Name: ______________________

Class: ______________________

Due Date: ______________________

D.1 Gravitational Fields

Understandings

- Kepler's three laws of orbital motion.
- Newton's universal law of gravitation as given by $F = \frac{Gm_1m_2}{r^2}$ for bodies treated as point masses.
- Conditions under which extended bodies can be treated as point masses.
- Gravitational field strength g at a point is the force per unit mass experienced by a small point mass at that point as given by $g = \frac{F}{m} = G\frac{M}{r^2}$.
- Gravitational field lines.

Equations

$$F = \frac{Gm_1m_2}{r^2}$$

$$g = \frac{F}{m} = G\frac{M}{r^2}$$

Additional HL Understandings

- The gravitational potential energy E_p of a system is the work done to assemble the system from infinite separation of the components of the system.
- The gravitational potential energy for a two-body system as given by $E_\mathrm{p} = -G\frac{m_1m_2}{r}$ where r is the separation between the center of mass of the two bodies.
- The gravitational potential V_g at a point is the work done per unit mass in bringing a mass from infinity to that point as given by $V_\mathrm{g} = -G\frac{M}{r}$.
- The gravitational field strength g as the gravitational potential gradient as given by $g = -\frac{\Delta V_\mathrm{g}}{\Delta r}$.
- The work done in moving a mass m in a gravitational field as given by $W = m\Delta V_\mathrm{g}$.

- Equipotential surfaces for gravitational fields.
- The relationship between equipotential surfaces and gravitational field lines.
- The escape speed v_{esc} at any point in a gravitational field as given by $v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$.
- The orbital speed v_{orbital} of a body orbiting a large mass as given by $v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$.
- The qualitative effect of a small viscous drag due to the atmosphere on the height and speed of an orbiting body.

Additional HL Equations

$$E_{\text{p}} = -\frac{Gm_1m_2}{r}$$

$$V_g = -\frac{GM}{r}$$

$$g = -\frac{\Delta V_{\text{g}}}{\Delta r}$$

$$W = m\Delta V_{\text{g}}$$

$$v_{\text{esc}} = \sqrt{\frac{2G}{r}}$$

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$$

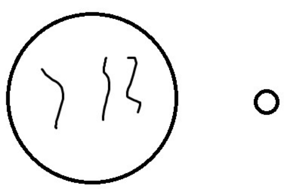
The solutions can be found on the YouTube channel Go Physics Go:

https://www.youtube.com/@gophysicsgo/playlists

1. State the names and describe the laws of Kepler's three laws of orbital motion.

2. Define *Newton's Law of Gravitation* $\vec{F}_{\mathrm{g}} = m_1\vec{g} = \frac{Gm_1m_2}{r^2}$. Units?

3. A satellite in space moves in a circle around the Earth with a constant speed at a radius *r* from the center of the Earth. Label the direction of the velocity, force, and acceleration of the satellite in the diagram below.



4. The force of gravity between a satellite circling the Earth at a distance *r* at a constant speed is F_g. What will happen to the magnitude of the force of gravity between the satellite and the Earth if the satellite moves a distance

   a. 2*r*. b. 3*r*. c. 4*r*. d. *r*/2. e. *r*/3. f. *r*/4.

a.	b.	c.
d.	e.	f.

5. A satellite is moving in a circle with a constant speed around the sun.
   a. Use Newton's second law of motion to obtain an equation for the speed of the satellite in terms of the mass of the sun M_{sun}, the mass of the satellite $M_{\text{satellite}}$, the distance of the satellite to the sun *r*, and the gravitational constant *G*.
   b. Use your solution to obtain an equation for the period *T* of the satellite.

6. The following problem refers to *gravitational field strength*.

   a. Define g*ravitational field strength*. Is it a scalar or a vector?

   b. What is the equation and what are the units for *gravitational field strength*? Define and give the units of each variable.

   c. Where is the *gravitational field strength* zero? Where is the *gravitational field strength* maximum?

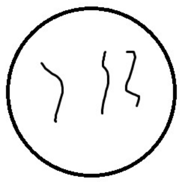
   d. What are the mathematical limits of *gravitational field strength*? Can *gravitational field strength* be positive? Negative? Zero?

7. Draw a *gravitational field strength vs. distance* graph for a planet with a radius *r*.

8. List some rules in drawing gravitational field lines.

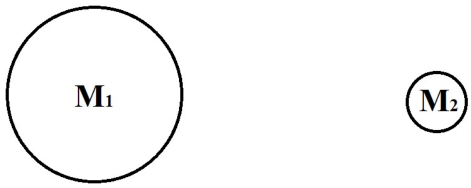
9. **Use a pencil and ruler!** Draw gravitational field lines for each figure.

