Name: _____

Class: _____

Due Date: _____

A.0 Math

Understandings
- Fundamental units
- Derived SI units
- Significant figures
- Unit conversions
- Random and systematic errors
- Absolute, fractional, and percentage uncertainties
- Error bars
- Uncertainty of gradient and intercepts
- Vector and scalar quantities
- Adding and subtracting vectors
- Combination and resolution of vectors

If you are interested in learning more about mathematical physics then please read the books *Mathematical Methods in the Physical Sciences* by Mary L. Boas and *div grad curl and all that* by H.M. Schey.

The solutions can be found on the YouTube channel: Go Physics Go

Part 1: State the fundamental units

Quantity	Unit
Length	
Mass	
Time	
Current	
Temperature	
Quantity	
Light intensity	

Memorize these two acronyms to memorize the SI fundamental units:

My **K**nuckles **G**row **S**tronger **A**nd **K**ill **M**ore **C**reatures

=

Meters **K**ilo**G**rams **S**econds **A**mperes **K**elvin **M**oles **C**andela

Part 2: Answer the following questions about fundamental units

1. What is the meaning and what are the fundamental units of *perimeter*?

2. What is the meaning, equation, and the fundamental units of *circumference*?

3. What is the meaning and what are the fundamental units of *area*?

4. What is the meaning and what are the fundamental units of *volume*?

5. A.1: Use the equation $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$ to solve for the fundamental units of *velocity*.

6. A.1: Use the equation $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ to solve for the fundamental units of *acceleration*.

7. A.2: What are the units of *force*? Use the equation $\sum \vec{F} = m\vec{a}$ to solve for the fundamental units of *force*.

8. A.2: Use the equation $\vec{F}_{\mathrm{H}} = -k\vec{x}$ to solve for the fundamental units of the *spring constant k*.

9. A.2: Use the equation $F_{\mathrm{d}} = 6\pi\eta r v$ to solve for the fundamental units of the *fluid viscosity* η.

10. A.2: Use the equation $\vec{p} = m\vec{v}$ to solve for the fundamental units of *momentum* $\vec{p}$.

11. A.3: What are the units of *energy*?

12. A.3: Use the equation $E_{\mathrm{k}} = \frac{1}{2}mv^2$ to solve for the fundamental units of *kinetic energy*.

13. A.3: Use the equation $E_{\mathrm{p}} = mg\Delta h$ to solve for the fundamental units of the *gravitational potential energy* near the surface of a planet.

14. A.3: What are the fundamental units of *energy*?

15. A.3: What are the units of *work*? Use the equation $W = Fs\cos\theta$ to solve for the fundamental units of *work W*.

16. A.3: What is the relationship between the fundamental units of *work* and *energy*?

17. A.3: What are the units of *power*? Use the equation $P = \frac{\mathrm{Work}}{t}$ to solve for the fundamental units of *power P*.

18. A.4 HL: Use the equation $\tau = rF\sin\theta$ to solve for the fundamental units of *torque* τ. The variable *r* has units of meters and the variable F represents the external force acting on an object.

19. A.4: HL: Use the equation $I = kMR^2$ to solve for the fundamental units of the *moment of inertia I*. The variable *k* is a unitless constant which depends on the physical dimensions of the object, the variable *M* is the mass of the object, and the variable *R* has units of meters.

20. A.4: HL: Use the equation $\Delta L = \tau \Delta t$ to solve for the fundamental units of angular momentum L.

21. A.4: HL: Use the equation $L = I\omega$ to solve for the fundamental units of angular speed ω.

22. A.4: HL: Use the equation $\tau = I\alpha$ to solve for the fundamental units of angular acceleration α.

23. B.1: What is the equation and what are the fundamental units of density ρ?

24. B.1: Use the equation $\overline{E_{\mathrm{k}}} = \frac{3}{2} k_{\mathrm{B}} T$ to determine the fundamental units for the Boltzmann's constant k_{B}.

25. B.1: Use the equation $Q = mc\Delta T$ to determine the fundamental units for the specific heat capacity c. The variable Q has the units of energy.

