Name: ______________________

Class: ______________________

Due Date: ______________________

A.4 Rigid Body Mechanics

Additional HL Understandings

- The torque τ of a force about an axis as given by $\tau = Fr\ \sin\theta$.
- Bodies in rotational equilibrium have a resultant torque of zero.
- An unbalanced torque applied to an extended, rigid body will cause angular acceleration.
- The rotation of a body can be described in terms of angular displacement, angular velocity, and angular acceleration.
- Equations of motion for uniform angular acceleration can be used to predict the body's angular position θ, angular displacement $\Delta\theta$, angular speed ω, and angular acceleration α as given by
  - $\Delta\theta = \frac{\omega_f + \omega_i}{2} t$
  - $\omega_f = \omega_i + \alpha t$
  - $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
  - ${\omega_f}^2 = {\omega_i}^2 + 2\alpha\Delta\theta$
- The moment of inertia I depends on the distribution of mass of an extended body about an axis of rotation.
- The moment of inertia for a system of point masses as given by $I = \sum mr^2$.
- Newton's second law for rotation as given by $\tau = I\alpha$ where τ is the average torque.
- An extended body rotating with an angular speed has an angular momentum L as given by $L = I\omega$.
- Angular momentum remains constant unless the body is acted upon by a resultant torque.
- The action of a resultant torque constitutes an angular impulse ΔL as given by $\Delta L = \tau\Delta t = \Delta(I\omega)$
- The kinetic energy of rotational motion as given by $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$.

Additional HL Equations

$\tau = Fr\ \sin\theta$

$\Delta\theta = \frac{\omega_f + \omega_i}{2} t$

$\omega_f = \omega_i + \alpha t$

$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

${\omega_f}^2 = {\omega_i}^2 + 2\alpha\Delta\theta$

$I = \sum mr^2$

$\tau = I\alpha$

$L = I\omega$

$\Delta L = \tau\Delta t$

$\Delta L = \Delta(I\omega)$

$E_k = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$

The solutions can be found on the YouTube channel Go Physics Go:

https://www.youtube.com/@gophysicsgo/playlists

1. C: Define, state the equation, and give the units of *angular position* θ.

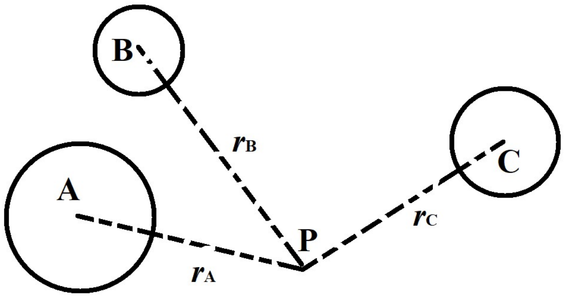
2. C: Define, state the equation, and give the units of *angular speed* ω.

3. C: Define, state the equation, and give the units of *angular acceleration* α.

4. C: Convert the *suvat* equations from linear motion to circular motion.

5. C: Define, state the equation, define each variable, and give the units for the *moment of inertia I*. What is the *moment of inertia I* equivalent to in translational motion?

6. E: Determine the moment of inertia of three objects rotating around a point P. Treat the three objects as point masses.

$m_A = 2.00$ kg $m_B = 3.00$ kg $m_C = 4.00$ kg $r_A = 7.00$ m $r_B = 8.00$ m $r_C = 9.00$ m	

7. C: Define, state the equation, define each variable, and give the units for *torque*.

8. C: State the equations for *Newton's second law of motion for linear motion* and *Newton's second law of motion for rotational motion*.

9. C: Define and state the conditions for *translational equilibrium* and *rotational equilibrium*.

10. C: State the equations for *power for linear motion* and *power for rotational motion*.

11. C: State the equations for *linear momentum* and *angular momentum*. Also state the equations for *linear impulse* and *angular impulse*.

12. C: State the equations for *translational kinetic energy* and *rotational kinetic energy*.

13.C: State the *law of conservation of linear momentum* and the *law of conservation of angular momentum*. Also state their equations.

