving with a velocity of 0.900c away from the Earth. According to the astronauts the boring online IB HL physics class takes 90.0 min. How long, in minutes, does the boring online IB HL physics class take according to an observer at rest on Earth?

44. E: A rocket is moving toward Epsilon Eridani, which is 11.3 ly away as measured by an observer at rest on the Earth, at 0.998c. When the astronaut leaves the Earth in 1991 he has just had his 22nd birthday and his young daughter has just turned 1.0 years old. The rocket travels to the star, remains there for 6.00 months, and finally returns to the Earth at the same speed.

a. In what year will the rocket return to the Earth?

b. How many years will the journey take according to the astronaut on board the rocket?

c. How old will the astronaut be when he returns to the Earth?

d. How old will his daughter be when he returns to the Earth?

45.E: A rocket is moving toward Luyten's Star, which is 12.3485 ly away as measured by an observer at rest on the Earth, at 0.855c. When the astronaut leaves the Earth in 1992 he has just had his 32nd birthday and his young son has just turned 2.00 years old. The rocket travels to the star, remains there for 3.00 years, and finally returns to the Earth at the same speed.

a. In what year will the rocket return to the Earth?

b. How many years will the journey take according to the astronaut on board the rocket?

c. How old will the astronaut be when he returns to the Earth?

d. How old will his son be when he returns to the Earth?

46.E: Two twins are 22.0 years old when one of them sets out on a journey through space with a constant speed. The twin in the spaceship measures time with an accurate watch. When he returns to Earth, he claims to be 28.0 years old while the twin left on Earth is 40.0 years old. What was the speed of the spaceship?

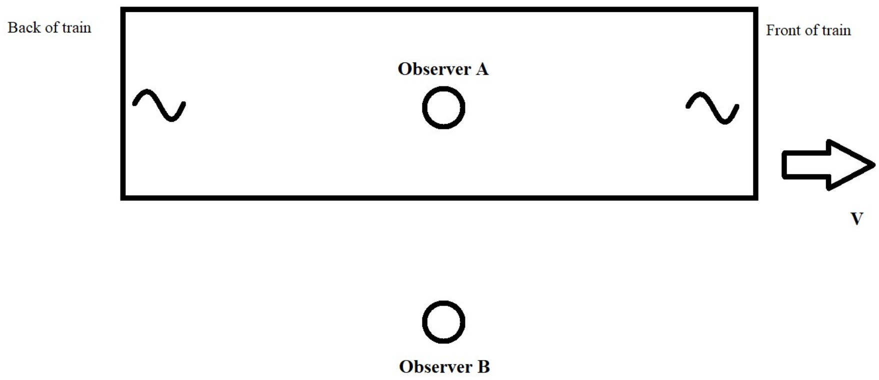
47.E: Two twins are 34.0 years old when one of them sets out on a journey through space with a constant speed. The twin in the spaceship measures time with an accurate watch. When he returns to Earth, he claims to be 38.0 years old while the twin left on Earth is 44.0 years old. What was the speed of the spaceship?

48.E: A neutron outside the confines of the nucleus of an atom is unstable and has a life expectancy (half-life) of 6.00 min. Suppose that a fast moving alpha particle collides with a block of beryllium and knocks a neutron out of the nucleus with a speed of $8.15 \times 10^7\ \frac{\text{m}}{\text{s}}$. What will be the expected lifetime of this neutron as measured by and observer in the rest frame?

49.E: A neutron outside the confines of the nucleus of an atom is unstable and has a life expectancy (half-life) of 6.00 min. Suppose that a fast moving alpha particle collides with a block of beryllium and knocks a neutron out of the nucleus with a speed of $7.25 \times 10^7\ \frac{\text{m}}{\text{s}}$. What will be the expected lifetime of this neutron as measured by and observer in the rest frame?

50.C: Describe the equation of the *space-time interval* $\Delta s$. Describe p*roper time interval* and *proper length* in terms of the *space-time interval* $\Delta s$.

51.C: Describe the *relativity of simultaneity*.



52.C: **Use a pencil and ruler!** Draw and label the axes of a space-time diagram.

53.C: **Use a pencil and ruler!** Draw a space-time graph where two events occur at the same time and another space-time graph where two events which occur at the same location.

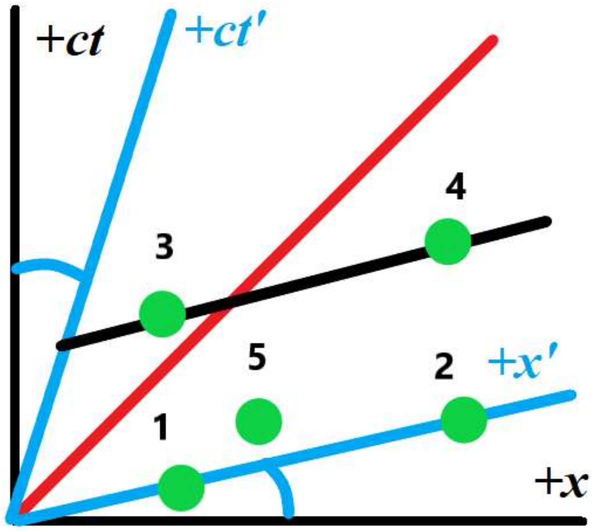
54.C: **Use a pencil and ruler!** Define the *world line*. Draw a *world line* in a space-time graph of an object at rest, a second object moving with a velocity of $0.4c$, and a third object moving with a velocity of $0.8c$.

55.C: Describe the relationship between the angle between the world line of a moving particle and the time axis on a space-time diagram.

56.C: **Use a pencil and ruler!** Add another reference frame to a space-time diagram. On the space time diagram label an event and describe how that event takes place at two different times in both reference frames.

57.C: **Use a pencil and ruler!** Draw a diagram with two reference frames. Label four stationary events in an $(x', ct')$ graph. Draw a second diagram with two reference frames. Label four events which occur at the same time in an $(x', ct')$ graph.

58.C: Below is a space-time diagram with two reference frames and five events. Determine the order of events according to the $(x, ct)$ graph and the $(x', ct')$ graph.




59.C: **Use a pencil and ruler!** Draw and describe a graph of $\Delta s^2 = -1$.

60.C: **Use a pencil and ruler!** Draw and describe a graph of $\Delta s^2 = +1$.

61.C: Three observers $L$, $M$, and $R$ are separated a distance $A$ from each other on a parallel line. They all travel from rest to a speed of $0.4c$ to the right and remain a distance $A$ from each other. The observer in the middle $M$ sends out a light pulse in all directions. The light gets reflected from observer $L$ and observer $R$ and returns to $M$. Does observer $M$ receive the light pulse from observer $L$ or observer $R$ first?

62.C: Describe *muon decay*.