26. B.1: Use the equation $Q = mL_{\mathrm{f}}$ to determine the fundamental units for the latent heat of fusion L_{f}. The variable Q has the units of energy.

27. B.1: Use the equation $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$ to determine the fundamental units for the thermal conductivity k. The variable Q has the units of energy.

28. B.1: Use the equation $L = \sigma A T^4$ to solve for the fundamental units of the *Stefan-Boltzmann constant* σ. The variable *L* represents the luminosity of an object and has units of Watts, the variable *A* represents the surface area of an object, and the variable *T* represents the temperature of an object.

29. B.1: Use the equation $b = \frac{L}{4\pi d^2}$ to solve for the fundamental units of the apparent brightness *b*. The variable *L* represents the luminosity of an object and has units of Watts. The variable *d* represents the distance from a light source and has units of meters.

30. B.2: Use the equation $e = \frac{\frac{P}{A}}{\sigma T^4}$ to solve for the fundamental units of emissivity *e*. The fundamental units for the Stefan-Boltzmann constant σ has been solved for earlier.

31. B.3: What are the units of *pressure*? Use the equation $P = \frac{F}{A}$ to solve for the fundamental units of *pressure P*.

32. B.3: Use the equation $PV = nRT$ to solve for the fundamental units of the *ideal gas constant R*.

33. B.4 HL: Use the equation $S = \frac{\Delta Q}{\Delta T}$ to solve for the fundamental units of *entropy S*.

34. B.4 HL: Use the equation $S = k_\mathrm{B} \ln \Omega$ to determine the fundamental units for the *Boltzmann's constant* k_B.

35. B.5: What are the units of *current I*? Use the equation $I = \frac{\Delta q}{\Delta t}$ to solve for the fundamental units of *current I*.

36. B.5: Use the equation $I = \frac{\Delta q}{\Delta t}$ to solve for the fundamental units of *charge q*.

37. B.5: Use the equation $V = W/q$ to solve for the fundamental units of *voltage V*.

38. B.5: What are the units for the *resistance* in a resistor R? Use the equation $V = IR$ to solve for the fundamental units of *resistance R*.

39. C.1: What is the meaning and what are the fundamental units of *period T*?

40. C.1: What is the meaning and what are the fundamental units of *frequency f*?

41. C.1: What is the meaning and what are the fundamental units of *wavelength λ*?

42. C.2: Intensity is defined as power per unit area. What are the fundamental units of *intensity I*?

43. D.1: Use the equation $F_{\text{gravity}} = \frac{Gm_1m_2}{r^2}$ to solve for the fundamental units of the *gravitational constant G*.

44. D.1 HL: Use the equation $V_{\text{g}} = \frac{Gm}{r}$ to solve for the fundamental units of the *gravitational potential* V_{g}.

45. D.2: Use the equation $F_{\text{electric}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ to solve for the fundamental units of the *permittivity of free space* ε_0.

46. D.2: Use the equation $F_{\text{electric}} = k\frac{q_1 q_2}{r^2}$ to solve for the fundamental units of the *Coulomb constant* k.

47. D.2: Use the equation $\vec{F}_{\text{e}} = q\vec{E}_{\text{ext}}$ to solve for the fundamental units of the *electric field E*.

48. D.2 HL: Use the equation $V_{\text{e}} = \frac{kQ}{r}$ to solve for the fundamental units of the *electric potential* V_{e}.

49. D.3: What are the units of *magnetic field* $\vec{B}$? Use the equation $\vec{F}_{\text{B}} = q\vec{v}\vec{B}_{\text{ext}}$ to solve for the fundamental units of the *magnetic field* $\vec{B}$.

50. D.3: Use the equation $\frac{F}{L} = \mu_0\frac{I_1 I_2}{2\pi r}$ to solve for the fundamental units of μ_0.

51. D.4 HL: What are the units for the *magnetic flux* Φ? HL: Use the equation $\Phi = BA\cos\theta$ to solve for the fundamental units for the *magnetic flux* Φ.

52. E.1: Use the equation $E = hf$ to solve for the fundamental units of *Planck's constant h*.

53. E.2: HL: Use the equation $\lambda = \frac{h}{p}$ to solve for the fundamental units of the *de Broglie wavelength* λ.

