Name: ______________________

Class: ______________________

Due Date: ______________________

A.4 Rigid Body Mechanics

Additional HL Understandings

- The torque τ of a force about an axis as given by $\tau = Fr\ \sin\theta$.
- Bodies in rotational equilibrium have a resultant torque of zero.
- An unbalanced torque applied to an extended, rigid body will cause angular acceleration.
- The rotation of a body can be described in terms of angular displacement, angular velocity, and angular acceleration.
- Equations of motion for uniform angular acceleration can be used to predict the body's angular position θ, angular displacement $\Delta\theta$, angular speed ω, and angular acceleration α as given by
  - $\Delta\theta = \frac{\omega_f + \omega_i}{2} t$
  - $\omega_f = \omega_i + \alpha t$
  - $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
  - ${\omega_f}^2 = {\omega_i}^2 + 2\alpha\Delta\theta$
- The moment of inertia I depends on the distribution of mass of an extended body about an axis of rotation.
- The moment of inertia for a system of point masses as given by $I = \sum mr^2$.
- Newton's second law for rotation as given by $\tau = I\alpha$ where τ is the average torque.
- An extended body rotating with an angular speed has an angular momentum L as given by $L = I\omega$.
- Angular momentum remains constant unless the body is acted upon by a resultant torque.
- The action of a resultant torque constitutes an angular impulse ΔL as given by $\Delta L = \tau\Delta t = \Delta(I\omega)$
- The kinetic energy of rotational motion as given by $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$.

Additional HL Equations

$\tau = Fr\ \sin\theta$

$\Delta\theta = \frac{\omega_{\mathrm{f}}+\omega_{\mathrm{i}}}{2}t$

$\omega_{\mathrm{f}} = \omega_{\mathrm{i}} + \alpha t$

$\Delta\theta = \omega_{\mathrm{i}}t + \frac{1}{2}\alpha t^2$

${\omega_{\mathrm{f}}}^2 = {\omega_{\mathrm{i}}}^2 + 2\alpha\Delta\theta$

$I = \sum mr^2$

$\tau = I\alpha$

$L = I\omega$

$\Delta L = \tau\Delta t$

$\Delta L = \Delta(I\omega)$

$E_{\mathrm{k}} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

The solutions can be found on the YouTube channel Go Physics Go:

https://www.youtube.com/@gophysicsgo/playlists

1. Define, state the equation, and give the units of *angular position* θ.

2. Define, state the equation, and give the units of *angular speed* ω.

3. Define, state the equation, and give the units of *angular acceleration* α.

4. Convert the *suvat* equations from linear motion to circular motion.

5. Define, state the equation, define each variable, and give the units for the *moment of inertia I*. What is the *moment of inertia I* equivalent to in translational motion?

6. Define, state the equation, define each variable, and give the units for *torque*.

7. State the equations for *Newton's second law of motion for linear motion* and *Newton's second law of motion for rotational motion*.

8. Define and state the conditions for *translational equilibrium* and *rotational equilibrium*.

9. State the equations for *power for linear motion* and *power for rotational motion*.

10. State the equations for *linear momentum* and *angular momentum*. Also state the equations for *linear impulse* and *angular impulse*.

11. State the equations for *translational kinetic energy* and *rotational kinetic energy*.

12. State the *law of conservation of linear momentum* and the *law of conservation of angular momentum*. Also state their equations.

13. A solid chocolate sphere with a mass of 9 kg and a diameter of 80 cm is placed on top of a rough incline ($\mu = 0.7$) with a length of 6 m at an angle of 50 degrees. The solid chocolate sphere begins from rest and rolls down the incline.

   a. Draw a figure.

   b. What is the initial height of the solid chocolate sphere?

   c. How many revolutions will it take for the solid chocolate sphere to reach the bottom of the incline?

   d. What will be the final linear speed of the solid chocolate sphere at the bottom of the incline?

e. What will be the final angular speed of the solid chocolate sphere at the bottom of the incline?

f. What will be the angular acceleration of the solid chocolate sphere?

g. What will be the linear acceleration of the solid chocolate sphere?

h. How long will it take for the solid chocolate sphere to reach the bottom of the incline?

14. A solid chocolate sphere with a mass of 8 kg and a diameter of 70 cm is rolling to the right on a frictionless horizontal surface with a linear speed of 6 m/s. The surface then becomes rough with a coefficient of dynamic friction of 0.15.

a. Draw a figure.

b. What is the angular speed of the solid chocolate sphere as it rolls along the frictionless horizontal surface?

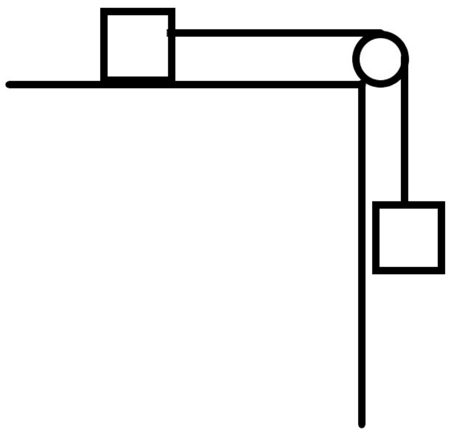
c. What is the angular acceleration of the solid chocolate sphere as it travels along the rough surface?

d. What is the linear acceleration of the solid chocolate sphere as it travels along the rough surface?

e. How many revolutions does the solid chocolate sphere complete as it travels along the rough surface?

f. How long does it take for the solid chocolate sphere to stop along the rough horizontal surface?

15. A block of mass $m_1 = 7$ kg sits at rest on a horizontal surface with $\mu = 0.2$. Mass m_1 is attached to a massless string which is wrapped around a pulley. Another massless string is wrapped around the same pulley and is holding another block of mass $m_2 = 47$ kg in the air. The pulley is a cylinder which has a mass of $m_C = 12$ kg and diameter of 10 cm.



a. Draw a free body diagram.

b. What is the common linear acceleration of the system?
c. What is the force of tension on the two massless strings?

16. A thin disk with a mass of 250 g and diameter of 40 cm is spinning with an angular speed of 3 rad/sec. A point mass with a mass of 350 g strikes and sticks to the top of the thin disk 4 cm from the edge. What is the final angular speed of the system?

17. A ladder has a mass of 20 kg and is 6 m long is leaning against a frictionless wall. The ladder is at rest and makes an angle of 30 degrees from the wall. Draw a figure and write down the equations for static equilibrium.

18. Write down the common terms and equations for rigid body mechanics.