A fixed uniform spherical mass

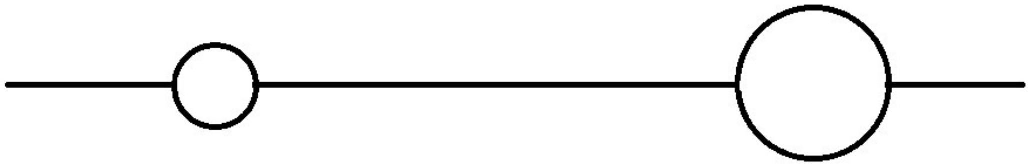


Two uniform spherical masses with equal mass and equal radius

Two fixed uniform spherical masses $M_1 \gg M_2$



10. The center of a planet of mass of $m_1 = 4M$ is a distance of $5d$ from the center of a larger planet of mass $m_2 = 9M$. Assume that the mass of the planets can be concentrated at their centers.



What will be the gravitational field strength (both magnitude and direction) at a point

a. $2d$ to the left of mass m_1?

b. $2d$ to the right of mass m_1?

c. $2d$ to the left of mass m_2?

d. $2d$ to the right of mass m_2?

Additional HL Understandings

11. The following problem refers to *gravitational potential energy* E_P.

    a. Define *gravitational potential energy* E_P. Is it a scalar or a vector?

    b. What is the equation of the *gravitational potential energy* E_P of an object when it is near the surface of a planet? What is the general equation of the *gravitational potential energy* E_P? What is the significance of the negative sign? What are the units of *gravitational potential energy* E_P?

12. The following problem refers to *gravitational potential* V_g.

    a. Define *gravitational potential* V_g. Is it a scalar or a vector?

    b. What is the equation for *gravitational potential* V_g? What are the units of *gravitational potential*?

13. What is the relationship between the gravitational field strength g and gravitational potential V_g?

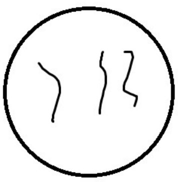
14. What is constant in a *gravitational equipotential surface*?

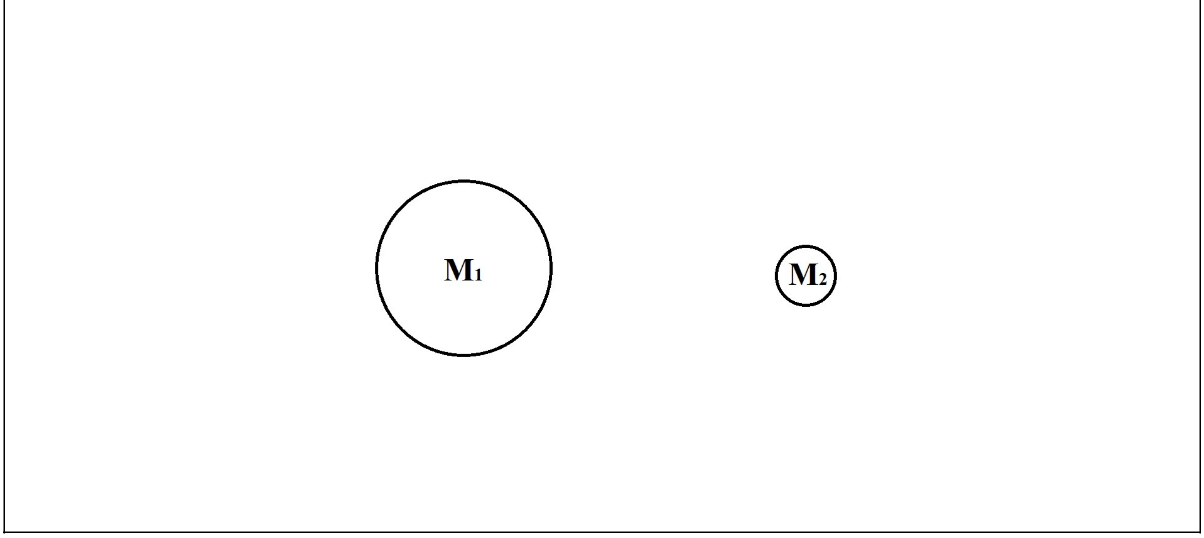
15. How much work is done in moving a mass along the same *equipotential surface*?