Part 3: Determine the number of significant figures

1. 1,000	21.0.00020	41.100.00
2. 1,000.	22.0.0205	42.300.0000
3. 1,000.00	23.0.2	43.301
4. 1,020	24.8,000	44.301.001
5. 1020.	25.8,070	45.301.0010000
6. 1,020.0	26.8.0	46.8,670
7. 1,000.001	27.8.007	47.80,600
8. 1,200	28.800,700	48.8,670.00
9. 1,200.	29.800,700.00	49.1,000,000
10.1,200.00	30.4	50.1,200,000
11.1,200.03	31.4.0	51.1,205,000
12.1,200.0300	32.4.000	52.4,000
13.100200	33.1.2	53.4,300
14.100200.	34.1.25	54.4,300.
15.100200.00	35.1.250000	55.4,030
16.4,500	36.10	56.4003
17.4,050	37.10.	57.4,003.
18.405	38.100	58.0.00867
19.0.0000405	39.101	59.0.00086000753
20.0.0002	40.100.	60.0.0230076

Part 4: Unit conversions. Give all your solutions to three significant figures.

1. A man has a mass of 80.0 kg. What is the mass of the man in pounds? Show all your work and place a box around your answer.

2. How many seconds are in 80.0 years? Show all your work and place a box around your answer.

3. In 2009 Usain Bolt ran 100. m in a record time of 9.58 s. If he continues to run at this constant rate then how many meters will he run in one day? Show all your work and place a box around your answer.

4. In 2018 Eliud Kipchoge ran a marathon (42.195 km) in a record time of 2:01:39. If he continues to run at this constant rate then how many meters will he run in one day? Show all your work and place a box around your answer.

5. The circumference of the Earth is about 40,075.017 km from the Equator. What is the circumference of the Earth in inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.

6. The surface area of Earth is about 510,064,472 square kilometers. What is the surface area of the Earth in square inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.

7. The volume of Earth is about 1,083,206,916,846 cubic kilometers. What is the volume of Earth in cubic inches? There are approximately 39.370 inches in one meter. Show all your work and place a box around your answer.

8. The speed of light is 299,792,458 m/s. What is the distance, in kilometers, light travels in one year? Show all your work and place a box around your answer.

9. The density of gold is 19.32 grams per cubic centimeters. What is the density of gold in kilograms per cubic meters? Show all your work and place a box around your answer.

10. The density of gold is 19.32 grams per cubic centimeters. What is the density of gold in pounds per cubic feet? There is approximately 0.454 kg in one pound. There is approximately 0.305 meters in one foot. Show all your work and place a box around your answer.

11. A man drinks 60.0 liters of water in a 30.0 day month. On average how many cubic meters of water does he drink per hour? Show all your work and place a box around your answer.