14.E: A uniform disc, which has a mass of 3.50 kg and a radius of 28.0 cm, is accelerated from rest by a tangential force of 28.0 N applied to the outer edge of the disc. The moment of inertia of a disc is $I = \frac{1}{2}mr^2$.

a. What is the moment of inertia of this disc?

b. What is the magnitude of the torque being applied to this disc?

c. What will be the angular acceleration of this disc?

d. What will be the angular velocity of this disc after 8.00 s?

e. What will be the linear velocity of a point on the outer edge of the disc at the end of 8.00 s?

f. What will be the angular displacement of this disc during the 8.00 s period?

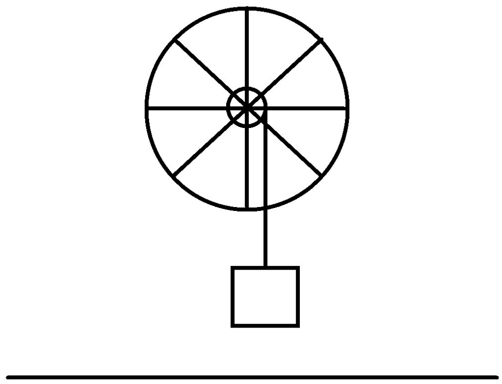
g. What will be the linear displacement of a point on the outer edge of this disc during the 8.00 s period?

h. How much work was done on this disc during the 8.00 s time period?

i. What will be the final angular momentum of this disc?

j. What is the final angular kinetic energy of this disc?

15.E: A bicycle wheel is mounted as in the lab and as shown below. This wheel has a mass of $m_\mathrm{w} = 6.55$ kg, a radius of $r_\mathrm{w} = 38.0$ cm, and is in the shape of a ring. A mass $M = 1.85$ kg is attached to the end of a string which is wrapped around an inner hub which has a radius $r_\mathrm{i} = 5.40$ cm. Initially the mass M is a distance $h = 72.0$ cm above the floor. Assume friction is negligible. The moment of inertia of a wheel is $I = mr^2$.



a. What will be the resulting angular acceleration of this wheel?

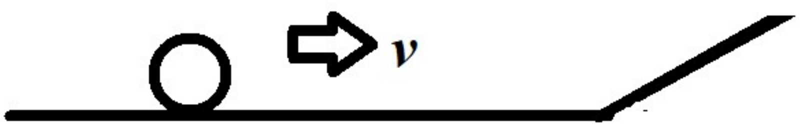
b. How long will it take for the mass M to reach the floor?

c. What will be the total angular displacement of the wheel during the time in which the mass M is falling to the floor?

d. How much work was done on the wheel by the external torque as the mass M falls to the floor?

e. What will be the angular kinetic energy of this wheel just as the mass M reaches the floor?

16.E: A sphere, which has a mass of 2.10 kg and a radius of 35.0 cm, is rolling along a horizontal surface with a velocity of 12.2 m/s when the sphere encounters an inclined plane which meets the horizontal at an angle of 22.0°. The moment of inertia of a sphere is $I = \frac{1}{2}mr^2$.



a. What is the angular velocity of this sphere?

b. What is the linear kinetic energy of this sphere?

c. What is the angular kinetic energy of this sphere?

d. What is the total kinetic energy of this sphere?

e. How far up this incline will the sphere roll before it comes to a halt?

f. How long will it take for this sphere to come to a halt?

g. What will be the linear acceleration of this sphere as it rolls up the incline?

h. What will be the angular acceleration of this sphere as it rolls up the incline?

i. What is the moment of inertia of this sphere?

j. What will be the net torque acting on this sphere as it rolls up the incline?

k. What will be the gravitational potential energy of this sphere just as it comes to a halt on the incline?