16. How much work is done in moving a mass along two different *equipotential surfaces*? State the equation.

17. What is the relationship between an objects *gravitational equipotential surfaces* and *gravitational field lines*?

18. **Use a pencil and ruler!** Draw gravitational field lines and equipotential surfaces for each figure.

A fixed uniform spherical mass	Two uniform spherical masses with equal mass and equal radius
	



Two fixed uniform spherical masses $M_1 \gg M_2$

19. The following problem refers to *escape speed*.

a. Define *escape speed*. This is also called *escape velocity*.

b. What are some assumptions made when defining *escape speed*?

c. For which objects does *escape speed* apply to?

d. For which objects does *escape speed* not apply to?

e. Use the law of conservation of energy $E_{\text{initial}} = E_{\text{final}}$ to solve for the equation of the *escape velocity* of an object leaving the gravitational pull of a planet. What is the minimum speed needed for an object to escape the gravitational pull of the Earth?

20. You might want to derive this equation with your teacher: A planet is orbiting in a circular motion with a constant speed around a star (like the Sun).

a. Use Newton's Law of Gravitation $F_g = m_{\text{planet}} a_{\text{planet}} = \frac{G m_{\text{star}} m_{\text{planet}}}{r^2}$ and the equation for centripetal acceleration $a_{\text{planet}} = \frac{{v_{\text{planet}}}^2}{r}$ to solve for the speed squared ${v_{\text{planet}}}^2$ of a planet moving around a star. Place a box around your answer. In both equations r is the distance from the planet to the star.

b. Take your solution from part a and multiply both sides by $\frac{m_{\text{planet}}}{2}$. This new equation is equal to the kinetic energy of the planet. Place a box around your answer.

c. Substitute your answer from part b to the total energy of the orbiting planet $E_{\text{total}} = KE + GPE = \frac{1}{2} m_{\text{planet}} {v_{\text{planet}}}^2 - \frac{G m_{\text{planet}} m_{\text{star}}}{r}$. Place a box around your answer. Your answer should have only one fraction.

d. Is the total energy positive, negative, or zero? Why?

e. Draw an *energy vs. distance* graph. On this graph draw the kinetic energy vs. distance, potential energy vs. distance, and total energy vs. distance.

21. An object is orbiting a planet. It encounters a small viscous drag due to the atmosphere. Describe the subsequent motion of the object.

22. A rocket with a mass of 64,000 kg is orbiting Jupiter's moon Callisto. Callisto has a radius of 2.40×10^6 m and a mass of 7.35×10^{22} kg.

   a. What would the velocity of the rocket have to be in order for it to orbit Callisto at an altitude of 4,600 km?

   b. What would be the gravitational force between this rocket and Callisto while orbiting at this altitude?

   c. What would be the kinetic energy of this rocket while orbiting Callisto at this altitude?

   d. What would be the gravitational potential energy of this rocket while orbiting Callisto at this altitude?

e. What would be the total energy of this rocket while orbiting Callisto at an altitude of 4,600 km?

f. What would be the total energy of this rocket while sitting at rest on the surface of Callisto?

g. How much kinetic energy would you have to give to this rocket while sitting on the surface of Callisto in order to put the rocket into orbit around Callisto at an altitude of 4,600 km?

h. With what velocity would this rocket have to be launched from the surface of Callisto in order to go into orbit around Callisto at an altitude of 4,600 km?

i. With what velocity would this rocket have to be launched from the surface of Callisto in order for the rocket to escape the gravitational effects of Callisto?

23. A rocket, which has a mass of 38,000 kg, is initially sitting at rest on the surface of the planet Venus. Venus has a radius of 6.05×10^{6} m and a mass of 4.87×10^{24} kg.

a. What is the total energy content of this rocket while sitting at rest on the surface of Venus?

b. What velocity would be required for this rocket to orbit Venus at an altitude of 550 km?

c. What total energy is required if this rocket is to orbit Venus at an altitude of 550 km?

d. With what velocity should this rocket be launched from the surface of Venus in order to go into orbit around Venus at an altitude of 550 km?

e. With what minimum velocity should this rocket be launched from the surface of Venus in order to escape the gravitational effects of Venus?

f. What will be the velocity of this rocket when it is very far from Venus if the rocket is launched from the surface of Venus with a velocity of $1.4 \times 10^4 \ \frac{\mathrm{m}}{\mathrm{s}}$?