Part 5: Answer the following questions

1. Define *random error* and give two examples.

2. Define *systematic error* and give two examples.

3. Define *accuracy* and give an example of high accuracy and low accuracy.

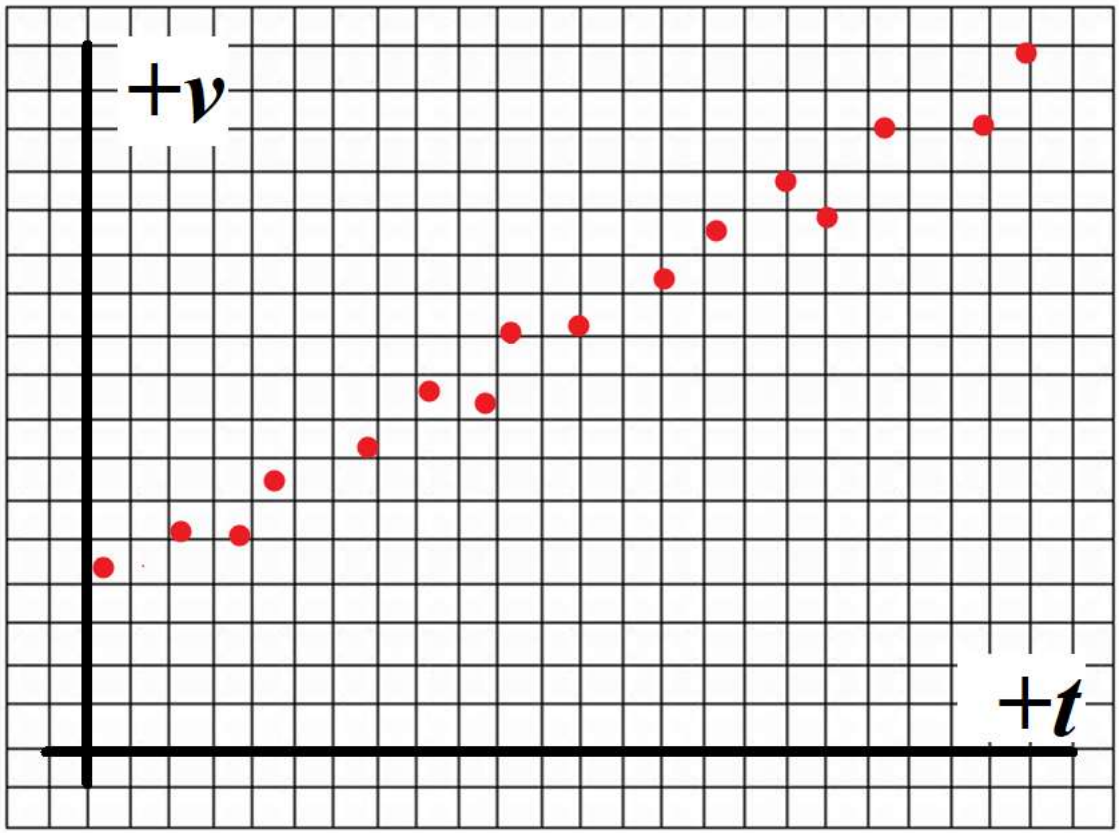
4. Define *precision* and give an example of high precision and low precision.

5. List some rules with regards to uncertainties in measurements.

6. State the equation and give the meaning of *standard deviation σ.*

7. Calculate the *absolute uncertainty*, *fractional uncertainty*, and *percent uncertainty* for a measured length of 87.65 ± 0.43 m.

8. **Use a pencil and ruler!** A *displacement vs. time* graph of an object moving with a constant speed in a straight line is given below. Draw a best fit line, estimate the slope and y-intercept, and finally give an equation of the speed of the object in the form $y = mx + b$ which is equivalent to $v(t) = mt + x_{\mathrm{i}}$. The horizontal blocks have units of 1 m and the vertical blocks have units of 1 m/s.



Part 6: Learn how to add, subtract, multiply, and divide uncertainties

1. 3.14 ± 0.15
   $+ \quad 9.26 \pm 0.53$

2. 6.26 ± 0.43
   $+ \quad 3.8 \ \pm 0.27$

3. 1.69 ± 0.39
   $+ \quad 9.37 \pm 0.51$

4. 5.89 ± 0.79
   $- \quad 3.23 \pm 0.84$

5. 9.50 ± 0.28
   $- \quad 8.4 \ \pm 0.97$

6. 5.82 ± 0.09
   $- \quad 4.94 \pm 0.45$

7. 3.14 ± 0.15
$\times \quad 9.26 \pm 0.53$

8. 6.26 ± 0.43
$\times \quad 3.8 \ \pm 0.27$

9. 1.69 ± 0.39
$\times \quad 9.37 \pm 0.51$

10. 5.89 ± 0.79
$\div \quad 3.23 \pm 0.84$

11. $\begin{array}{rl} & 9.50 \pm 0.28 \\ \div & 8.4 \ \pm 0.97 \end{array}$

12. $\begin{array}{rl} & 5.82 \pm 0.09 \\ \div & 4.94 \pm 0.45 \end{array}$

13. $(3.14 \pm 0.15)^2$

14. $(9.26 \pm 0.53)^3$

15. $(6.26 \pm 0.43)^4$

16. $\sqrt{(3.14 \pm 0.15)}$

17. $\sqrt[3]{(9.26 \pm 0.53)}$

18. $\sqrt[4]{(6.26 \pm 0.43)}$

19. What is the percent uncertainty of the perimeter of a rectangle if has a length of 2.45 ± 0.3 m and a width of 3.56 ± 0.4 m?