17.E: A disc, which has a mass of 12.0 kg and a radius of 65.0 cm, sits at the top of an inclined plane, which is 8.40 m long and 1.50 m high. At $t = 0.00$ s the disc is released and is allowed to roll to the bottom of the incline without slipping. The moment of inertia of a disc is $I = \frac{1}{2}mr^2$.

a. What is the gravitational potential energy of this disc as it sits at the top of this incline?

b. What will be the total kinetic energy of this disc as it reaches the bottom of the incline?

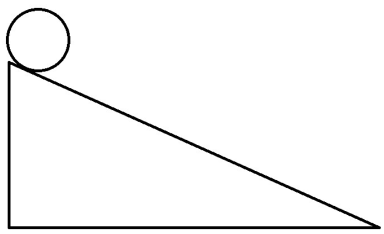
c. What will be the linear velocity of this disc when it reaches the bottom of the inclined plane?

d. What would the linear velocity be if the disc is replaced by a sphere?

e. What would the velocity be if the object was a ring?

18.E: A solid chocolate sphere with a mass of 9.00 kg and a diameter of 80.0 cm is placed on top of a rough incline ($\mu = 0.700$) with a length of 6.00 m at an angle of 50.0°. The solid chocolate sphere begins from rest and rolls down the incline. The moment of inertia of a sphere is $I = \frac{2}{5}mr^2$.

a. Draw a figure.



b. What is the initial height of the solid chocolate sphere?

c. How many revolutions will it take for the solid chocolate sphere to reach the bottom of the incline?

d. What will be the final linear speed of the solid chocolate sphere at the bottom of the incline?

e. What will be the final angular speed of the solid chocolate sphere at the bottom of the incline?

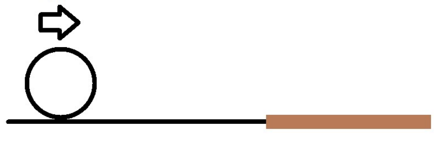
f. What will be the angular acceleration of the solid chocolate sphere?

g. What will be the linear acceleration of the solid chocolate sphere?

h. How long will it take for the solid chocolate sphere to reach the bottom of the incline?

19. E: A solid chocolate sphere with a mass of 8.00 kg and a diameter of 70.0 cm is rolling to the right on a frictionless horizontal surface with a linear speed of 6.00 m/s. The surface then becomes rough with a coefficient of dynamic friction of 0.150. The moment of inertia of a sphere is $I = \frac{2}{5}mr^2$.

a. Draw a figure.



b. What is the angular speed of the solid chocolate sphere as it rolls along the frictionless horizontal surface?

c. What is the angular acceleration of the solid chocolate sphere as it travels along the rough surface?

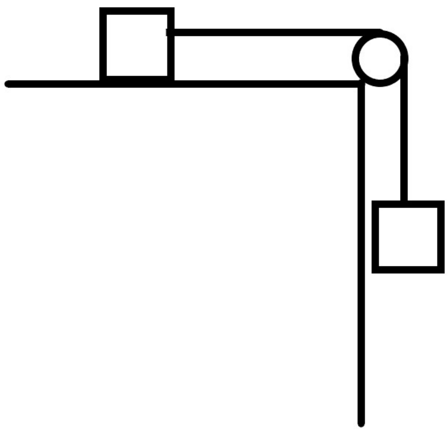
d. What is the linear acceleration of the solid chocolate sphere as it travels along the rough surface?

e. How many revolutions does the solid chocolate sphere complete as it travels along the rough surface?

f. How long does it take for the solid chocolate sphere to stop along the rough horizontal surface?

20.E: A block of mass $m_1 = 7.00$ kg sits at rest on a horizontal surface with $\mu = 0.200$. Mass m_1 is attached to a massless string which is wrapped around a pulley. Another massless string is wrapped around the same pulley and is holding another block of mass $m_2 = 47.0$ kg in the air. The pulley is a cylinder which has a mass of $m_C = 12.0$ kg and diameter of 10.0 cm. The moment of inertia of the rotating pulley is $I = \frac{1}{2}mr^2$.

a. Draw a free body diagram.



b. What is the common linear acceleration of the system?
c. What is the force of tension on the two massless strings?