20. What is the percent uncertainty of the area of a rectangle if its length is uncertain by 3% and its width is uncertain by 4%?

21. What is the percent uncertainty of the volume of a box if its length is uncertain by 3%, its width is uncertain by 4%, and its height is uncertain by 5%?

22. What is the percent uncertainty of the perimeter/circumference of a circle if its radius is uncertain by 7%?

23. What is the percent uncertainty of the area of a circle if its radius is uncertain by 7%?

24. What is the percent uncertainty of the volume of a sphere if its radius is uncertain by 7%?

25. Mustafa has a height of (172 ± 0.2) cm. Nour has a height of (167 ± 0.35) cm. How much taller, including uncertainty, is Mustafa taller than Nour?

26. Twelve identical square tiles each have a length of $45.62\ \text{cm}$ with an uncertainty of $0.2\ \text{cm}$. What is the total length, including uncertainty, of the 12 tiles if they are each placed side-by-side?

27. What is the perimeter, including uncertainty, of a rectangle with a length of $(3.14 \pm 0.15)\ \text{cm}$ and a width of $(9.26 \pm 0.53)\ \text{cm}$?

28. What is the area, including uncertainty, of a rectangle with a length of $(3.14 \pm 0.15)\ \text{cm}$ and a width of $(9.26 \pm 0.53)\ \text{cm}$?

29. What is the volume, including uncertainty, of a box with a length of $(3.14 \pm 0.15)\ \text{cm}$, a width of $(9.26 \pm 0.53)\ \text{cm}$, and a height of $(6.26 \pm 0.43)\ \text{cm}$?

31. What is the area, including uncertainty, of a circle with radius of $(3.83 \pm 0.27)\ \text{cm}$?

32. What is the volume, including uncertainty, of a sphere with radius of (3.83 ± 0.27) cm?

33. What is the speed, including uncertainty, of a boat which travels (31.41 ± 0.59) m in (2.65 ± 0.35) s?

Part 7: Define the following terms

1. magnitude

2. scalar

3. vector (What is the symbol for a vector?)

Part 8: Determine if the following quantities are *scalars* or *vectors*.

1. Money 2. Perimeter 3. Circumference 4. Area 5. Volume 6. Angle 7. Time 8. Length 9. Distance 10. Displacement 11. Speed 12. Velocity 13. Acceleration 14. Force 15. Linear momentum 16. Impulse 17. Work 18. Calories 19. Energy 20. Kinetic energy 21. Potential energy 22. Power 23. Angular speed (HL) 24. Angular acceleration (HL) 25. Moment of inertia (HL) 26. Torque (HL) 27. Angular momentum (HL) 28. Density 29. Temperature	30. Specific heat capacity 31. Latent heat of fusion 32. Luminosity 33. Brightness 34. Emissivity 35. Albedo 36. Pressure 37. Moles 38. Entropy (HL) 39. Charge 40. Current 41. Voltage 42. Resistance 43. Electromotive force 44. Wavelength 45. Period 46. Frequency 47. Gravitational field strength 48. Gravitational potential (HL) 49. Electric field strength 50. Electric Potential (HL) 51. Magnetic field strength 52. Magnetic flux (HL) 53. Activity 54. Half-life

Part 9: Drawing vectors. Use a pencil and ruler!

1. Let the vectors $\vec{A} = (x_1, y_1) = (3, -2)$ and $\vec{B} = (x_2, y_2) = (-1,4)$

   a. Draw a horizontal and vertical axis on the graph below. Label the horizontal axis x and the vertical axis y.

   b. Draw $\vec{A}$ on the graph below.

   c. What is the magnitude of the horizontal component of $\vec{A}$?

   d. What is the magnitude of the vertical component of $\vec{A}$?

   e. What is the magnitude of $\vec{A}$?

   f. Draw $\vec{B}$ on the graph below.

   g. What is $\vec{A} + \vec{B}$? Draw it on the graph below.

   h. What is the magnitude of the horizontal component of $\vec{A} + \vec{B}$?

   i. What is the magnitude of the vertical component of $\vec{A} + \vec{B}$?

   j. What is the magnitude of $\vec{A} + \vec{B}$?

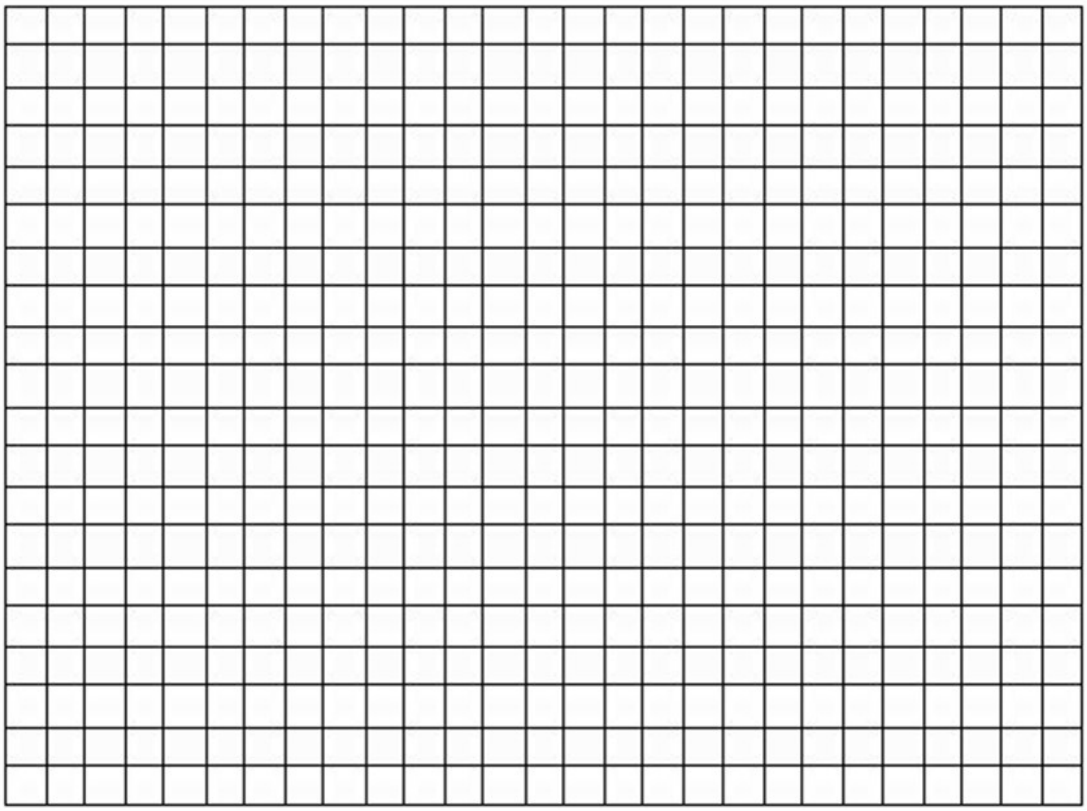
   k. What is $\vec{B} + \vec{A}$? Draw it on the graph below.

   l. What is $\vec{A} - \vec{B}$? Draw it on the graph below.

   m. What is $\vec{B} - \vec{A}$? Draw it on the graph below.

   n. What is $-\vec{A} - \vec{B}$? Draw it on the graph below.

   o. What is $-\vec{B} - \vec{A}$? Draw it on the graph below.



Part 10: The Classic "Boat Crossing a River" Problem

1. Adam is on a boat. It is moving from south to north on a river at a speed of 9.00 m/s. The water in the river is moving from east to west with a speed of 4.00 m/s. The river is 81.0 m wide.

    a. Draw a figure.
    b. How long will it take for the boat to reach the other side?
    c. How many meters will the boat have traveled westward?
    d. What will be the total displacement of the boat?

2. Enoch is on a boat. It is moving from north to south on a river at a speed of 6.00 m/s. The water in the river is moving from west to east with a speed of 3.00 m/s. The river is 99.0 m wide.

   a. Draw a figure.
   b. How long will it take for the boat to reach the other side?
   c. How many meters will the boat have traveled westward?
   d. What will be the total displacement of the boat?