21.E: A thin disk with a mass of 250. g and diameter of 40.0 cm is spinning with an angular speed of 3.00 rad/s. A point mass with a mass of 350. g strikes and sticks to the top of the thin disk 4.00 cm from the edge. What is the final angular speed of the system? The moment of inertia of a disc is $I = \frac{1}{2}mr^2$.

22.E: A star, which has a mass of 4.60×10^{30} kg and a radius of 9.30×10^{8} m, rotates on its axis once every 16.0 days. The moment of inertia of a sphere is $I = \frac{2}{5}mr^2$.

a. What is the moment of inertia of this star?

b. What is the angular velocity of this star?

c. What will be the linear velocity of a sunspot that is located on the equator of this star?

d. What is the angular momentum of this star?

As this star finally consumes most of its hydrogen fuel it starts to contract and heat up until it gets hot enough to start burning helium as a fuel. The burning of the helium fuel is very intense and the star quickly becomes a red giant as it expands to a new radius of $\mathbf{4.65 \times 10^9}$ m.

e. What will be the angular momentum of this star after it has expanded into a red giant?

f. What will be the angular velocity of the star after it has expanded into a red giant?

g. What will be the new rotational period of this star (in days) after it has become a red giant?

After a few million years the helium fuel is all used up and the star again begins to contract, becomes extremely hot, and then undergoes a massive supernova explosion. After the explosion the remaining neutron star has a radius of only 1,200 km. Assume that the mass of the star remains unchanged.

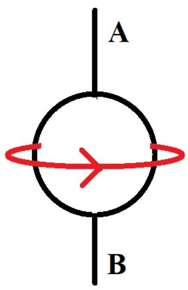
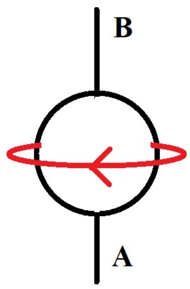
h. What will be the angular momentum of this star after it has collapsed into a neutron star?

i. What will be the moment of inertia of this star after it has collapsed into a neutron star?

j. What will be the angular velocity of this star after it has collapsed?

k. What will be the rotational period of this star (in seconds) after it has collapsed?

23.E: You are holding a sphere, which has a mass of 3.55 kg and a radius of 0.380 m, by an axle inserted vertical through its center as shown below. Initially, the sphere is rotating about its vertical axis 12.0 times every 5.00 seconds. The moment of inertia of a sphere is $I = \frac{2}{5}mr^2$.

Before	After
	

a. What is the moment of inertia of this sphere?

b. What is the magnitude of the angular velocity of this sphere?

c. What is the magnitude of the angular momentum of this sphere?

d. What is the direction of this spheres angular momentum?

Assume that you are sitting on a freely rotating chair, that your mass is 65.5 kg, and that your radius of gyration is 14.0 cm. Suddenly, you invert the sphere. The sphere continues to rotate with the same linear speed.

e. What will be the spheres final angular momentum?

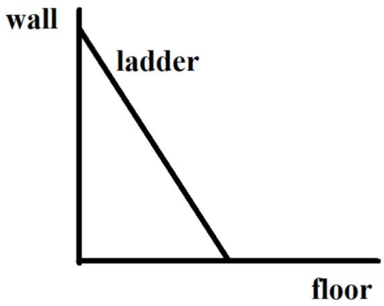
f. What will be the direction of the spheres final angular momentum?

g. What will be the magnitude of your final angular momentum?

h. What is the magnitude and direction of the impulse delivered to the sphere?

i. How much work was done in inverting the sphere?

24. E: A ladder has a mass of 20.0 kg and is 6.00 m long is leaning against a frictionless wall. The ladder is at rest and makes an angle of 30.0° from the wall. Draw a figure and write down the equations for static equilibrium.



25.C: Write down the common terms and equations for rigid body mechanics.